Some New Divisor Cordial Graphs

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Abstract. A divisor cordial labeling of a graph G with vertex set V is a bijection f from V to {1, 2, 3, … , |V|} such that when each edge uv is assigned the label 1 if f(u) divides f(v) or f(v) divides f(u), and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph is called divisor cordial if, it admits divisor cordial labeling. In this paper we prove that the graphs such as $DS(K_{1, n, n})$, DS(Gl(n)), W-graph W_n, the graph $B(m)\Theta u_n K_1$, Herschel graph H_s and switching of an apex vertex in the Hershel graph H_s are divisor cordial graphs.

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1. Introduction. All graphs in this paper are simple, finite, connected and undirected graphs. Let G = $(V(G), E(G))$ be a graph with p vertices and q edges. For standard terminology and notations related to graph theory we refer to Harary [3] while for number theory we refer to Burton [2]. Graph labeling is a technique in which the vertices and edges are assigned real values subject to certain conditions. For a dynamic survey on various graph labeling problems we refer to Gallian [4]. The concept of cordial labeling was introduced by Cahit [1]. After this many labeling schemes are also introduced with minor variations in cordial labeling.

The concept of divisor cordial labeling was introduced by Varatharajan et al. [8], further they have proved that the path, cycle, wheel, star, $K_{2, n}$ and $K_{3, n}$ are divisor cordial graphs. The divisor cordial labeling of full binary trees, $G*K_{2, n}$, $G*K_{3, n}$, $<K_{1,n}^{(1)}, K_{1,n}^{(2)}>$ and $<K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)}>$ are reported by the same authors in [9]. Vaidya et al. [6, 7] have proved that degree splitting graph of $B_{n, n}$, shadow graph of $B_{n, n}$, square graph of $B_{n, n}$, Helm H_n, flower graph Fl_n, Gear graph G_n, switching of a vertex in C_n, switching of a rim vertex in W_n, switching of the apex vertex in Helm H_n are divisor cordial graphs. The divisor cordial labeling of some cycle related graphs are reported by Maya et al. [5]. In this paper we have discussed divisor cordial labeling of $DS(K_{1, n, n})$, DS(Gl(n)), W-graph W_n, the graph $B(m)\Theta u_n K_1$, Herschel graph H_s and switching of an apex vertex in the Hershel graph H_s. We will provide a brief summary of definitions and other information which are necessary for the present work.

Definition 1.1 A mapping $f : V(G) \rightarrow \{0, 1\}$ is called a *binary vertex labeling* of G and f(v) is called the *label of the vertex* v of G under f. If for an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$.

Notation 1.2 We denote $v_f(i)$ = number of vertices having label i and $e_f(i)$ = number of edges having label i, where i = 0, 1.

Definition 1.3 A binary vertex labeling of a graph G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called *cordial* if it admits cordial labeling.

Definition 1.4 A *divisor cordial labeling* of a graph G with vertex set V is a bijection f from V to {1, 2, 3, … , |V|} such that each edge uv is assigned the label 1 if either f(u) or f(v) divides the other and 0 otherwise. In addition, the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph is called *divisor cordial* if it admits divisor cordial labeling.

Definition 1.5 The *graph* $K_{1, n, n}$ is obtained from the star graph $K_{1, n}$ by subdividing each and every edge of $K_{1, n}$ by inserting a new vertex.

Definition 1.6 Let G be a simple (p, q) graph. For each i, let S_i $(1 \leq i \leq m)$ be the set of vertices having the same degree in G. The *degree splitting graph* of G denoted by DS(G) is the graph obtained from G by adding new vertices $w_1, w_2, w_3, \ldots, w_m$ such that joining w_i to each vertex of S_i respectively, for each i = 1, 2, … , m.

Definition 1.7 A *globe* is a graph obtained from two isolated vertices are joined by n paths of length 2 and it is is denoted by Gl(n).

Definition 1.8 The *vertex switching* G_v of graph G is obtained by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertices which are not adjacent to v in G.

Definition 1.9 Let S_m, S_n be any two shell graphs with orders m and n respectively. If the apex vertices of S_m and S_n are adjoined in to a single vertex, then the resulting graph is called a *bow* graph and it is denoted by B(m, n). If bow graph in which each shell is of same order m, then it is called a *uniform bow graph* and it is denoted by B(m, m) or simply B(m).

Definition 1.10 Let v be a vertex in graph G. The *graph* $G\odot_v K_1$ is obtained from G by adding a new vertex u and connect u and v by an edge.

Definition 1.11 The *Herschel graph H_s* is a bipartite undirected graph with 11 vertices and 18 edges. The Herschel graph H_s is shown in Figure 1.

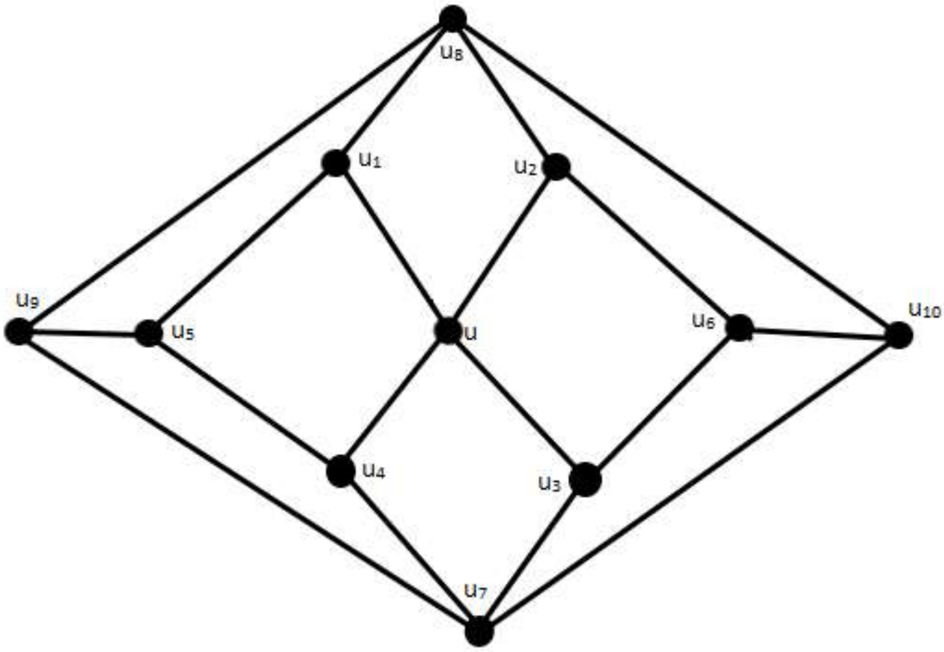

Figure 1. Herschel graph H_s

2. Main results

Theorem 2.1 The graph $DS(K_{1, n, n})$ is a divisor cordial graph.

Proof. Let $V(K_{1, n, n}) = \{u, u_i, v_i : 1 \le i \le n\}$, where $u_i's$ are vertices of degree 2 and $v_i's$ are the pendant vertices in $K_{1, n, n}$. Let u be an apex vertex of $K_{1, n, n}$. Let x, y, z be the new vertices of $DS(K_{1, n, n})$ corresponding to set of vertices of degrees 1, 2 and n respectively.

Let $G = DS(K_{1, n, n})$. Note that $|V(G)| = 2n+4$ and $|E(G)| = 4n+1$. We define a vertex labeling $f : V(G) \to \{1, 2, \dots, 2n+4\}$ as follows:

$f(u) = 1$, $f(x) = 2$,

$f(u_i) = 2i + 1$ for $1 \le i \le n$,

$f(v_i) = 2i + 2$ for $1 \le i \le n$,

$f(z) = 2n + 4$.

It is clear that all edges incident to u will take 1. i.e., n + 1 edges have been assigned the label 1. All edges u_iv_i $(1 \le i \le n)$ are assigned the value 0, since each label of u_i is relatively prime to label v_i. i.e., n edges have been assigned the label 0. All edges xv_i ($1 \leq i \leq n$) are assigned the value 1, since the labels of both x and v_i are even. i.e., n edges have been assigned the label 1.

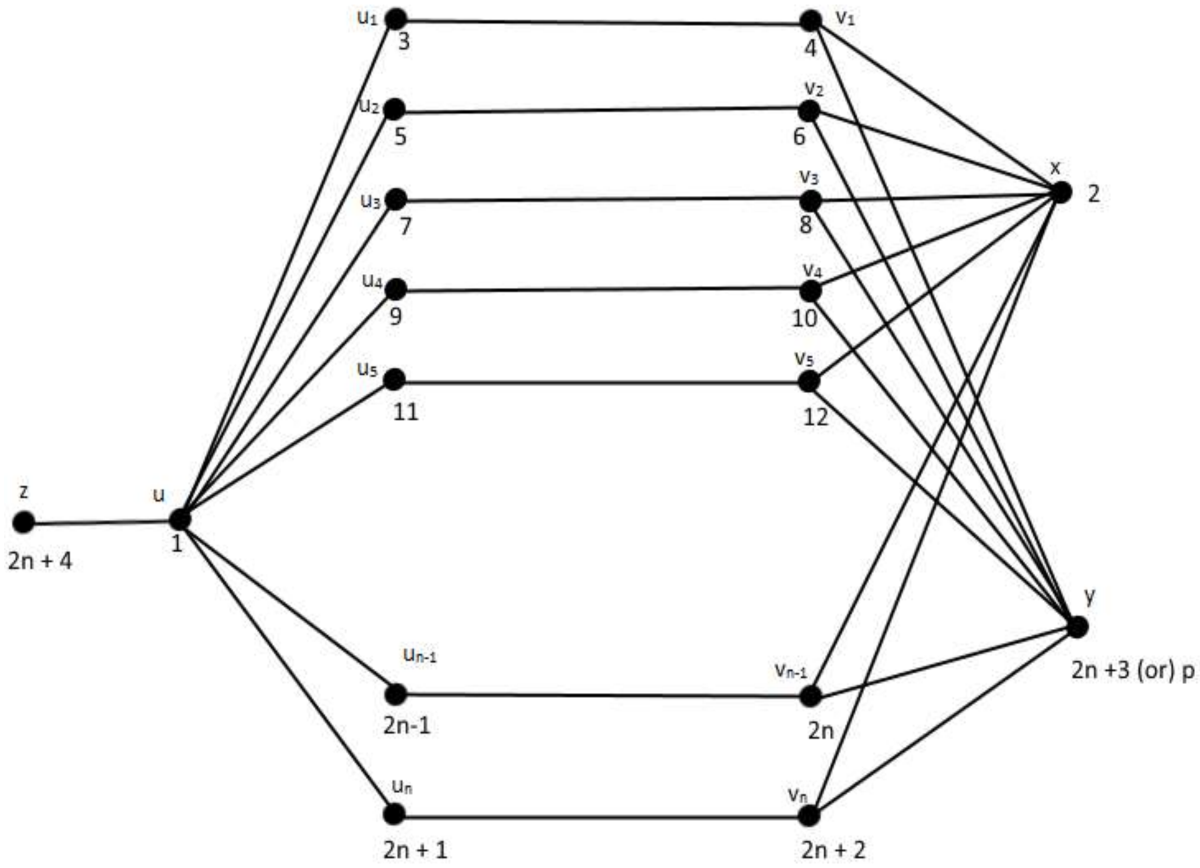

Figure 2. Divisor cordial labeling of $DS(K_{1, n, n})$

We need to assign a label y such that each edge yv_i ($1 \leq i \leq n$) must take the value 0, so we assign f(y) = 2n + 3, if 2n + 3 is a prime number. If 2n +3 is not a prime number, in that case let p be the largest prime number in {1, 2, … , 2n+4}. In such a case, we interchange the labels p & 2n + 3 such that f(y) = p. Since p is a largest prime, so that each edge yu_i ($1 \leq i \leq n$) will take the value 0. Thus $|e_f(0) - e_f(1)| \leq 1$. Therefore, G is a divisor cordial graph. The divisor cordial labeling of $DS(K_{1, n, n})$ is shown in Figure 2.

Theorem 2.2 The W-graph W_n admits divisor cordial labeling.

Proof. Let $K_{1,n}^1$ and $K_{1,n}^2$ be the first and second copies of star graph with apex vertices a and b respectively. Let $V(K_{1,n}^1) = \{a, x_1, x_2, x_3, \ldots, x_n\}$ and $V(K_{1,n}^2) = \{b, y_1, y_2, y_3, \ldots, y_n\}$. We obtain the W-graph by adjoining x_n and y_1. Let G = W_n. Then |V(G)| = 2n + 1 and |E(G)| = 2n. Now we define vertex labeling $f : V(G) \to \{1, 2, \ldots , 2n+4\}$ as follows:

f(a) = 1, f(b) = Largest prime number p in the set {1, 2, … , 2n+1}.

Label the remaining vertices $x_1, x_2, x_3, \ldots, x_n$ and $y_1, y_2, y_3, \ldots, y_n$ from the set $\{1, 2, \ldots , 2n+1\} - \{1, p\}$ uniquely in any order. By the above labeling pattern, we have $|e_f(0) - e_f(1)| = 0$. Hence the W-graph admits divisor cordial labeling. The divisor cordial labeling of W_n is shown in Figure 3.

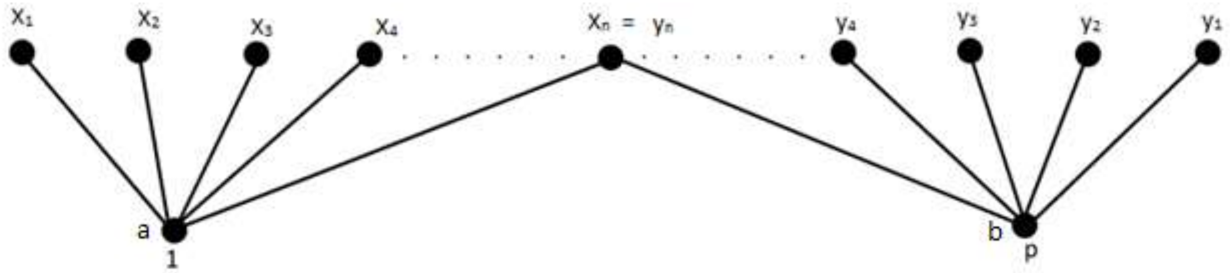

Figure 3. Divisor cordial labeling of W_n

Theorem 2.3 Degree splitting graph of globe Gl(n) is a divisor cordial graph.

Proof. Let G = DS(Gl(n)). We need to consider the following three cases.

Case 1. When n = 1

Now we define the vertex labeling of DS(Gl(1)) as shown in Figure 4.

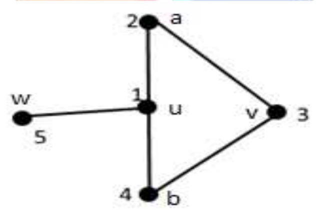

Figure 4. Divisor cordial labeling of DS(Gl(1))

Let v be the new vertex corresponding to the pendant vertices a and b. Also let w be the new vertex corresponding to the vertex u of degree 2.

Case 2. When n = 2

Let v be the new vertex in DS(Gl(2)), corresponding to the vertices a, b, u_1 and u_2 (since each vertex in Gl(2) is of degree 2). Now we define the vertex labeling as shown in Figure 5.

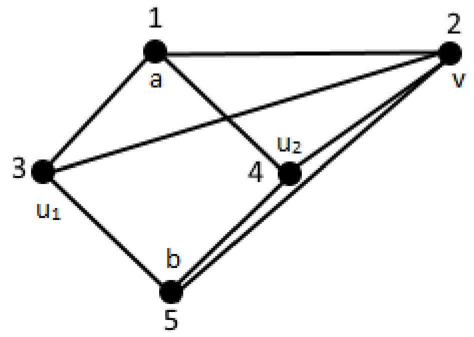

Figure 5. Divisor cordial labeling of DS(Gl(2))

Case 3. When n ≥ 3

In G, let Let v be the new vertex corresponding to the vertices a and b. Also let w be the new vertex corresponding to the vertices u_1, u_2, u_3, … , u_n of degree 2. Note that |V(G)| = n + 4 and |E(G)| = 3n + 2. We define vertex labeling f : V(G) → {1, 2, … , n + 4} as follows:

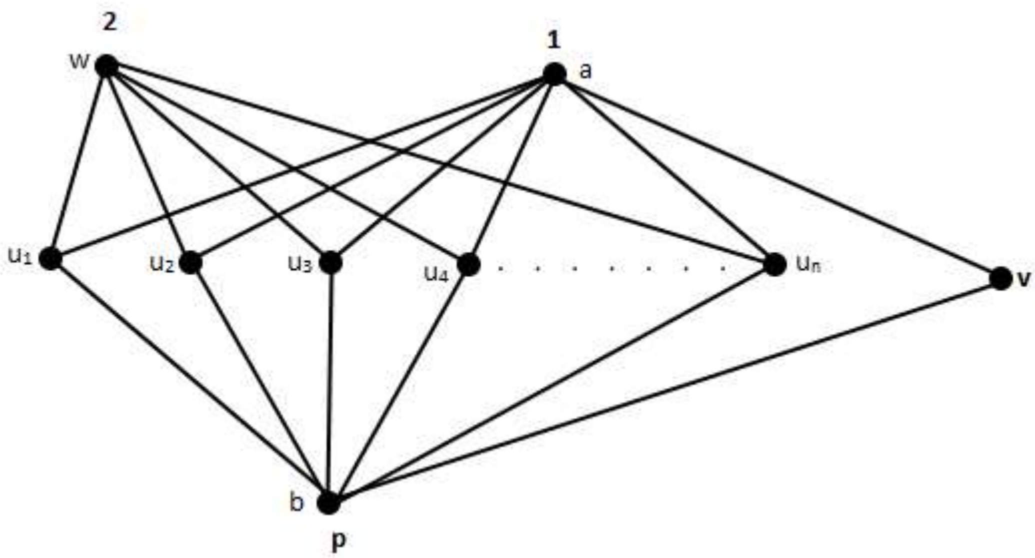

Figure 6. Divisor cordial labeling of DS(Gl(n))

f(a) = 1, f(b) = Largest prime number p in the set {1, 2, … , n+4}, f(w) = 2.

We label the remaining vertices u_1, u_2, u_3, …, u_n and v from the set {1, 2, … , n+4} – {1, 2, p} uniquely in any order. In view of the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$. Hence DS(Gl(n)) admits divisor cordial labeling. The divisor cordial labeling of DS(Gl(n)) is shown in Figure 6.

Theorem 4. The graph B(n)Θu_mK$_1$ is a divisor cordial graph, where B(n) is a uniform bow graph.

Proof. Let the graph G = B(n)Ͽu_mK$_1$. Let the vertex set V(G) = {w_0, u_1, u_2, u_3, …, u_n, v_1, v_2, v_3, …, v_n}, where w_0 is the apex vertex. {u_1, u_2, u_3, …, u_n} are the vertices of first shell adjacent to w_0 and {v_1, v_2, v_3, …, v_n} are the vertices of second shell adjacent to w_0. In G, the new vertex u is connected with u_n by an edge. Note that |V(G)| = 2n + 2 and |E(G)| = 4n - 1. We define vertex labeling f : V(G) → {1, 2, … , 2n + 2} as follows:

$f(w_0) = 1$,

$f(v_i) = i + 1$ for $1 \le i \le n$,

$f(u_i) = i + 1$ for $n + 1 \le i \le 2n + 1$,

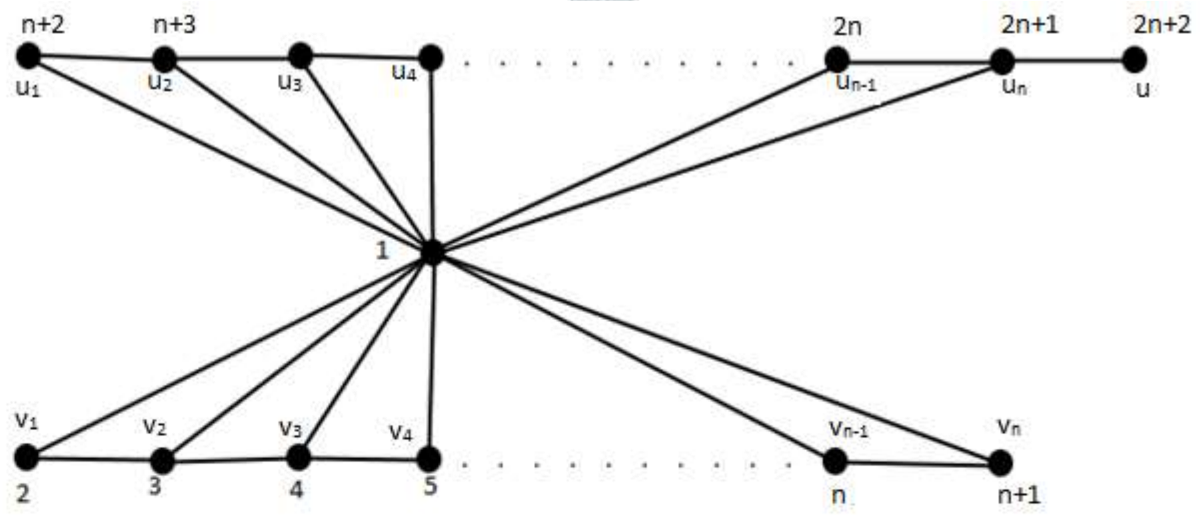

Figure 7. Divisor cordial labeling of B(n)Ͽu_mK$_1$

By definition of f, we obtain $e_f(0) = 2n - 1$ and $e_f(1) = 2n$. Thus $|e_f(0) - e_f(1)| = 1$. Hence G admits divisor cordial labeling. The divisor cordial labeling of B(n)Ͽu_mK$_1$ is shown in Figure 7.

Theorem 5. The Herschel graph H_s is a divisor cordial graph.

Proof. Consider the Herschel graph H_s. We see that $|V(H_s)| = 11$ and $|E(H_s)| = 18$. We define vertex labeling function f : V(G) → {1, 2, … , 11} as follows:

$f(u) = 1$, $f(u_i) = 2i + 1$; $1 \le i \le 4$,

$f(u_5) = 6$, $f(u_6) = 11$, $f(u_7) = 10$, $f(u_8) = 4$, $f(u_9) = 2$, $f(u_{10}) = 8$.

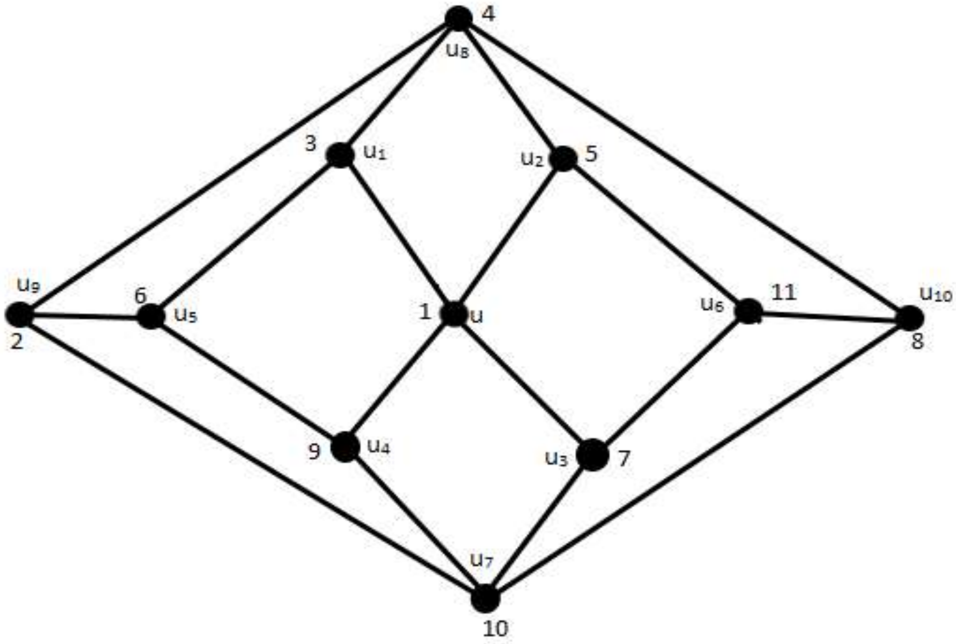

Figure 8. Divisor cordial labeling of Herschel graph H_s

In view of the above labeling pattern, we have $|e_f(0) - e_f(1)| = 0$. Hence Herschel graph H_s is a divisor cordial graph. The divisor cordial labeling of Herschel graph H_s is shown in Figure 8.

Theorem 6. Switching of an apex vertex in the Herschel graph H_s is a divisor cordial graph.

Proof. Let G be the Switching of an apex vertex in the Herschel graph H_s. Note that $|V(G)| = 11$ and $|E(G)| = 20$. We define $f : V(G) \rightarrow \{1, 2, \dots, 11\}$ as follows:

$f(u) = 1$, $f(u_1) = 5$, $f(u_2) = 6$, $f(u_3) = 11$, $f(u_4) = 9$, $f(u_5) = 3$, $f(u_6) = 7$, $f(u_7) = 10$, $f(u_8) = 4$, $f(u_9) = 2$, $f(u_{10}) = 8$.

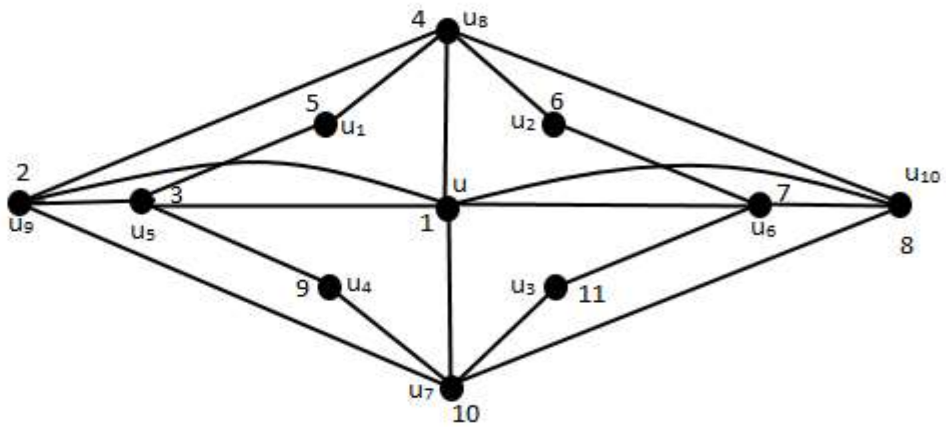

Figure 9. Divisor cordial labeling of switching of an apex vertex in Herschel graph H_s

In view of the above labeling pattern, we have $|e_f(0) - e_f(1)| = 0$. Hence, switching of an apex vertex in Herschel graph H_s is a divisor cordial graph.

The divisor cordial labeling of switching of an apex vertex in Herschel graph H_s is shown in Figure 9.

3. Conclusion

Labeling of graphs connecting graphs with number theory. Divisor cordial labeling is an active area of research at present. It is not necessary that all graphs are divisor cordial graphs. In literature many graphs are shown as divisor cordial graphs. In this paper we have investigated some new divisor cordial graphs such as $DS(K_{1,n,n})$, DS(Gl(n)), W-graph W_n, the graph $B(m)\Theta u_n K_1$, Herschel graph H_s and switching of an apex vertex in the Herschel graph H_s.

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5. References

1. I. Cahit. 1987. Cordial graphs: A weaker version of graceful and harmonious graphs, Ars Combinatorial, vol. 23, pp. 201 – 207.

2. David M. Burton. 1980. Elementary number theory, Second edition, Wm. C. Brown Company Publishers.

3. F. Harary. 1972. Graph theory, Addison – Wesley, Reading, Mass.

4. J. A. Gallian. 2015. A dynamic survey of graph labeling, The Electronic Journal of Combinatorics.

5. P. Maya and T. Nicholas. 2014. Some new families of divisor cordial graph, Annals of Pure and Applied Mathematics, vol. 5, No. 2, pp. 125 – 134.

6. S. K. Vaidya and N. H. Shah. 2013. Further results on divisor cordial labeling, Annals of Pure and Applied Mathematics, vol. 4, No. 2, pp. 150 – 159.

7. S. K. Vaidya and N. H. Shah. 2013. Some star and bistar related divisor cordial graphs, Annals of Pure and Applied Mathematics, vol. 3, No. 1, pp. 67 – 77.

8. R. Varatharajan, S. Navaneethakrishnan and K. Nagarajan. 2011. Divisor cordial graphs, International J. Math. Combin., vol. 4, pp. 15 – 25.

9. R. Varatharajan, S. Navaneethakrishnan and K. Nagarajan. 2012. Special classes of divisor cordial graphs, International Mathematical Forum, vol. 7, No. 35, pp. 1737 – 1749.