Analytical solution of Fully developed combined free and forced Convection through a Vertical Channel with heat generation/absorption and First order Chemical Reaction.

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Abstract:
Fully developed combined free and forced Convection through a vertical channel with heat generation/absorption with first order chemical reaction is presented. The boundary conditions for temperature are Isothermal-Isothermal, Isoflux – Isothermal and isothermal – Isoflux for left and right walls of channel. The effect of thermal Grashoff parameter, concentration Grashoff parameter and heat generation/absorption parameter are studied. Also the flow field with the presence of first order chemical reaction is particularly analyzed. The governing equations are solved analytically. Velocity, temperature and concentration profiles are investigated for different values of the flow parameters and presented.

Key words: Combined Free and Forced Convection, Laminar flow, Vertical channel, Chemical reaction, Analytical.

1. Introduction.

There is an enhanced interest in flow through vertical channels among the researchers. The common cases are on heating or cooling of channel walls, and at the small velocities of fluid flow that are characteristic of laminar flow, combined free and forced convection is realized. Forced convection (Laminar) may be obtained in capillaries. Channel flows have been studied substantially for their increased importance in the various fields of science and engineering. Combined free and forced convection is normally observed in high-powered-output devices, like nuclear reactor technology, electronic cooling, etc. In electronic cooling, the components are assembled on the circuit card, an array of which is placed vertically so that forming the vertical channel through this a coolant is passed. Also combined free and forced convection has applications in solar systems, heat exchangers and chemical industries.

Literature survey reveals that lot of papers are available in this area, Win Aung and G.Worku[1] have basically analyzed the theory of fully developed, combined convection including the flow reversal, the numerical simulation of forced convective incompressible flow in a channel an array of heated obstacles attached to one was studied by Timothy J. et.al[2], papers by Aung and Worku[3] and Habachi and Acharya[4] have given ideas about fluid flow and thermal characteristics inside a vertical channel with symmetrical or asymmetrical thermal boundary conditions. Cheng,Kou and Huaung[5] has concentrated on the analysis of flow reversal and heat transfer of fully developed mixed convection in vertical channel, Hamdah and Wirtz[6], Berletta [7,8] pointed out the relevant effects of laminar mixed convection in a vertical channel and flow reversal in vertical duct with uniform heat fluxes. Gill and Casal[9] showed that the buoyancy significantly affects the flow of low prandtl number fluids which is highly sensitive to gravitational force and the extent to which the buoyancy force influences a forced flow. A Berletta [10] has worked on the fully developed mixed convection and flow reversal of power law fluid in vertical channel.

Flow of fluid with internal heat generation/absorption is of great experimental as well as theoretical importance. The volumetric heat generation/absorption term exerts strong effect on the heat transfer and flow when the temperature difference is significantly high. This analysis is of much importance in view of chemical reaction concerned with disassociating fluids. Combined natural and forced convection in vertical channels is commonly arises in nuclear reactor applications. Md.Mamun Molla et.al[11] have investigated the effect of internal heat generation/absorption on a steady two dimensional natural convection flow of viscous incompressible fluid alone uniformly heated vertical wavy surface. Umavathi et.al[12] discussed the laminar magneto convection flow in a vertical channel in the presence of heat generation or absorption.

The increased scope for chemical reaction in chemical and hydrometallurgical industries, the study of heat transfer with chemical reaction has become the need of the hour. These are commonly encountered in nuclear safety and combustion systems, solar collectors, metallurgical and chemical engineering. Anjalidevi and Kandaswamy[13] investigated the effect of chemical reaction and heat and mass transfer on laminar flow along a semi infinite horizontal plate. Fully developed laminar mixed convection flow in a vertical channel in the presence of first order chemical reactions has been studied by J Prathap Kumar et.al[14]. An analytical solution of dispersion of solute with first order chemical reaction in a vertical double passage channel filled with porous medium has been analyzed by J. Prathap Kumar et.al[15], Muthucumaraswamy and Ganeshan[16] have investigated the impulsive motion of a vertical plate with heat flux/mass, flux/suction and diffusion of chemically reactive species. J.Prathap Kumar et.al[17] studied the effect of homogeneous and heterogeneous reaction on solute dispersion in composite porous medium, R.Muthucumaraswamy, P.Ganesan,[18] have analyzed the first order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux.

In view of this the analytical solution of fully developed Mixed Convection through Vertical Channel with heat generation/absorption and First order Chemical Reaction is considered. The aim of this study is to analyze the fully developed mixed convection through vertical channel with heat generation/absorption and first order chemical reaction. This analysis adds very useful inputs to the existing literature. Also this can be used in chemical and metallurgical industries.

2. Mathematical Formulation

The nature of the fluid considered is laminar and Newtonian which flows steadily in a parallel plate vertical channel. It is assumed that Boussinesq approximation holds, hence the thermal conductivity, thermal diffusivity, dynamic viscosity and thermal expansion coefficients are considered constant. Equation of state under Boussinesq approximation is

$$\rho = \rho_0[1-\beta(T-T_0)] \qquad 2.1$$

Fig 1 shows the spatial coordinate system for the fluid flow. The X-axis is parallel to the gravitational field but with opposite direction and Y-axis is parallel to the horizontal direction.

For fully developed flow, the only nonzero component of velocity is the X component, thus equation of continuity equation is obtained as

$$\frac{\partial u}{\partial X} = 0 \qquad 2.2$$

that is U depends only Y.

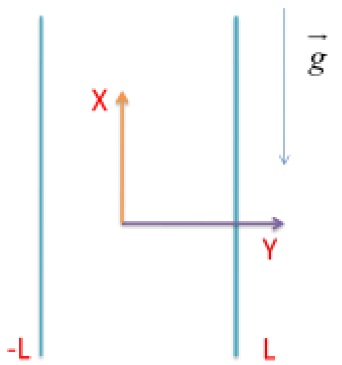

Fig 1 Physical configuration

X-Momentum balance equation:

$$g\beta_T\left(T-T_0\right)+g\beta_C\left(C-C_0\right)-\frac{1}{\rho_0}\frac{dP}{dX}+\nu\frac{d^2U}{dY^2}=0 \qquad 2.3$$

Y-Momentum balance equation:

$$\frac{\partial P}{\partial Y}=0 \qquad 2.4$$

Energy balance equation:

$$\frac{d^2T}{dY^2}\pm\frac{Q\left(T-T_0\right)}{K}=0 \qquad 2.5$$

Concentration equation:

$$D\frac{d^2C}{dY^2}-KC=0 \qquad 2.6$$

Boundary conditions:

Velocity, temperature and concentration

$$U(-L)=U(L)=0$$
$$at\,Y=-L \quad T=T_1\ ,\ C=C_1$$
$$at\,Y=L \quad T=T_2\ ,\ C=C_2 \qquad 2.7$$

It is assumed that $T_2\geq T_1$ and $C_2\geq C_1$

Above dimensional equations are expressed in non-dimensional form using the following variables

$$u=\frac{U}{U_0},\ y=\frac{Y}{H},\quad \theta=\frac{T-T_0}{T_1-T_2}=\frac{T-T_0}{\Delta T},\quad \phi=\frac{C-C_0}{C_1-C_2}=\frac{C-C_0}{\Delta C},\quad Gr_T=\frac{g\beta_T\Delta TH^3}{\nu^2},$$

$$Gr_C=\frac{g\beta_C\Delta CH^3}{\nu^2},\quad \mathrm{Re}=\frac{U_0H}{\nu^2},\quad \lambda_T=\frac{Gr_T}{\mathrm{Re}},\quad \lambda_C=\frac{Gr_C}{\mathrm{Re}},\quad p=-\frac{H^2}{\mu U_0}\frac{dP}{dX},$$

$$\Delta T=T_2-T_1,\ \Delta C=C_2-C_1,\ T_0=\frac{T_1+T_2}{2}\quad C_0=C_1 \qquad 2.8$$

Thus, non-dimensional equations for momentum, temperature and concentration are

$$\frac{d^2u}{dy^2}+\lambda_T\theta+\lambda_C\phi+p=0 \qquad 2.9$$

$$\frac{d^2\theta}{dy^2}\pm\psi\theta=0 \qquad 2.10$$

$$\frac{d^2\phi}{dy^2}-\alpha^2\phi=0 \qquad 2.11$$

Where $\psi=\frac{QH^2}{K}$, $\alpha^2=\frac{KH^2}{D}$

Non-dimensional boundary conditions are

At $Y=-L$, $y=-\frac{1}{2}$, $\theta=-\frac{1}{2}$, $\phi=0.$

$$Y=L\ ,\quad y=\frac{1}{2},\quad \theta=\frac{1}{2},\quad \phi=1. \qquad 2.12$$

3. Solutions

The governing equations are solved analytically. The effects of thermal buoyancy λ_T and concentration buoyancy λ_C, heat generation/absorption parameter ψ, concentration parameter ϕ are all studied. Velocity, temperature, concentration profiles are presented graphically and analyzed.

Case 1 Isothermal – Isothermal $\left(T_1 - T_2\right)$

From 2.11 solution of concentration is

$$\phi = A_3 \cosh(\alpha y) + A_4 \sinh(\alpha y) \qquad 3.1$$

Equation 2.10 has the solution for heat generation as

$$\theta = A_2 \sin\left(\sqrt{\psi}\, y\right) \qquad 3.2$$

Using 3.1 and 3.2 in 2.9 we get the solution of the velocity as

$$u = R_1 \sin\left(\sqrt{\psi}\, y\right) - R_2 \cosh\left(\alpha y\right) - R_3 \sinh\left(\alpha y\right) - \frac{py^2}{2} - A_5 y - A_6 \qquad 3.3$$

Case 2 Isoflux – Isothermal $\left(q_1 - T_2\right)$

In this we consider the thermal boundary conditions for the left and right walls are in dimensional form are

$$q_1 = -K \left.\frac{dT}{dY}\right|_{Y=-L} \qquad 3.4$$

$$T\left(Y = L\right) = T_2 \qquad 3.5$$

And the corresponding dimensionless equations are,

$$\left.\frac{d\theta}{dY}\right|_{y=-\frac{1}{2}} = -1 \qquad 3.6$$

$$\theta\left(y = \frac{1}{2}\right) = R_{qt}, \qquad R_{qt} = \frac{T_2 - T_0}{\Delta T} \qquad 3.7$$

For heat generation from 2.10, 3.6 and 3.7 we obtain

$$\theta = A_7 \cos(\sqrt{\psi}\, y) + A_8 \sin(\sqrt{\psi}\, y) \qquad 3.8$$

Substituting 3.1 & 3.8 in equation 2.9 and integrating twice we get velocity expression as

$$u = R_5 \cos(\sqrt{\psi}\, y) + R_6 \sin(\sqrt{\psi}\, y) - R_7 \cosh(\alpha y) - R_8 \sinh(\alpha y) - p\frac{y^2}{2} - A_9 y - A_{10} \qquad 3.9$$

Case 3 Isothermal – Isoflux $\left(T_1 - q_2\right)$

In this we consider the thermal boundary conditions for the left and right walls are in dimensional form are

$$T(Y=-L)=T_1 \quad 3.10$$

$$q_2=-K\left.\frac{dT}{dY}\right|_{Y=L} \quad 3.11$$

And the corresponding dimensionless equations are,

$$\theta\left(y=-\frac{1}{2}\right)=R_{tq}, \qquad R_{tq}=\frac{T_1-T_0}{\Delta T} \quad 3.12$$

$$\left.\frac{d\theta}{dY}\right|_{y=\frac{1}{2}}=-1 \quad 3.13$$

For heat generation from 2.10, 3.12 and 3.13 one obtains

$$\theta=A_{11}\cos(\sqrt{\psi}\,y)+A_{12}\sin(\sqrt{\psi}\,y) \quad 3.14$$

Substituting 3.14 & 3.1 in equation 2.9 and integrating twice we get velocity expression as

$$u=R_9\cos(\sqrt{\psi}\,y)+R_{10}\sin(\sqrt{\psi}\,y)-R_{11}\cosh(\alpha y)-R_{12}\sinh(\alpha y)-p\frac{y^2}{2}-A_{13}y-A_{14} \quad 3.15$$

4. Results and Discussions

Fully developed combined free and forced convection through a vertical channel with heat generation/ absorption and first order chemical reaction is studied analytically. The velocity, temperature and concentration profiles are presented graphically and interpreted. The effects of λ_T, λ_C, ψ and α on velocity, temperature are analyzed.

In Fig 2, we observe that for different value of λ_T there is a considerable change in the velocity profile. For $\lambda_T=0$ the Poiseulle parabolic profile is obtained. With increase in the value of λ_T the flow accelerates near the hot wall (more and more channeled towards the hot wall) and for further higher values of λ_T the flow reversal that is the back flow is observed near the cooler wall. Fluid velocities are larger adjacent to the hot wall. Velocity profiles for different values intersect at $y=0$.

The effect λ_C on the velocity profile is presented in the figure 3, where the values of the other parameters are $\lambda_T=10$, $\alpha=0.5$, $p=12$. Increase in λ_C there is an increase in the flow field. The concentration buoyancy is the ratio of species buoyancy force to the viscous force, therefore increase in its values increase the buoyancy forces of the species which results in the increase in the velocity range.

A first order reaction depends on the concentration of only one reactant (a **unimolecular reaction**). Other reactants can be present, but each will be zero order. α has significant effect on velocity profile which is analyzed in the figure 4. Increase in the values of α decreases the flow field, this is due to increase in concentration of the reactant, the number of molecules per unit volume is increased, and thus the collision frequency is increased.

The effect of ψ on the temperature is accessible in figures 5. In case of heat generation, temperature is nearly linear for lower values of ψ. Increased value of ψ gives the nonlinear profile. The profile has point of intersection at the middle of the channel at $y=0$

Concentration profiles on the basis of α are shown in the figure 6. For $\alpha=0.5$ the concentration profiles is nearly linear and further increase in α decrease the profile.

Figure 7 displays the influence of ψ on the temperature profile. For the case of isoflux – isothermal walls $R_{qt} = 0$ it is noticed that the temperature at the wall with constant heat flux decreases as the heat generation parameter increases.

Velocity profile for isoflux – isothermal case is considered in figure 8. The effect of ψ is significant and increase in value of ψ decreases the flow field for positive value of thermal buoyancy constant and flow field increases for the negative value. This is due to the first order reaction in which increased molecular collision takes place.

Figure 9 demonstrate the influence of ψ on the temperature profiles for the case of isothermal – isoflux walls for $R_{tq} = 0$ .It is observed that the temperature at the wall with constant heat flux increases as the heat generation parameter increaeses.

Velocity profile for isothermal – isoflux case is well thought-out in figure 10. The effect of ψ is similar to isoflux- isothermal case.

Nomenclature

$U \rightarrow$ Dimensional velocity along the X -axis

$u \rightarrow$ Dimensionless velocity along X-axis

$\mathrm{Re} \rightarrow$ Reynolds Number

$Gr_T \rightarrow$ Thermal Grashof constant

$\rho_0 \rightarrow$ Fluid density

$T_0 \rightarrow$ Reference Temperature

$T_2 \rightarrow$ Temperature at right wall

$C_2 \rightarrow$ Concentration at right wall

$\beta_C \rightarrow$ Thermal expansion coefficient

$\phi \rightarrow$ Dimensionless concentration

$U_0 \rightarrow$ Reference velocity

$H \rightarrow$ Hydraulic diameter

$\mu \rightarrow$ Dynamic viscosity

$Gr_C \rightarrow$ Concentration Grashof constant

$C_0 \rightarrow$ Reference concentration

$T_1 \rightarrow$ Temperature at left wall

$C_1 \rightarrow$ Concentration at left wall

$\beta_T \rightarrow$ Thermal expansion coefficient

$\theta \rightarrow$ Dimensionless temperature

$g \rightarrow$ Acceleration due to gravity

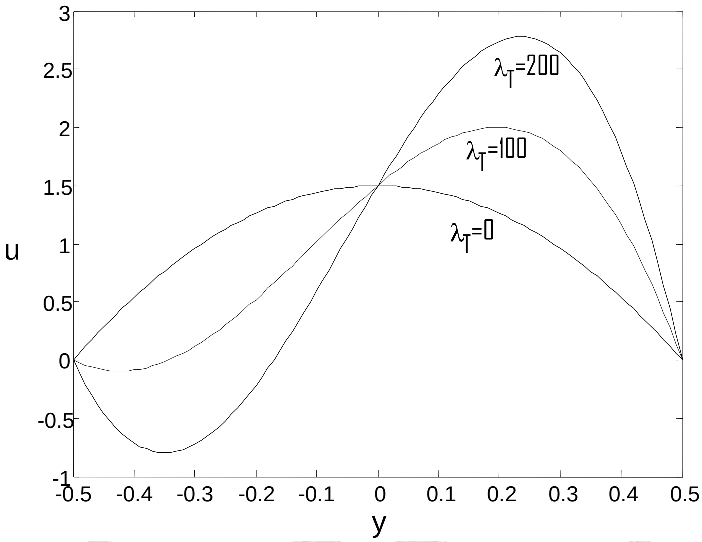

Fig2: Velocity profiles for different values of λ_T

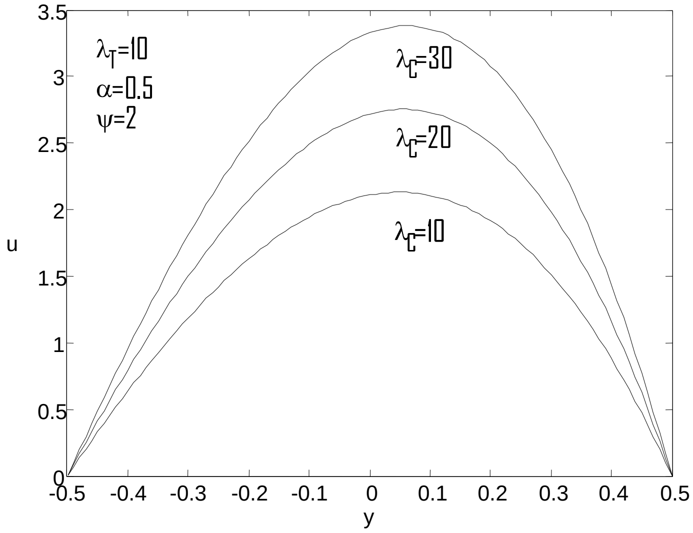

Fig 3: Velocity profiles for different values of λ_C

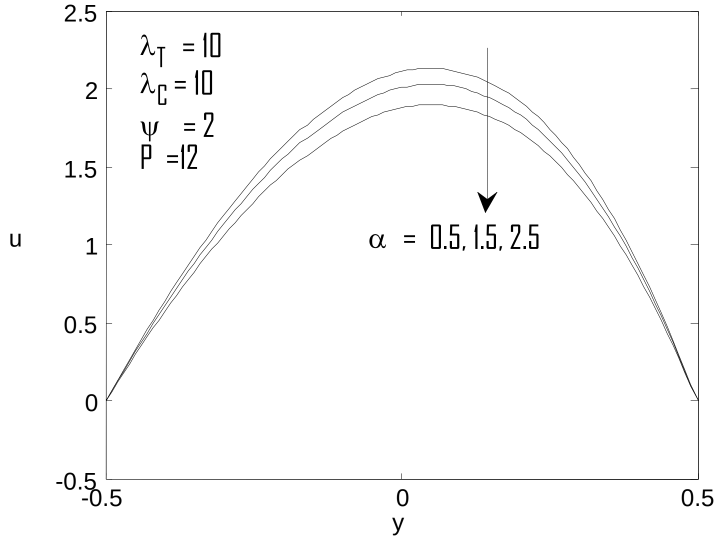

Fig4: Velocity profiles for different values of α

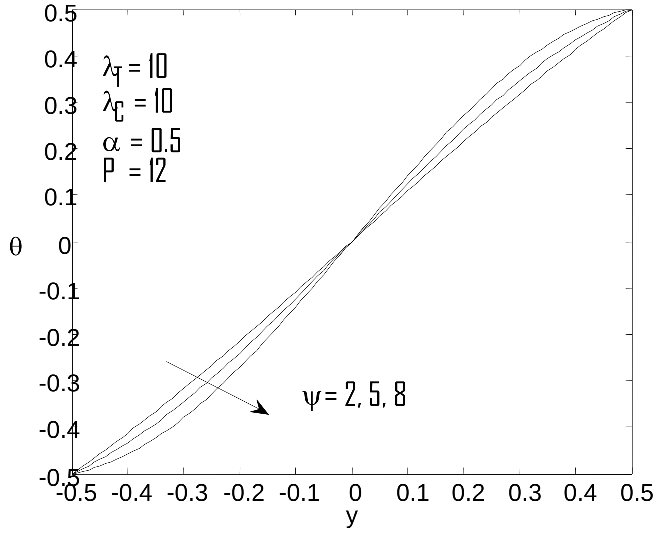

Fig 5: Temperature profiles for different values of ψ

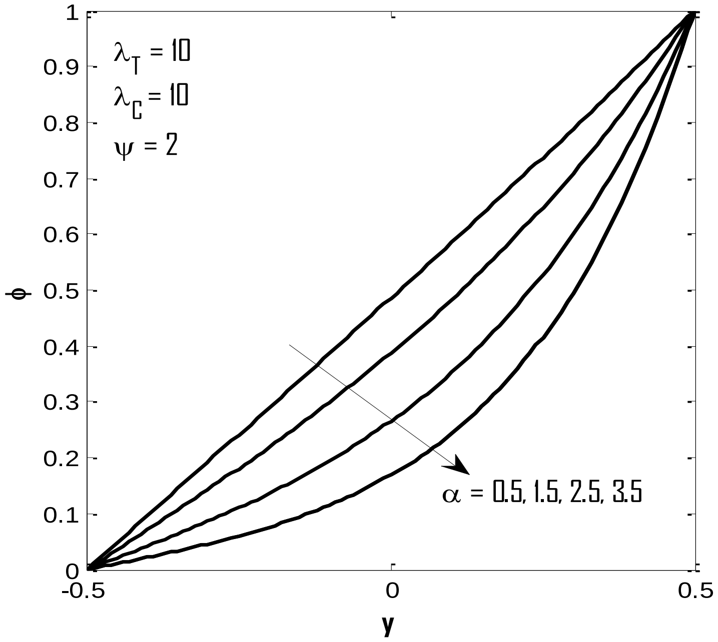

Fig 6: Concentration profiles for different values α

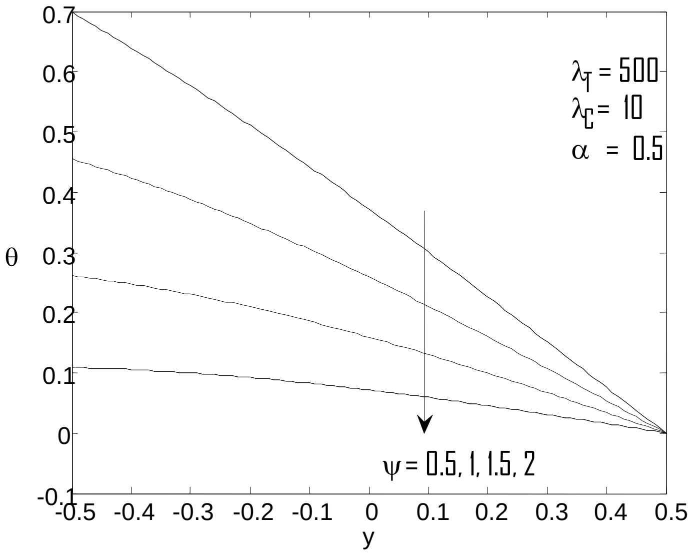

Fig 7 Temperature profiles for Isoflux-Isothermal case

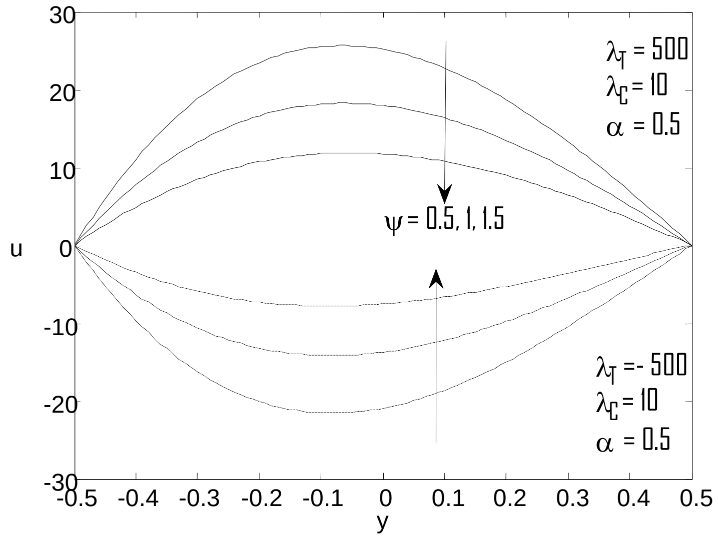

Fig 8 Velocity profiles for Isoflux- Isothermal case

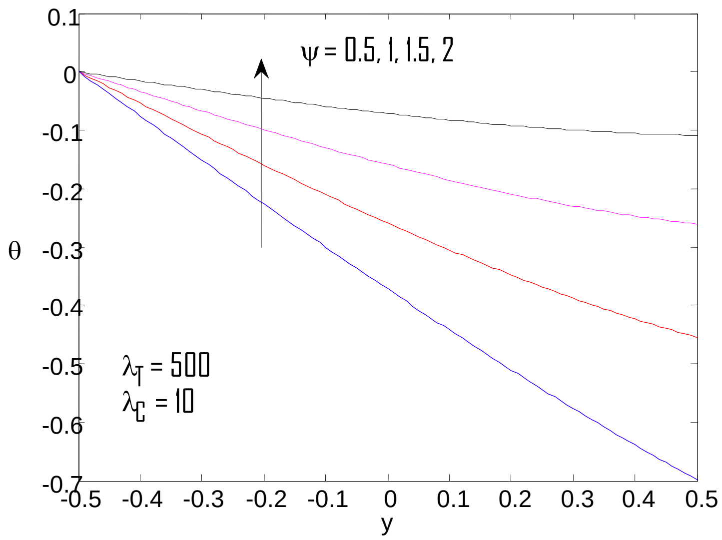

Fig 9 Temperature profiles for Isothermal – Isoflux case.

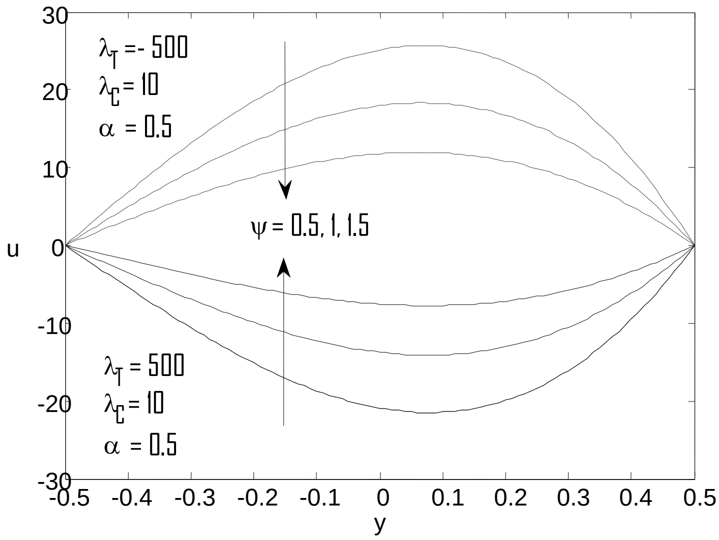

Fig 10 Velocity profiles for Isothermal – Isoflux cases.

5 Conclusions

The problem of fully developed mixed convection through a vertical channel in the presence of heat generation/absorption with first order chemical reaction is analyzed by finite element method and following conclusions are drawn.

1) Thermal buoyancy constant increases the velocity field with flow reversal.
2) Concentration buoyancy number enhances the velocity flow field.
3) Heat generation/absorption parameter increases the velocity flow field.
4) The effect of concentration parameter is that it decreases the flow field.
5) Increase in the values of concentration parameter, decreases the concentration profile.

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