BALANCED INTUITIONISTIC TRIPOLAR FUZZY GRAPHS

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ABSTRACT:

In this paper, we discuss about balanced intuitionistic tripolar fuzzy graphs and study some of their properties.

KEYWORDS:

Tripolar Fuzzy Graphs, Tripolar intuitionistic Fuzzy Graphs, Balanced intuitionistic on Tripolar Fuzzy Graphs, Complete Balanced Tripolar Fuzzy Graph.

1. INTRODUCTION

Graph theory is developed when Euler[1] gave the solution to the famous Konigsberg bridge problem in 1736. Graph theory is very useful as a branch of combinatorics in the field of geometry, algebra, number theory, topology, operations research, optimization and computer science. Rosenfield[2] developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Later on Bhattacharya[3] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson and Peng[4]. The complement of a fuzzy graph was defined by Mordeson and Nair[5] and further studied by Sunitha and Vijayakumar[6]. Akram[7] has introduced Bipolar Fuzzy Graphs, and investigated their ptoperties. Balanced graph first arose in the study of random graphs and balanced IFG defined here is based on density functions. A graph with maximum density is complete and graph with minimum density is a null graph. There are several papers written on balanced extension of graph[8] which has tremendous applications in artificial intelligence, signal processing robotics, computer networks and decision making Al-hawary[9] introduced the concept of balaqnced fuzzy graphs and studied some operations of fuzzy graphs. Shannon and Atanassov[10] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs, and investigated some of their properties. Parvathi etal[11] defined operations on intuitionistic fuzzy graphs. Karunambigai et al[12] introduced Balanced intuitionistic fuzzy graphs and studied some of their properties. In 2015, D.Ezhilmaran and K.Sankar[16] have introduced Bipolar intuitionistic fuzzy graphs and 2016, Balanced bipolar intuitionistic fuzzy graphs. Jon Arockiaraj and Obed Issac[17] have introduced Tripolar fuzzy Graphs in 2018.

In this Paper, We discussed balanced tripolar inutionistic fuzzy graphs and study some of their properties.

2. PRELIMINARIES

Definition 2.1:

Let X be a non-empty set. A tripolar fuzzy set B in X is an object having the form B = { (x, $\mu^P(x)$, $\mu^N(x)$, $\mu^T(x)$ ⁄ x∊X }
Where μ^P: X→[0,1] , μ^N : X→[-1,0] and μ^T: X→[-1,1] are mappings.

Definition 2.2:

Let X be a non-empty set. An intuitionistic fuzzy set B = { (x, μ(x), γ(x) ⁄ x∊X }
Where μ: X→[0,1] , γ: X→[0,1] are mapping such that $0\leq \mu(x)+\gamma(x)\leq 1$.

3. TRIPOLAR INTUITIONISTIC FUZZY GRAPH

Let X be a non-empty set. A Tripolar intuitionistic fuzzy set
B = { (x, $\mu^P(x)$, $\mu^N(x)$, $\mu^T(x)$, $\gamma^P(x)$ $\gamma^N(x)$, $\gamma^T(x)$ ⁄ x∊X and $\mu^T(x)= \mu^P(x)+\mu^N(x)$, $\gamma^T(x)= \gamma^P(x)+\gamma^N(x)$},Where μ^P: X→[0,1] , μ^N: X→[-1,0] and μ^T: X→[-1,1] and γ^P: X→[0,1] γ^N: X→[-1,0] and γ^T: X→[-1,1] are the mappings such that $0\leq \mu^P(x)+\gamma^P(x)\leq 1$, $-1\leq \mu^N(x)+\gamma^N(x)\leq 0$ and $-1\leq \mu^T(x)+\gamma^T(x)\leq 1$.

We use the positive membership degree $\mu^P(x)$ to denote the satisfaction degree of an element X to the property corresponding to a tripolar intuitionistic fuzzy set B, and the negative membership degree $\mu^N(x)$ to denote the satisfaction degree of an element X to some implicit counter property corresponding to a tripolar fuzzy set B and positive or negative degree $\mu^T(x)$ to denote the satisfaction degree of an element X to some properties corresponding to a tripolar fuzzy set B.

If $\mu^P(x)\neq 0$, $\mu^N(x)=0$ and $\mu^T(x)=0$ then $\gamma^P(x)=0$, $\gamma^N(x)=0$, $\gamma^T(x)=0$ it is the situation that X regarded as, having only the positive membership property of a tripolar intuitionistic fuzzy set.

If $\mu^P(x)=0$, $\mu^N(x)\neq 0$ and $\mu^T(x)=0$ then $\gamma^P(x)=0$, $\gamma^N(x)=0$, $\gamma^T(x)=0$ it is the situation that X regarded as, having only the negative membership property of a tripolar intuitionistic fuzzy set.

If $\mu^P(x)=0$, $\mu^N(x)=0$ and $\mu^T(x)\neq 0$ then $\gamma^P(x)\neq 0$, $\gamma^N(x)=0$, $\gamma^T(x)=0$ it is the situation that X regarded as, having only the positive or negative membership property of a tripolar intuitionistic fuzzy set.

If $\mu^P(x)=0$, $\mu^N(x)=0$ and $\mu^T(x)=0$ then $\gamma^P(x)=0$, $\gamma^N(x)=0$, $\gamma^T(x)\neq 0$ it is the situation that X regarded as, having only the positive or negative non-membership property of a tripolar intuitionistic fuzzy set.

If $\mu^P(x)=0$, $\mu^N(x)=0$ and $\mu^T(x)=0$ then $\gamma^P(x)=0$, $\gamma^N(x)\neq 0$, $\gamma^T(x)=0$ it is the situation that X regarded as, having only the negative non-membership property of a tripolar intuitionistic fuzzy set.

If $\mu^P(x)=0$, $\mu^N(x)=0$ and $\mu^T(x)=0$ then $\gamma^P(x)\neq 0$, $\gamma^N(x)=0$, $\gamma^T(x)=0$ it is the situation that X regarded as, having only the positive non-membership property of a tripolar intuitionistic fuzzy set. It is possible for an element X to be such that $\mu^P(x)\neq 0$, $\mu^N(x)\neq 0$ and $\mu^T(x)\neq 0$ then $\gamma^P(x)\neq 0$, $\gamma^N(x)\neq 0$, $\gamma^T(x)\neq 0$ when the membership and non-membership function of the property overlaps with its counter properties over some portion of X.

For the sake of simplicity, we shall use the symbol B = ($\mu^P(x)$, $\mu^N(x)$, $\mu^T(x)$, $\gamma^P(x)$ $\gamma^N(x)$, $\gamma^T(x)$) for the tripolar intuitionistic fuzzy set,

B = { (x, $\mu^P(x)$, $\mu^N(x)$,$\mu^T(x)$,$\gamma^P(x)$,$\gamma^N(x)$, $\gamma^T(x)$ ⁄ x∊X} where $\mu^T(x) = \mu^P(x) + \mu^N(x)$, and $\gamma^T(x) = \gamma^P(x) + \gamma^N(x)$.

Definition 3.1:

Let X be a non-empty set. Then we call a mapping $(\mu^P_A, \mu^N_A, \mu^T_A, \gamma^P_A, \gamma^N_A, \gamma^T_A): X\times X \to [0,1] \times [-1,0] \times [-1,1] \times [0,1] \times [-1,0] \times [-1,1]$ a tripolar intuitionistic fuzzy relation on X such that $\mu^P_A(x,y) \in [0,1]$, $\mu^N_A(x,y) \in [-1,0]$, $\mu^T_A(x,y) \in [-1,1]$, $\gamma^P_A(x,y) \in [0,1]$, $\gamma^N_A(x,y) \in [-1,0]$, and $\gamma^T_A(x,y) \in [-1,1]$

Definition 3.2:

Let $A = (\mu^P_A(x), \mu^N_A(x), \mu^T_A(x), \gamma^P_A(x), \gamma^N_A(x), \gamma^T_A(x))$

And $B = (\mu^P_B(x), \mu^N_B(x), \mu^T_B(x), \gamma^P_B(x), \gamma^N_B(x), \gamma^T_B(x))$ be tripolar intuitionistic fuzzy set on x.

If $A = (\mu^P_A(x), \mu^N_A(x), \mu^T_A(x), \gamma^P_A(x), \gamma^N_A(x), \gamma^T_A(x))$ is a tripolar intuitionistic fuzzy relation on $B = (\mu^P_B(x), \mu^N_B(x), \mu^T_B(x), \gamma^P_B(x), \gamma^N_B(x), \gamma^T_B(x))$.

If $\mu^P_A(x,y) \leq \mu^P_B(x) \wedge \mu^P_B(y)$,

$\mu^N_A(x,y) \geq \mu^N_B(x) \vee \mu^N_B(y)$,

$\mu^T_A(x,y) = \text{minmax}(\mu^T_B(x), \mu^T_B(y))$ and

$\gamma^P_A(x,y) \geq \gamma^P_B(x) \vee \gamma^P_B(y)$

$\gamma^N_A(x,y) \leq \gamma^N_B(x) \wedge \gamma^N_B(y)$and

$\gamma^T_A(x,y) = \text{maxmin}(\gamma^T_B(x)\ \gamma^T_B(y))$ for all $x,y \in X$.

A tripolar intuitionistic fuzzy relation A on X is called symmetric if

$\mu^P_A(x,y) = \mu^P_A(y,x)$

$\mu^N_A(x,y) = \mu^N_A(y,x)$

$\mu^T_A(x,y) = \mu^T_A(y,x)$

$\gamma^P_A(x,y) = \gamma^P_A(y,x)$

$\gamma^N_A(x,y) = \gamma^N_A(y,x)$

$\gamma^T_A(x,y) = \gamma^T_A(y,x)$ for all $x,y \in X$.

Definition 3.3:

For any two tripolar intuitionistic fuzzy sets,

$A = (\mu^P_A(x), \mu^N_A(x), \mu^T_A(x), \gamma^P_A(x), \gamma^N_A(x), \gamma^T_A(x))$ and

$B = (\mu^P_B(x), \mu^N_B(x), \mu^T_B(x), \gamma^P_B(x), \gamma^N_B(x), \gamma^T_B(x))$

$(A\cap B)(x) = (\mu^P_A(x) \wedge \mu^P_B(x), \mu^N_A(x) \vee \mu^N_B(x), \text{minmax}(\mu^T_A(x), \mu^T_B(x))$

$(A\cup B)(x) = (\mu^P_A(x) \vee \mu^P_B(x), \mu^N_A(x) \wedge \mu^N_B(x), \text{maxmin}(\mu^T_A(x), \mu^T_B(x))$

$(A\cap B)(x) = (\gamma^P_A(x) \vee \gamma^P_B(x), \gamma^N_A(x) \wedge \gamma^N_B(x), \text{maxmin}(\gamma^T_B(x)\ \gamma^T_B(x))$

$(A\cup B)(x) = (\gamma^P_A(x) \wedge \gamma^P_B(x), \gamma^N_A(x) \vee \gamma^N_B(x), \text{minmax}(\gamma^T_B(x)\ \gamma^T_B(x))$

Definition 3.4:

A tripolar intuitionistic fuzzy graph of a graph G^* (V,E) is pair G(A,B) where $A = (\mu^P_A, \mu^N_A, \mu^T_A, \gamma^P_A, \gamma^N_A, \gamma^T_A)$ is a tripolar intuitionistic fuzzy set in V and $(\mu^P_B, \mu^N_B, \mu^T_B, \gamma^P_B, \gamma^N_B, \gamma^T_B)$ is a tripolar intuitionistic fuzzy set in V×V such that,

$\mu^P_B(x,y) \leq \mu^P_A(x) \wedge \mu^P_A(y)$,

$\mu^N_B(x,y) \geq \mu^N_A(x) \vee \mu^N_A(y)$,

$\mu^T_B(x,y) = \text{minmax}(\mu^T_A(x), \mu^T_A(y))$ and

$\gamma^{P}{}_{B}(x,y) \geq \gamma^{P}{}_{A}(x) \vee \gamma^{P}{}_{A}(y)$

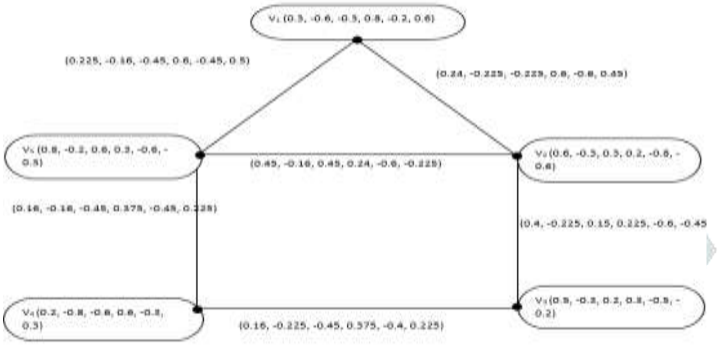
$\gamma^{N}{}_{B}(x,y) \leq \gamma^{N}{}_{A}(x) \wedge \gamma^{N}{}_{A}(y)$ and

$\gamma^{T}{}_{B}(x,y) = \text{maxmin}\,(\gamma^{T}{}_{A}(x)\ \gamma^{T}{}_{A}(y))$ for all $x,y \in V \times V$

$\mu^{P}{}_{B}(x,y) = \mu^{N}{}_{B}(x,y) = \mu^{T}{}_{B}(x,y) = 0$ for all $x,y \in V \times V-E$

$\gamma^{P}{}_{B}(x,y) = \gamma^{N}{}_{B}(x,y) = \gamma^{T}{}_{B}(x,y) = 0$ for all $x,y \in V \times V-E$

Example 3.5:



A tripolar intuitionistic fuzzy graph(TIFG) is of the form G = (V,E) said to be mini-max(TIFG)

(i) $V = \{ v_0, v_1, \ldots v_n \}$ such that $\mu^{P}{}_{1}: V \rightarrow [0,1]$, $\mu^{N}{}_{1} : V \rightarrow [-1,0]$ and $\mu^{T}{}_{1}: V \rightarrow [-1,1]$ and $\gamma^{P}{}_{1}: V \rightarrow [0, 1]$ $\gamma^{N}{}_{1}: V \rightarrow [-1, 0]$ and $\gamma^{T}{}_{1}: V \rightarrow [-1, 1]$ denotes the degree of positive membership negative membership and degree of positive non-membership, negative non-membership and positive (or) negative membership and positive (or) negative non-membership of the element respectively $0 \leq \mu^{P}{}_{1}+\gamma^{P}{}_{1} \leq 1$, $-1 \leq \mu^{N}{}_{1}+\gamma^{N}{}_{1} \leq 0$ and $-1 \leq \mu^{T}{}_{1}+\gamma^{T}{}_{1} \leq 1$ for every $v_i \in V(i=1, 2, 3, \ldots n)$

(ii) $E \subseteq V \times V$ where $\mu^{P}{}_{2}: V \times V \rightarrow [0, 1]$, $\mu^{N}{}_{2} : V \times V \rightarrow [-1, 0]$ and $\mu^{T}{}_{2}: V \times V \rightarrow [-1, 1]$ and $\gamma^{P}{}_{2}: V \times V \rightarrow [0, 1]$ $\gamma^{N}{}_{2}: V \times V \rightarrow [-1, 0]$ and $\gamma^{T}{}_{2}: V \times V \rightarrow [-1, 1]$ are such that,

$\mu^{P}{}_{2}(v_i, v_j) \leq (\mu^{P}{}_{1}(v_i) \wedge \mu^{P}{}_{1}(v_j))$

$\mu^{N}{}_{2}(v_i, v_j) \geq (\mu^{N}{}_{1}(v_i) \vee \mu^{N}{}_{1}(v_j))$

$\mu^{T}{}_{2}(v_i, v_j) = \text{minmax}(\mu^{T}{}_{1}(v_i), \mu^{T}{}_{1}(v_j))$

$\gamma^{P}{}_{2}(v_i, v_j) \leq (\gamma^{P}{}_{1}(v_i) \vee \gamma^{P}{}_{1}(v_j))$

$\gamma^{N}{}_{2}(v_i, v_j) \geq (\gamma^{N}{}_{1}(v_i) \wedge \gamma^{N}{}_{1}(v_j))$

$\gamma^{T}{}_{2}(v_i, v_j) = \text{maxmin}(\gamma^{T}{}_{1}(v_i), \gamma^{T}{}_{1}(v_j))$

denotes the degree of positive membership negative membership and degree of positive non-membership, negative non-membership and positive (or) negative membership and positive (or) negative non-membership of the edge $(v_i, v_j) \in V$ respectively $0 \leq \mu^{P}{}_{2}(v_i, v_j)+\gamma^{P}{}_{2}(v_i, v_j) \leq 1$, $-1 \leq \mu^{N}{}_{2}(v_i, v_j)+\gamma^{N}{}_{2}(v_i, v_j) \leq 0$, $-1 \leq \mu^{T}{}_{2}(v_i, v_j)+\gamma^{T}{}_{2}(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in V$

A Tripolar Intuitionistic Fuzzy Graph(ATIFG) H = (V', E') is said to be TIF subgraph of G(V, E) if, (i) $V' \subseteq V$ where $\mu^{P}{}_{1}(v_i') = \mu^{P}{}_{1}(v_i)$, $\mu^{N}{}_{1}(v_i') = \mu^{N}{}_{1}(v_i)$ and $\mu^{T}{}_{1}(v_i') = \mu^{T}{}_{1}(v_i)$,

$\gamma^{P}{}_{1}(v_i') = \gamma^{P}{}_{1}(v_i)$, $\gamma^{N}{}_{1}(v_i') = \gamma^{N}{}_{1}(v_i)$, $\gamma^{N}{}_{1}(v_i') = \gamma^{N}{}_{1}(v_i)$ for all $v_i' \in V'$, $v_i' = v_i \in V$, i=1,2,3,…..n.

(ii) $\mu^{P}{}_{2}(v_i', v_j') = \mu^{P}{}_{2}(v_i, v_j)$, $\mu^{N}{}_{2}(v_i', v_j') = \mu^{N}{}_{2}(v_i, v_j)$, and $\mu^{T}{}_{2}(v_i', v_j') = \mu^{T}{}_{2}(v_i, v_j)$, then $\gamma^{P}{}_{2}(v_i', v_j') = \gamma^{P}{}_{2}(v_i, v_j)$, $\gamma^{N}{}_{2}(v_i', v_j') = \gamma^{N}{}_{2}(v_i, v_j)$, and $\gamma^{T}{}_{2}(v_i', v_j') = \gamma^{T}{}_{2}(v_i, v_j)$

for all $(v_i', v_j') \epsilon E'$, $(v_i', v_j') = (v_i, v_j) \epsilon E$ i,j=123…n.

A Tripolar Intuitionistic Fuzzy Graph(ATIFG) G = (V, E) is said to be complete TIFG if,

$\mu^P{}_2(v_i, v_j) = (\mu^P{}_1(v_i) \wedge \mu^P{}_1(v_j))$

$\mu^N{}_2(v_i, v_j) = (\mu^N{}_1(v_i) \vee \mu^N{}_1(v_j))$

$\mu^T{}_2(v_i, v_j) = \text{minmax}(\mu^T{}_1(v_i), \mu^T{}_1(v_j))$

$\gamma^P{}_2(v_i, v_j) = (\gamma^P{}_1(v_i) \vee \gamma^P{}_1(v_j))$

$\gamma^N{}_2(v_i, v_j) = (\gamma^N{}_1(v_i) \wedge \gamma^N{}_1(v_j))$

$\gamma^T{}_2(v_i, v_j) = \text{maxmin}(\gamma^T{}_1(v_i), \gamma^T{}_1(v_j))$ for every $v_i, v_j \epsilon V$

A Tripolar Intuitionistic Fuzzy Graph(ATIFG) G = (V, E) is said to be strong TIFG if,

$\mu^P{}_2(v_i, v_j) = (\mu^P{}_1(v_i) \wedge \mu^P{}_1(v_j))$

$\mu^N{}_2(v_i, v_j) = (\mu^N{}_1(v_i) \vee \mu^N{}_1(v_j))$

$\mu^T{}_2(v_i, v_j) = \text{minmax}(\mu^T{}_1(v_i), \mu^T{}_1(v_j))$

$\gamma^P{}_2(v_i, v_j) = (\gamma^P{}_1(v_i) \vee \gamma^P{}_1(v_j))$

$\gamma^N{}_2(v_i, v_j) = (\gamma^N{}_1(v_i) \wedge \gamma^N{}_1(v_j))$

$\gamma^T{}_2(v_i, v_j) = \text{maxmin}(\gamma^T{}_1(v_i), \gamma^T{}_1(v_j))$ for every $v_i, v_j \epsilon E$

The complement of a TLFG, G = (V, E) is a TIF $\overline{G} = (\overline{V}, \overline{E})$ where

(i) $\overline{V} = V$

(ii) $\bar{\mu}^P{}_1(v_i) = \mu^P{}_1(v_i)$

$\bar{\mu}^N{}_1(v_i) = \mu^N{}_1(v_i)$

$\bar{\mu}^T{}_1(v_i) = \mu^T{}_1(v_i)$

$\bar{\gamma}^P{}_1(v_i) = \gamma^P{}_1(v_i)$

$\bar{\gamma}^N{}_1(v_i) = \gamma^N{}_1(v_i)$

$\bar{\gamma}^T{}_1(v_i) = \gamma^T{}_1(v_i)$

(iii) $\bar{\mu}^P{}_2(v_i, v_j) = (\mu^P{}_1(v_i) \wedge \mu^P{}_1(v_j)) - \mu^P{}_2(v_i, v_j)$

$\bar{\mu}^N{}_2(v_i, v_j) = (\mu^N{}_1(v_i) \vee \mu^N{}_1(v_j)) - \mu^N{}_2(v_i, v_j)$

$\bar{\mu}^T{}_2(v_i, v_j) = \text{minmax}(\mu^T{}_1(v_i), \mu^T{}_1(v_j)) - \mu^T{}_2(v_i, v_j)$

$\bar{\gamma}^P{}_2(v_i, v_j) = (\gamma^P{}_1(v_i) \vee \gamma^P{}_1(v_j)) - \gamma^P{}_2(v_i, v_j)$

$\bar{\gamma}^N{}_2(v_i, v_j) = (\gamma^N{}_1(v_i) \wedge \gamma^N{}_1(v_j)) - \gamma^N{}_2(v_i, v_j)$

$\bar{\gamma}^T{}_2(v_i, v_j) = \text{maxmin}(\gamma^T{}_1(v_i), \gamma^T{}_1(v_j)) - \gamma^T{}_2(v_i, v_j)$ for every $v_i, v_j \epsilon V$

A TIFG, G=(V,E) is said to be regular TIFG if all the vertices have the same closed neighborhood degree.

The density of a complete fuzzy graph G(σ,μ) is $D(G) = 2\left(\dfrac{\sum_{u,v\in E} \mu(u,v)}{\sum_{u,v\in V} \sigma(u)\wedge\sigma(v)}\right)$

Consider the two TIFG's $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. An isomorphism between two TIFG's G_1 and G_2, denoted by $G_1 \cong G_2$ is a bijective map $h:V_1 \rightarrow V_2$ which satisfies the following
$\mu^P{}_1(v_i) = \mu^P{}_1(h(v_i))$, $\mu^N{}_1(v_i) = \mu^N{}_1(h(v_i))$, $\mu^T{}_1(v_i) = \mu^T{}_1(h(v_i))$, and $\gamma^P{}_1(v_i) = \gamma^P{}_1(h(v_i))$,
$\gamma^N{}_1(v_i) = \gamma^N{}_1(h(v_i))$, $\gamma^T{}_1(v_i) = \gamma^T{}_1(h(v_i))$, and
$\mu^P{}_2(v_i, v_j) = \mu^P{}_2(h(v_i), h(v_j))$
$\mu^N{}_2(v_i, v_j) = \mu^N{}_2(h(v_i), h(v_j))$
$\mu^T{}_2(v_i, v_j) = \mu^T{}_2(h(v_i), h(v_j))$
$\gamma^P{}_2(v_i, v_j) = \gamma^P{}_2(h(v_i), h(v_j))$
$\gamma^N{}_2(v_i, v_j) = \gamma^N{}_2(h(v_i), h(v_j))$
$\gamma^T{}_2(v_i, v_j) = \gamma^T{}_2(h(v_i), h(v_j))$ for every $(v_i, v_j) \in V$

4. BALANCED TRIPOLAR INTUITIONISTIC FUZZY GRAPH

Definition 4.1:
The density of a balanced intuitionistic fuzzy graph (BIFG) $G = (V, E)$ is
$D(G) = (D\mu^P(G), D\mu^N(G), D\mu^T(G), D\gamma^P(G), D\gamma^N(G), D\gamma^T(G))$ where $D\mu^P(G)$ is defined by,

$$D\mu^P(G) = 2\left(\frac{\sum_{u,v\in V} (\mu^P{}_2(u,v))}{\sum_{u,v\in E} (\mu^P{}_1(u)\wedge\mu^P{}_1(v))}\right) \text{ for every } u,v \in V$$

$D\mu^N(G)$ is defined by,

$$D\mu^N(G) = 2\left(\frac{\sum_{u,v\in V} (\mu^N{}_2(u,v))}{\sum_{u,v\in E} (\mu^N{}_1(u)\vee\mu^N{}_1(v))}\right) \text{ for every } u,v \in V$$

$D\mu^T(G)$ is defined by,

$$D\mu^T(G) = 2\left(\frac{\sum_{u,v\in V} (\mu^T{}_2(u,v))}{\sum_{u,v\in E} \text{minmax}(\mu^T{}_1(u), \mu^N{}_1(v))}\right) \text{ for every } u,v \in V$$

$D\gamma^P(G)$ is defined by,

$$D\gamma^P(G) = 2\left(\frac{\sum_{u,v\,\epsilon V}(\gamma^P{}_2(u,v))}{\sum_{u,v\,\epsilon E}(\gamma^P{}_1(u)\vee\gamma^P{}_1(v))}\right) \text{ for every } u,v\ \epsilon V$$

$D\gamma^N(G)$ is defined by,

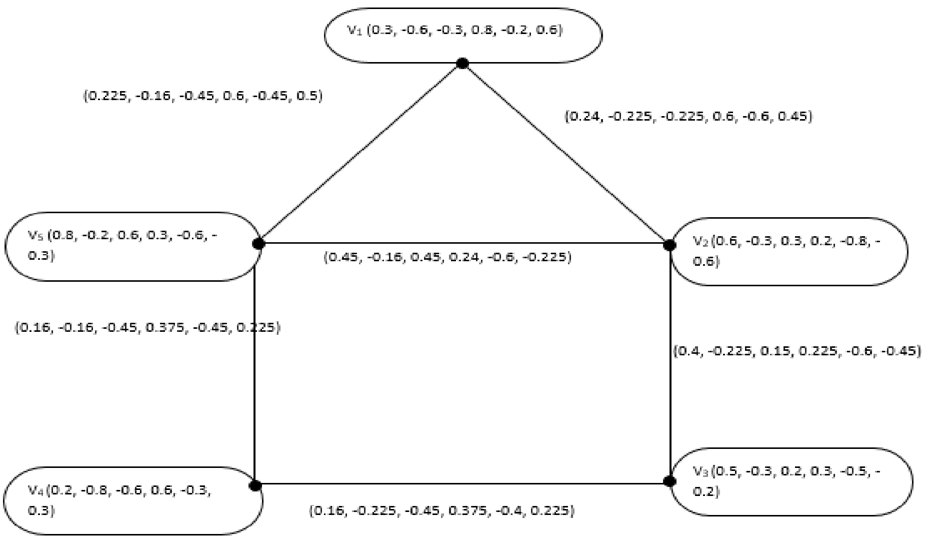
$$D\gamma^N(G) = 2\left(\frac{\sum_{u,v\,\epsilon V}(\gamma^N{}_2(u,v))}{\sum_{u,v\,\epsilon E}(\gamma^N{}_1(u)\wedge\gamma^N{}_1(v))}\right) \text{ for every } u,v\ \epsilon V$$

$D\gamma^T(G)$ is defined by,

$$D\gamma^T(G) = 2\left(\frac{\sum_{u,v\,\epsilon V}(\gamma^T{}_2(u,v))}{\sum_{u,v\,\epsilon E}\operatorname{maxmin}(\gamma^T{}_1(u),\gamma^T{}_1(v))}\right) \text{ for every } u,v\ \epsilon V$$

Definition 4.2:
A TIFG G = (V,E) is balanced if $D(H) \le D(G)$ that is $D\mu^P(H) \le D\mu^P(G)$, $D\mu^N(H) \le D\mu^N(G)$, $D\mu^T(H) \le D\mu^T(G)$, $D\gamma^P(H) \le D\gamma^P(G)$, $D\gamma^N(H) \le D\gamma^N(G)$, $D\gamma^T(H) \le D\gamma^T(G)$
For all sub graphs of G.

Example 4.3:
Consider a BTIFG (Balanced Tripolar Intuitionistic Fuzzy Graph) G = (V,E) such that, $V = \{v_1, v_2, v_3, v_4, v_5\}$ $E = \{(v_1,v_2), (v_1,v_5), (v_2,v_3), (v_3,v_4), (v_4,v_5), (v_5,v_2)\}$



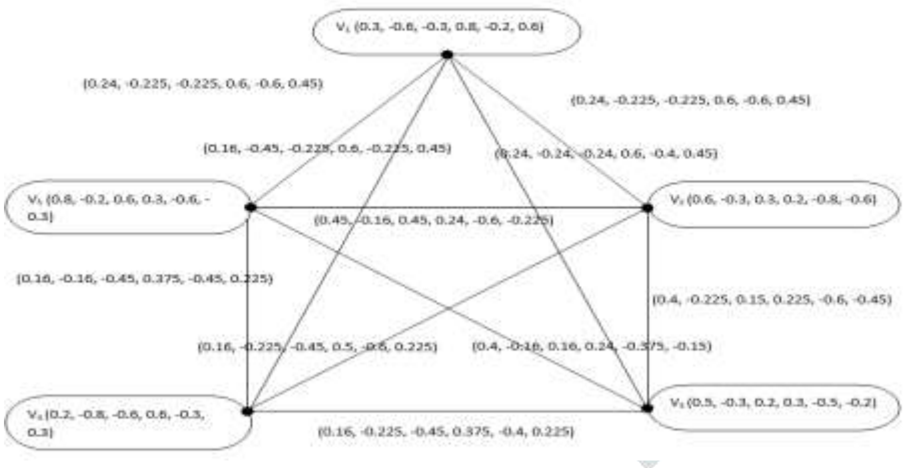
Density of subgraphs,

$H_1 = (v_1, v_2) \Rightarrow D(H_1) = \{ 1.6, 1.5, 1.5, 1.5, 1.5, 1.5 \}$
$H_2 = (v_1, v_3) \Rightarrow D(H_2) = \{ 0, 0, 0, 0, 0, 0 \}$
$H_3 = (v_1, v_4) \Rightarrow D(H_3) = \{ 0, 0, 0, 0, 0, 0 \}$
$H_4 = (v_1, v_5) \Rightarrow D(H_4) = \{ 1.6, 1.5, 1.5, 1.5, 1.5, 1.5 \}$
$H_5 = (v_2, v_3) \Rightarrow D(H_5) = \{ 1.6, 1.5, 1.5, 1.5, 1.5, 1.5 \}$
$H_6 = (v_2, v_4) \Rightarrow D(H_6) = \{ 0, 0, 0, 0, 0, 0 \}$
$H_7 = (v_2, v_5) \Rightarrow D(H_7) = \{ 1.6, 1.5, 1.5, 1.5, 1.5, 1.5 \}$
$H_8 = (v_3, v_4) \Rightarrow D(H_8) = \{ 1.6, 1.5, 1.5, 1.5, 1.5, 1.5 \}$
$H_9 = (v_3, v_5) \Rightarrow D(H_9) = \{ 0, 0, 0, 0, 0, 0 \}$
$H_{10} = (v_4,v_5) \Rightarrow D(H_{10}) = \{ 1.6, 1.5, 1.5, 1.5, 1.5, 1.5 \}$
$D(G) = (D\mu^P(G), D\mu^N(G) , D\mu^T(G), D\gamma^P(G), D\gamma^N(G), D\gamma^T(G)) =\{ 1.6, 1.5, 1.5, 1.5, 1.5, 1.5 \}$

Let $H_1 = (v_1,v_2)$, $H_2 = (v_1,v_3)$,……$H_{10} = (v_4,v_5)$ be non-empty sub graphs of G.
Density $(D\mu^P(H), D\mu^N(H) , D\mu^T(H), D\gamma^P(H), D\gamma^N(H), D\gamma^T(H))$ is $D(H_1) = \{ 1.6, 1.5, 1.5, 1.5, 1.5, 1.5 \}$, $D(H_2) = \{ 0, 0, 0, 0, 0, 0 \}$,………..$D(H_{10}) = \{ 1.6, 1.5, 1.5, 1.5, 1.5, 1.5 \}$.
So, $D(H) \leq D(G)$ for all subgraphs H of G. Hence G is Balanced Tripolar Intuitionistic Fuzzy Graph.

Definition 4.4:

A TIFG G = (V, E) is strictly balanced if for every u,v ϵV, D(H) = D(G) such that, $V = \{v_1, v_2, v_3, v_4, v_5\}$
$E = \{ (v_1,v_2), (v_1,v_3), (v_1,v_4), (v_1,v_5), (v_2,v_3), (v_2,v_4), (v_2,v_5), (v_3,v_4), (v_3,v_5), (v_4,v_5)\}$



$D(G) = (D\mu^P(G), D\mu^N(G) , D\mu^T(G), D\gamma^P(G), D\gamma^N(G), D\gamma^T(G)) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5 \}$

Let $H_1 = (v_1,v_2)$, $H_2 = (v_1,v_3)$,……$H_{10} = (v_4,v_5)$ be non-empty sub graphs of G.
Density $(D\mu^P(H), D\mu^N(H) , D\mu^T(H), D\gamma^P(H), D\gamma^N(H), D\gamma^T(H))$ is
$H_1 = (v_1, v_2) \Rightarrow D(H_1) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5 \}$
$H_2 = (v_1, v_3) \Rightarrow D(H_2) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5 \}$
$H_3 = (v_1, v_4) \Rightarrow D(H_3) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5 \}$
$H_4 = (v_1, v_5) \Rightarrow D(H_4) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5 \}$
$H_5 = (v_2, v_3) \Rightarrow D(H_5) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5 \}$
$H_6 = (v_2, v_4) \Rightarrow D(H_6) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5 \}$
$H_7 = (v_2, v_5) \Rightarrow D(H_7) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5 \}$
$H_8 = (v_3, v_4) \Rightarrow D(H_8) =\{1.6, 1.5, 1.5, 1.6, 1.5, 1.5 \}$
$H_9 = (v_3, v_5) \Rightarrow D(H_9) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5 \}$

$H_{10} = (v_4, v_5) \Rightarrow D(H_{10}) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5\}$

Hence D(G) = D(H) for all non-empty subgraphs H of G. Hence G is strictly balanced BTIFG.

Theorem 4.5:

Every complete tripolar intuitionistic fuzzy graph is balanced.

Proof:

Let G = (V, E) be a complete TIFG, then by the definition of complete TIFG, we have $\mu^P{}_2(u,v) = (\mu^P{}_1(u)\wedge\mu^P{}_1(v))$, $\mu^N{}_2(u,v) = (\mu^N{}_1(u)\vee\mu^N{}_1(v))$, $\mu^T{}_2(u,v) = \text{minmax}(\mu^T{}_1(u), \mu^N{}_1(v))$ and $\gamma^P{}_2(u,v) = (\gamma^P{}_1(u)\vee\gamma^P{}_1(v))$, $\gamma^N{}_2(u,v) = (\gamma^N{}_1(u)\wedge\gamma^N{}_1(v))$, $\gamma^T{}_2(u,v) = \text{maxmin}(\gamma^T{}_1(u),\gamma^T{}_1(v))$ for every u,v ϵV.

Therefore

$$\sum_{u,v\,\epsilon V} (\mu^P{}_2(u,v)) = \sum_{u,v\,\epsilon E} (\mu^P{}_1(u)\wedge\mu^P{}_1(v))$$

$$\sum_{u,v\,\epsilon V} (\mu^N{}_2(u,v)) = \sum_{u,v\,\epsilon E} (\mu^N{}_1(u)\vee\mu^N{}_1(v))$$

$$\sum_{u,v\,\epsilon V} (\mu^T{}_2(u,v)) = \sum_{u,v\,\epsilon E} \text{minmax}(\mu^T{}_1(u), \mu^N{}_1(v))$$

Then,

$$\sum_{u,v\,\epsilon V} (\gamma^P{}_2(u,v)) = \sum_{u,v\,\epsilon E} (\gamma^P{}_1(u)\vee\gamma^P{}_1(v))$$

$$\sum_{u,v\,\epsilon V} (\gamma^N{}_2(u,v)) = \sum_{u,v\,\epsilon E} (\gamma^N{}_1(u)\wedge\gamma^N{}_1(v))$$

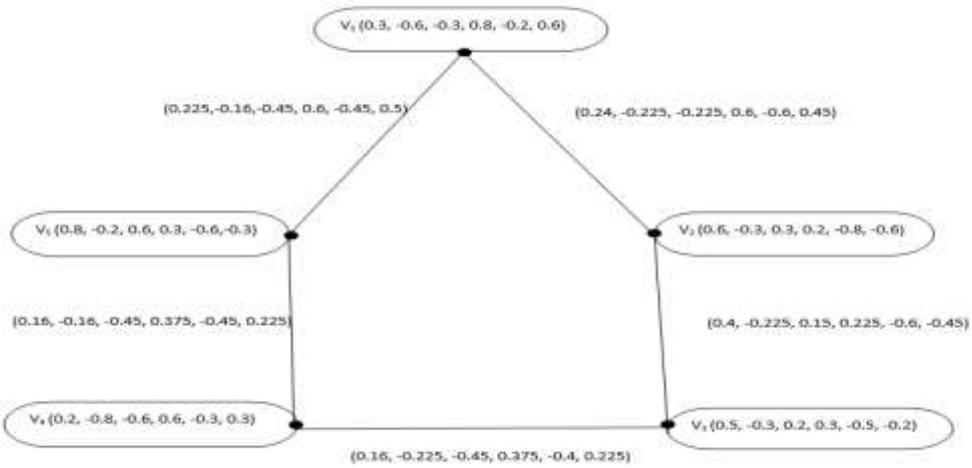
$$\sum_{u,v\,\epsilon V} (\gamma^T{}_2(u,v)) = \sum_{u,v\,\epsilon E} \text{maxmin}(\gamma^T{}_1(u),\gamma^T{}_1(v))$$

Now

$$D(G) = \begin{pmatrix} \dfrac{2\sum_{u,v\,\epsilon V} (\mu^P{}_2(u,v))}{\sum_{u,v\epsilon E}(\mu^P{}_1(u)\wedge\mu^P{}_1(v))} & \dfrac{2\sum_{u,v\,\epsilon V} (\mu^N{}_2(u,v))}{\sum_{u,v\epsilon E} (\mu^N{}_1(u)\wedge\mu^N{}_1(v))} & \dfrac{2\sum_{u,v\,\epsilon V} (\mu^T{}_2(u,v))}{\sum_{u,v\epsilon E} \text{minmax}(\mu^T{}_1(u),\mu^T{}_1(v))} \\ \dfrac{2\sum_{u,v\,\epsilon V} (\gamma^P{}_2(u,v))}{\sum_{u,v\epsilon E}(\gamma^P{}_1(u)\vee\gamma^P{}_1(v))} & \dfrac{2\sum_{u,v\,\epsilon V} (\gamma^N{}_2(u,v))}{\sum_{u,v\epsilon E}(\gamma^N{}_1(u)\vee\gamma^N{}_1(v))} & \dfrac{2\sum_{u,v\,\epsilon V} (\gamma^T{}_2(u,v))}{\sum_{u,v\,\epsilon E} \text{maxmin}(\gamma^T{}_1(u),\gamma^T{}_1(v))} \end{pmatrix}$$

D(G) = (2, 2, 2, 2, 2, 2). Let H be a non-empty subgraph of G then, D(H) = (2, 2, 2, 2, 2, 2) for every H⊆G. Thus G is balanced.

Note 4.6: The converse part of the above theorem is need not be true. Every Balanced Tripolar Intuitionistic Fuzzy Graph need not be complete.



$D(G) = (D\mu^P(G), D\mu^N(G), D\mu^T(G), D\gamma^P(G), D\gamma^N(G), D\gamma^T(G)) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5\}$
Where,
$D\mu^P(G) = 1.6$, $D\mu^N(G) = 1.5$, $D\mu^T(G) = 1.5$, $D\gamma^P(G) = 1.6$, $D\gamma^N(G) = 1.5$, $D\gamma^T(G) = 1.5$.
This $D(G) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5\}$
$H_1 = (v_1, v_2) \Rightarrow D(H_1) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5\}$
$H_2 = (v_1, v_3) \Rightarrow D(H_2) = \{0, 0, 0, 0, 0, 0\}$
$H_3 = (v_1, v_4) \Rightarrow D(H_3) = \{0, 0, 0, 0, 0, 0\}$
$H_4 = (v_1, v_5) \Rightarrow D(H_4) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5\}$
$H_5 = (v_2, v_3) \Rightarrow D(H_5) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5\}$
$H_6 = (v_2, v_4) \Rightarrow D(H_6) = \{0, 0, 0, 0, 0, 0\}$
$H_7 = (v_2, v_5) \Rightarrow D(H_7) = \{0, 0, 0, 0, 0, 0\}$
$H_8 = (v_3, v_4) \Rightarrow D(H_8) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5\}$
$H_9 = (v_3, v_5) \Rightarrow D(H_9) = \{0, 0, 0, 0, 0, 0\}$
$H_{10} = (v_4, v_5) \Rightarrow D(H_{10}) = \{1.6, 1.5, 1.5, 1.6, 1.5, 1.5\}$
Hence $D(G) = D(H)$ for all non-empty subgraphs H of G. Hence $D(H) \leq D(G)$ for all subgraphs H of G. So G is Balanced Intuitionistic Tripolar Fuzzy Graphs. From the above graph it is easy to see that $\sum_{u,v \in V} (\mu^P_2(u,v)) \neq \sum_{u,v \in E} (\mu^P_1(u) \wedge \mu^P_1(v))$

$\sum_{u,v \in V} (\mu^N_2(u,v)) \neq \sum_{u,v \in E} (\mu^N_1(u) \vee \mu^N_1(v))$

$\sum_{u,v \in V} (\mu^T_2(u,v)) \neq \sum_{u,v \in E} \text{minmax}(\mu^T_1(u), \mu^N_1(v))$

Then,

$\sum_{u,v \in V} (\gamma^P_2(u,v)) \neq \sum_{u,v \in E} (\gamma^P_1(u) \vee \gamma^P_1(v))$

$\sum_{u,v \in V} (\gamma^N_2(u,v)) \neq \sum_{u,v \in E} (\gamma^N_1(u) \wedge \gamma^N_1(v))$

$\sum_{u,v \in V} (\gamma^T_2(u,v)) \neq \sum_{u,v \in E} \text{maxmin}(\gamma^T_1(u), \gamma^T_1(v))$. Hence G is balanced but not complete.

5. CONCLUSION

In this paper we have introduced a balanced intuitionistic tripolar fuzzy graph. We have derived the condition for intuitionistic tripolar fuzzy graph. Justified our definitions and results through illustrating few examples

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