Chromatic Number Of Various Central Neighbourly Irregular Chemical Graph among s-block and p-block Elements

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ABSTRACT

In this paper, we find the dominator chromatic number of central Neighbourly Irregular Chemical tadpole graph and central Neighbourly Irregular Chemical graph. We could derive some Dominator Coloring of Central Neighbourly Irregular Chemical Graphs with the molecular structure which is derived only among the p-block Elements in the area of Inorganic Chemistry. Also these parameters are compared withthe dominator chromatic number of their respective graph families.

KEYWORDS

Regular graph, Irregular graph, Chromatic number of Central Neighbourly Irregular Chemical graph, Color Classes, Tadpole graph,

INTRODUCTION

We concerned with finite, undirected, connected graph G with vertex set and edge set. If v_i and v_j are vertices of G, then the edge connecting them will be denoted by v_iv_j. A graph is said to be regular if all its vertices have same degree. A connected graph G is said to be highly irregular if each neighbor has different degree.

The graph G is said to be Neighbourly irregular graph abbreviated as NI graph, if no two adjacent vertices of G have the same degree. This concept was introduced by Gnana Bhragasam and Ayyasamy who constructed NI graph.

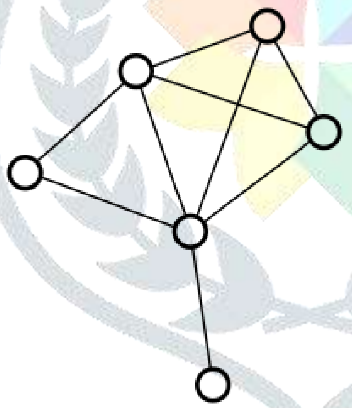
In this paper, A path graph is a graph whose vertices can be listed in the order $v_1, v_2, v_3,\ldots, v_n$ such that the edges are $\{ v_iv_j\}$ where $i = 1,2,3,\ldots,n-1$. A Tadpole graph C_n is a graph n vertices containing a through all the vertices.

Chemical term	Mathematical (graph-theoretical) term
Atom	Vertex
Molecule	Molecular graph
Covalent bond	Edge
Acyclic hydrocarbon	Tree
Alternant structure	bipartite Graph
Valence of a atom	Vertex degree (number of lines at that vertex)
Skeletal structure	Hydrogen-depleted graph
Number of rings	Cyclamate number
[n] Annulene	n-vertex cycle
Huckel theory	Spectral theory
Topological matrix	Adjacency matrix

DEFINITIONS

1. Neighbourly Irregular Graphs.

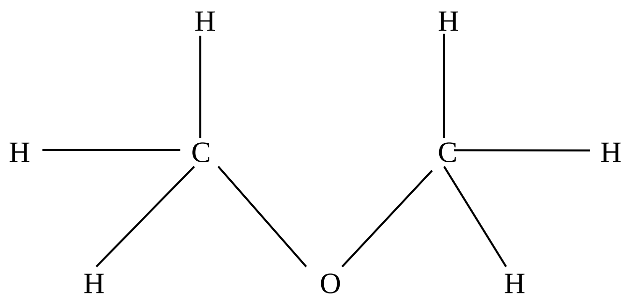
A connected graph G is said to be neighbourly irregular Graph (NIG) if no two adjacent vertices of G have the same degree.



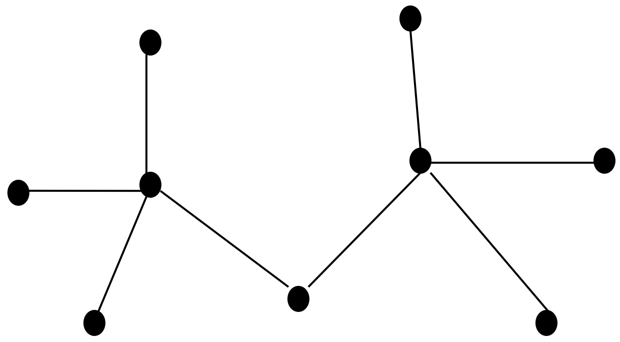
2.NEIGHBOURLY IRREGULAR CHEMICAL GRAPHS (NICG):

A graph is said to be a Neighbourly Irregular Chemical Graph (NICG) for the molecular structure of corresponding element of the atoms has different valency bond in its adjacent atoms.

Eg:



Dietyleter(CH_3OCH_3)



Definition 1.1

Let G be simple and undirected graph and let its vertex set and edge set be denoted by V(G) and E(G) the central graph of G, denoted by C(G) is obtained by subdividing each edge of G exactly once and joing all the non- adjacent vertices of G in C(G).

Definition 1.2

A Proper coloring of a graph G is a an assignment of colors to the vertices of G in such a way that no two adjacent vertices receive the same color. The chromatic number $\chi(G)$, is the minimum number of colors required for a proper coloring of G. A color class is the set of all vertices, having the same color. The color class corresponding to the color i is denoted by V_i.

Definition 1.3

A dominator coloring of a graph G is a proper coloring in which every vertex of G dominates every vertex of at least one color class. The convention is that if {v} is a color class, then V dominates the color class {V}. The dominator chromatic number $\chi_d(G)$ is the minimum number of color required for a dominator coloring of G.

DOMINATOR CHROMATIC NUMBER OF NEIGHBOURLY IRREGULAR CHEMICAL CENTRAL GRAPH (NICCG)

Dominator chromatic number of Neighbourly Irregular Chemical Central Graph (NICCG) of various classes of graphs is obtained in this section.

1.Thearom

For path graph P_n of order $n \leq 2$

$$\chi_d\,[C(P_n)\,] = \begin{cases} \lceil n/2 \rceil + 1 & \text{when n is odd} \\ \lceil n/2 \rceil + 2 & \text{when n is even} \end{cases}$$

Proof:

Let P_n be a path of order $n \leq 3$ and let $V(P_n) = \{ v_1, v_2, v_3, \ldots\ldots v_n \}$. The (NICCG) Nighbour Irregular Chemical Central Graph $C(P_n)$ is obtained by subdividing each $v_i v_{i+1}$, $1 \leq i \leq n-1$ of P_n exactly once by adding a new vertex c_i in $C(P_n)$ and connected each vertex v_j, $1 \leq j \leq n-2$ with each vertex v_k, $j+2 \leq k \leq n$. Let $V_1 \cup V_2$. Relabel the vertices of $C(P_n)$ by $w_1 = v_1$, $w_2 = c_1$, $w_3 = v_2, \ldots\ldots\ldots u_{2n-1}$ consecutively.

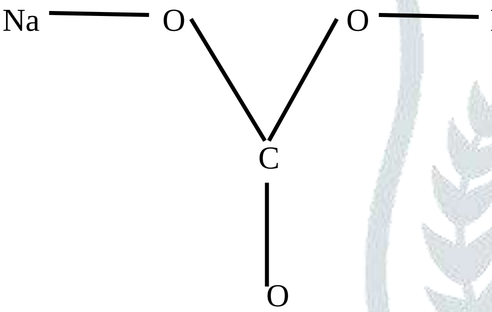
The procedure following given a dominator coloring of $C(P_n)$ when $n \le 6$, the vertex w_1 is colored by color 1 , w_2 is colored by color 2 and the vertex w_i , i= 4,6,8…….2n-2 is colored by color 1. If n is odd the vertex w_i, i= 5,9,13,……..2n-1 is colored by color 2 and the vertices w_j, j=3,7,11….2n-3 are respectively colored by individual colors 3,4,5…………$\lceil n/2 \rceil$ + 1. If n is even the vertex w_i , i=5,9,13,……..2n-3 is colored by color 2 and the vertices u_j, j= 3,7,11……..2n-1 are colored respectively by individual colors 3,4,5,……….$\lceil n/2 \rceil$ +2. When n=3,4 or 5, the vertices of $C(P_n)$ are colored by the color sequences (1,2,1,3,2), (1,2,3,1,2,4,2) or (1,2,3,1,2,1,4,2,1) in order to get a dominator coloring.

When $n \ge 6$, the vertex w_1 dominates the color class of w_7. If n is odd, vertices u_i and w_{1+2},

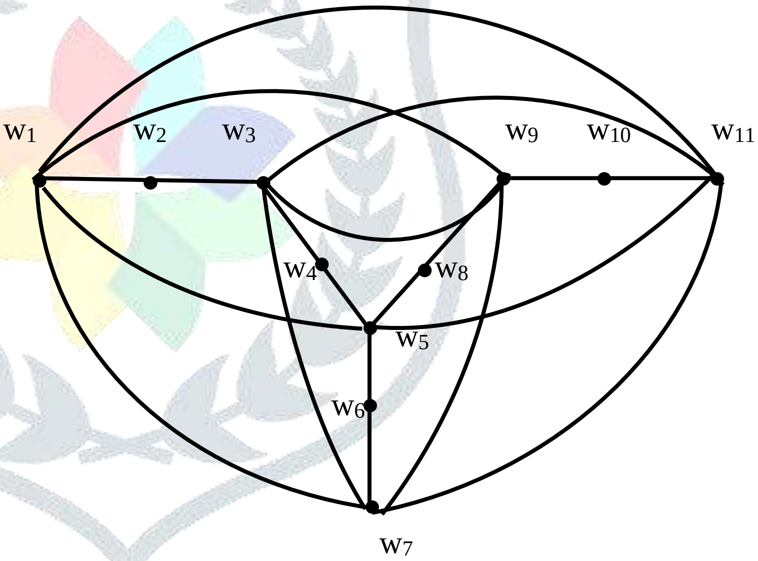
i=2,6,10,14……2n-3 dominate the color class of w_{i+1}. The vertex w_j, for j=3,7,11,…..2n-3 dominates itself. If n is even, vertices w_i and w_{i+2}, i=2,6,10,14,….2n-6 dominate the color class of u_{i+1} and u_{2n-2} dominates the color class of w_{2n-1}. The vertex w_j, for j=3,7,11,….2n-3 dominates itself. The vertex w_j, for j=5,9,13,…2n-1 dominate either w_{j+6}, w_{j-6} or both.

$$\text{Hence } \chi_d[C(P_n)] = \begin{cases} \lceil n/2 \rceil + 1 & \text{when n is odd} \\ \lceil n/2 \rceil + 2 & \text{when n is even} \end{cases}$$

The following example illustrate the procedure discussed in the above result.



Sodium Bicarbanate ($NaHCO_3$)



The color classes of $C(P_6)$ are

$v_1 = \{w_1, w_4, w_6, w_8, w_{10}\}$, $v_2=\{w_2, w_5, w_9\}$, $v_3=\{w_3\}$, $v_4= \{w_7\}$, $v_5 = \{w_{11}\}$ dominator chromatic number is $\chi_d[C(P_6)] = 5$.

NEIGHBOURLY IRREGULAR CHEMICAL DOMINATOR CHROMATIC NUMBER OF CENTRAL AND MIDDLE GRAPH OF TADPOLE GRAPH

Theorem

For the Neighbourly Irregular Chemical Tadpole Graph $T_{m,n}$ where n=1and $m \ge 3$,then

$$\chi_d[C(T_{m,1})] = \begin{cases} \left\lfloor \frac{m}{2} \right\rfloor + 2 & \text{when m is odd} \\ \frac{m+2}{2}, & \text{When m is even} \end{cases}$$

Proof:

Let $T_{(m,3)}$ be the tadpole graph with connected the cycle C_m and path P_n with m=5, n=3.

Let $V(T_{m,3})$ = $\{w_1,w_2,w_3,\ldots..w_n\}$. By the definition of central graph $C(T_{m,3})$ is obtained by subdividing the each edge $v_i v_{i+1}$, $1 \leq i \leq n-1$ of $T_{m,3}$ exactly once by adding a new vertex c_i in $C(T_{m,3})$ and joining each vertex w_j with each vertex w_k , $j+2\leq k\leq n$.

Let $V_1 = \{w_1,w_2,w_3,\ldots..v_n\}$ and $V_2= \{c_1,c_2,c_3,\ldots\ldots c_n\}$. Then $V[C(T_{m,1})] = V_i U\ C_j U\ V_{i+1}$ for $1\leq j\leq n-1$, $1\leq j\leq n$.

By applying the definition of dominator coloring of $C(T_{m,3})$ is as follows. Let $\chi_d[C(T_{m,3})]$ be the dominator chromatic number of the central graph of tadpole graph.

Case(i):

If m is odd

Suppose m=5, the graph is defined as $T_{5,3}$. Let us consists vertices $w_1,w_2,w_3\ldots\ldots\ldots w_4$ and the newly added vertices c_1,c_2,c_3,c_4. Let S= $\{w_1,w_2,w_3\}$ be the dominating set of $T_{5,3}$.

The assign a dominator coloring to the graph $C(T_{5,3})$. Let us assign the color 1 to the vertices $\{w_1,w_2,w_3\}$ of the dominating set S and assign the minimum color classes from 2,3,….n to the remaining non-adjacent vertices of the graph.

The definition of dominator coloring all the vertices in the set S dominates the color class of $\{w_1,w_2,w_3\}$ dominates all the color classes.

Therefore the dominator chromatic number of $C(T_{5,3})$ is 5.

By proceeding this way for order m.

we get the successive sequence of dominator chromatic number $\left\lfloor \frac{m}{2} \right\rfloor$ +2 colors for dominator coloring in $C(T_{5,3})$.

Therefore the dominator chromatic number of $C(T_{m,3})$ is $\left\lfloor \frac{m}{2} \right\rfloor$ +2, when m is odd.

Case (ii) :

If m is even

Suppose m=6, the graph id defined as $T_{6,6}$. Let us consists vertices w_1,w_2,w_3,w_4,w_5,w_6 and the newly added vertices $c_1,c_2,c_3,c_4,c_5,c_6,c_7$. Let $S=\{v_1,v_2,c_3,c_4,c_5,c_7\}$ be the dominating set of $C(T_{6,6})$.

Now assign the dominator coloring to the graph $C(T_{6,6})$.

Let us assign the color 1 to the vertices $\{w_1,w_2,c_3,c_4,c_5,c_7\}$ of the dominating set S and assign the minimum color classes from 2,3,……….n to the remaining non-adjacent vertices of the graph.

By the definition of dominator coloring all the vertices in the Set S Dominates the color class of $\{w_1,w_2,c_3,c_4,c_5,c_7\}$ but $\{w_1,w_2,c_3,c_4,c_5,c_7\}$ dominates all the color classes. Therefore the dominator chromatic number of $C(T_{6,6})$ is 6.

By Proceeding this way for order m. we get the successive sequence of dominator chromatic number $\frac{m+2}{2}$ colors for dominator coloring in $C(T_{m,6})$.

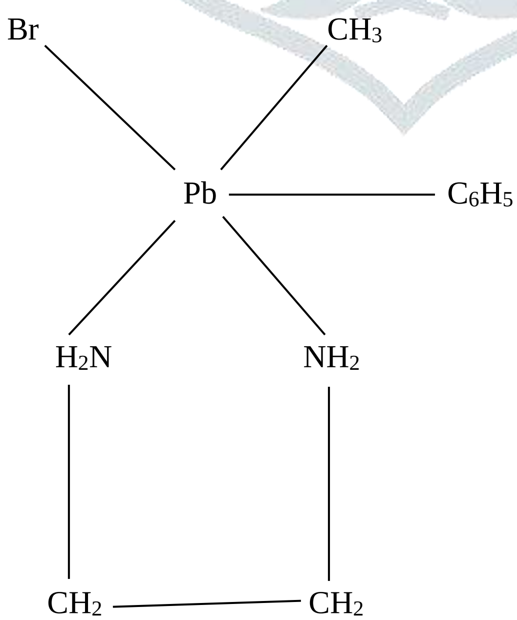
Therefore the dominator chromatic number of $C(T_{m,1})$ is $\frac{m+2}{2}$, when m is even. Hence

$$\chi_d\,[C(T_{m,6})] = \begin{cases} \left\lfloor \frac{m}{2} \right\rfloor + 2, \text{ when m is odd} \\ \left\lfloor \frac{m+2}{2} \right\rfloor, \quad \text{when m is even.} \end{cases} \text{, where n = 6}$$

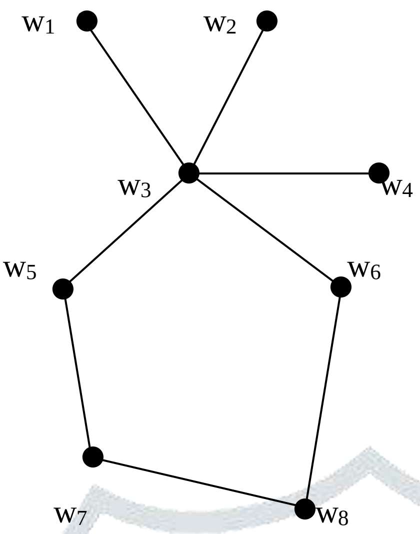
Example:

In figure, Tadople graph of $T_{5,3}$ and its central is depicted with a dominator coloring.

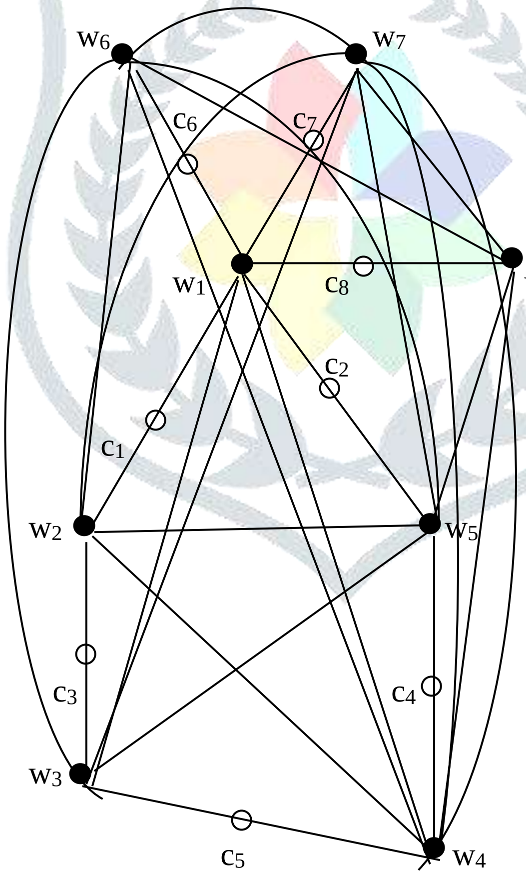
For $C(T_{5,3})$ when m is odd.



$(NH_4)_2 [Pd (SCH)_4]$



(a) $T_{(5,3)}$



(b) C $(T_{(5,3)}$

the color classes of $C(T_{5,3})$

$V_1 = 1 = \{w_1, w_2, w_3, w_4, w_5\}$,

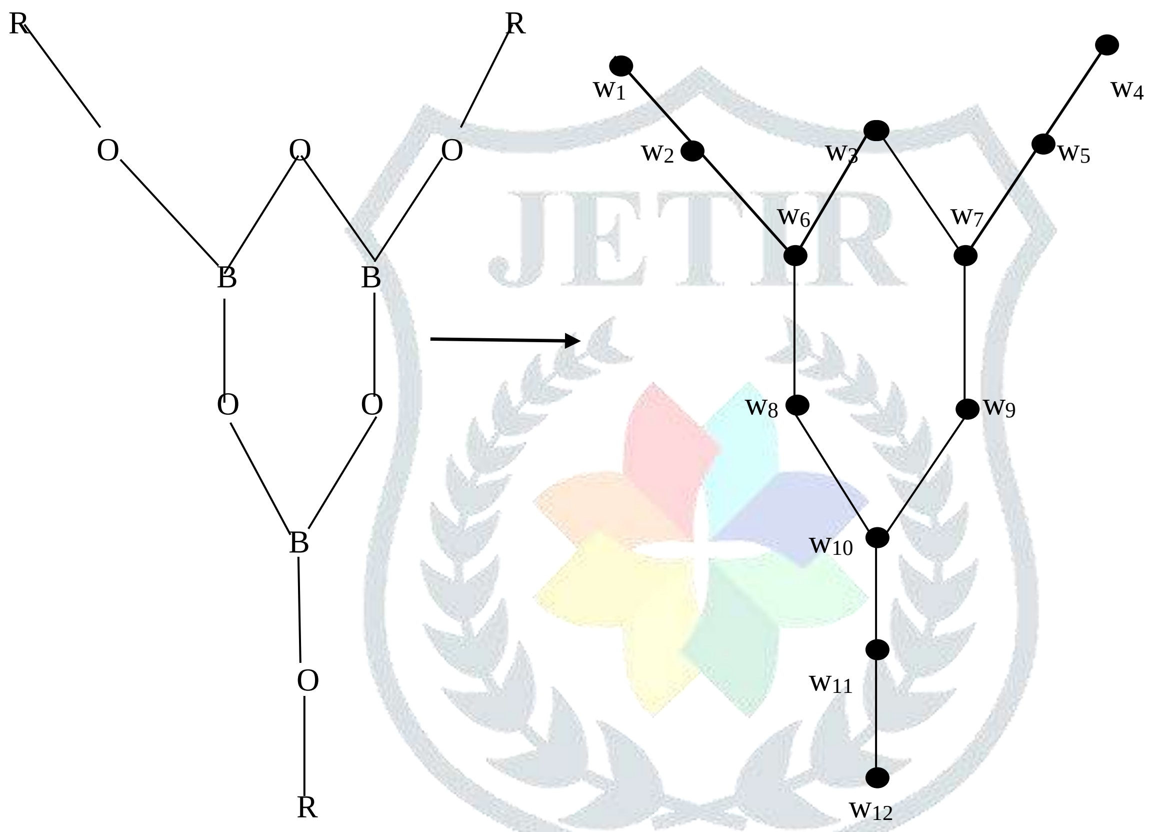
$V_2 = 2 = \{c_1, c_2, c_3, c_4, c_5\}$,

$V_3 = 3 = \{w_4, w_7, w_8\}$,

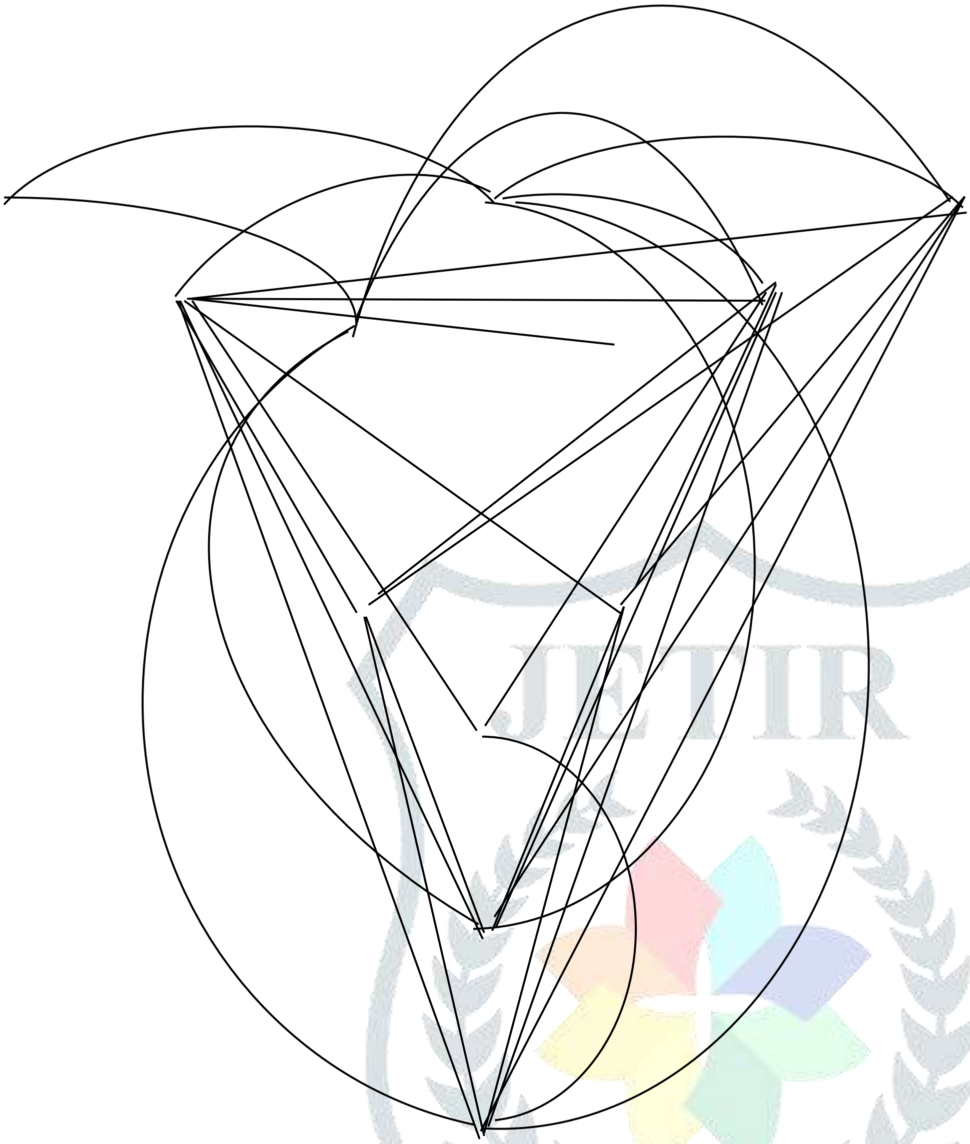
$V_4 = 4 = \{c_6, c_7, c_8\}$.

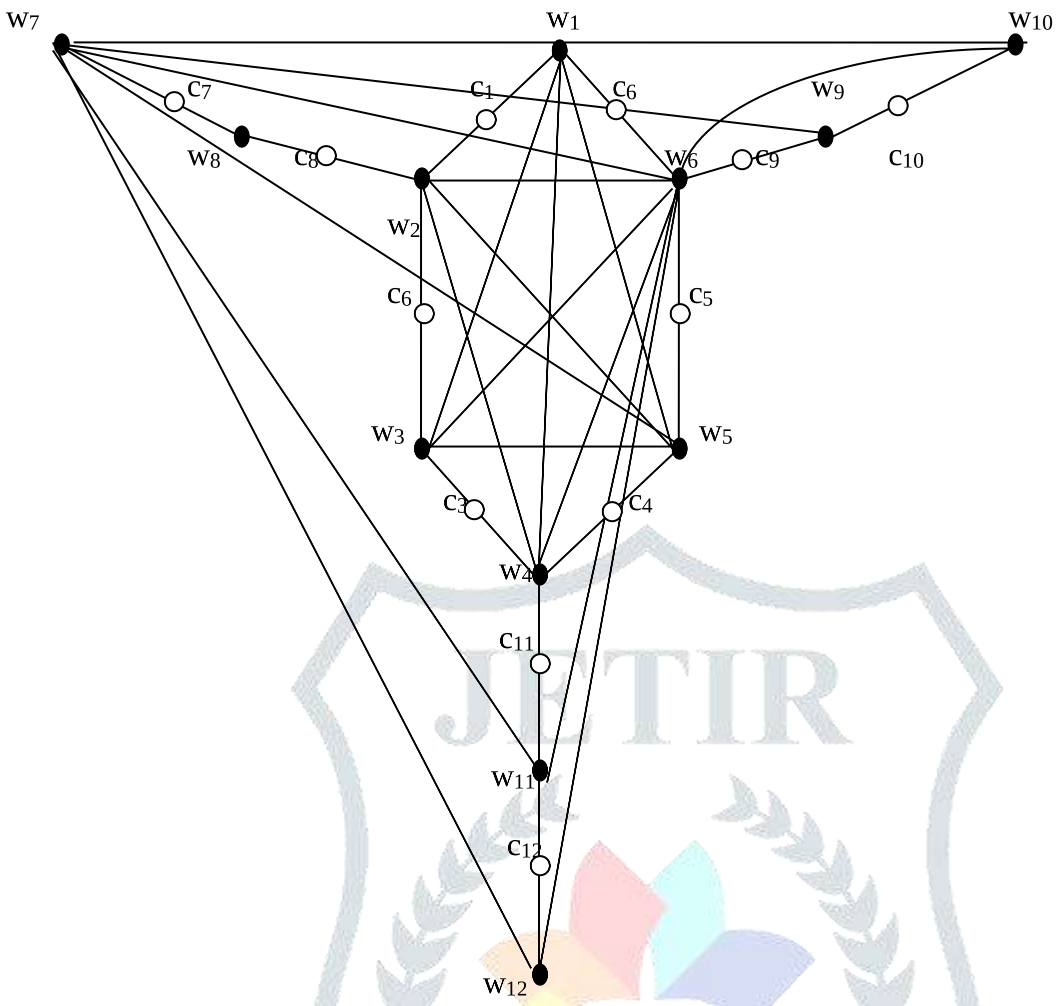
The dominator chromatic number is, therefore ,$\chi_d\, [T_{3,1}] = 3$.

In figure, tadpole graph of $T_{6,6}$ and its central is depicted with a dominator coloring. For $C(T_{6,6})$ when m is even.



Boric Acid Esters (a) $T_{(6,6)}$





(b) C ($T_{(6,6)}$)

W_1 = 1= { $w_1, w_2, c_3, c_4, c_5, c_9, c_8, c_{10}$ }

W_2 = 2 = {w_4, c_1, c_2, w_6}

W_3 = 3 = { w_5, c_6, w_3}

W_4 = 4 = { w_7, w_{12}, w_{10}}

W_5 = 5 = { w_9, c_{11}, c_{12}, c_7}

W_6 = 6 = {w_8, w_{11}}

CONCLUSION:

In this paper we have constructer " chromatic number of various central Neighbourly Irregular Chemical Graph among s-block and p-block Elements " in inorganic chemistry.

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