# DOMINATOR CHROMATIC NUMBER OF VARIOUS CENTRAL NEIGHBOURLY IRREGULAR CHEMICAL GRAPH among p-BLOCK ELEMENTS

J. AROCKIA ARULDOSS
Assistant professor,
Department of Mathematics,
St. Joseph's college of Arts and Science ( Autonomous )
Cuddalore-1

M.ARUNAMBIGAI
Research Scholor
Department of Mathematics
St. Joseph's college of Arts and Science ( Autonomous )
Cuddalore-1

## ABSTRACT

In this paper, we find the dominator chromatic number for central Neighbourly Irregular Chemical cycle graph and central Neighbourly Irregular Chemical path graph. We could derive some Dominator Chromatic of Central Neighbourly Irregular Chemical Graphs with the molecular structure which is derived only among the p-block Elements in the area of Inorganic Chemistry. Also these parameters are compared with the dominator chromatic number of their respective graph families.

## KEYWORDS

Regular graph, Irregular graph, NIC Graph, Chromatic number of Central NIC graph, Proper coloring and Dominator coloring.

## INTRODUCTION

We concerned with finite, undirected, connected graph G with vertex set V( G ) = { $v_1$, $v_2$, $v_3$,…,$v_n$} and edge set E( G ) = { $e_1$,$e_2$,$e_3$,…,$e_m$}. If $v_i$ and $v_j$ are vertices of G, then the edge connecting them will be denoted by $v_iv_j$. A graph is said to be regular if all its vertices have same degree. A connected graph G is said to be highly irregular if each neighbor has different degree.The graph G is said to be Neighbourly irregular graph abbreviated as NI graph, if no two adjacent vertices of G have the same degree. This concept was introduced by Gnaana Bhragasam and Ayyasamy who constructed NI graph.

In this paper, A path graph is a graph whose vertices can be listed in the order $v_1$, $v_2$, $v_3$,…, $v_n$ such that the edges are { $v_iv_j$ } where i = 1,2,3,…,n-1. A cycle graph $C_n$ is a graph n vertices containing a single cycle through all the vertices

| Chemical Term | Mathematical( Graph -theoretical) Term |
|---|---|
| Atom | Vertex |
| Molecule | Molecular graph |
| Covalent bond | Edge |
| Valency of a atom | Degree of the vertex |

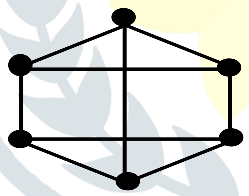
**Definition 1.1**

Let G be a simple and Undirected graph and let its vertex and edge set be denoted by V(G) and E(G).

**Definition 1.2**

In graph theory, a regular graph is a graph where every vertex has same number of degrees of valancy. A regular directed graph must be satisfy the stronger condition of that the indegree and outdegree of each vertex are equal to each other.

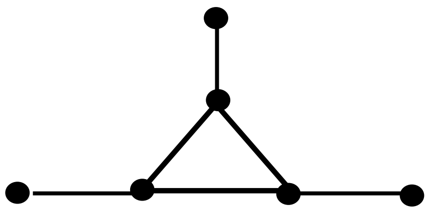
Example:



Definition 1.3

A graph G is said to be Irregular if for each vertex V of G the Neighbours of V having distinct degree.

Example:



Definition 1.4

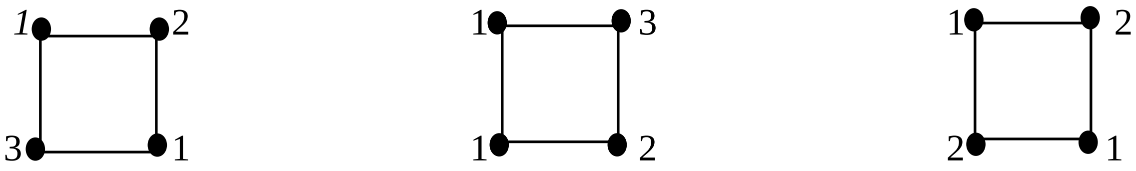
The Central graph of G denoted by C(G) is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G in C(G).

Definition 1.5

A proper (vertex) coloring of a simple graph G(v) is defined as a vertex coloring from a set of colors such that no two adjacent vertices share a common color.

$F: V(G) \rightarrow \{1,2,3,\ldots,k\}$

*Eaxmples:*




a proper coloring with 3 colors | not a proper coloring | a proper coloring with 2 colors

Definition 1.6

The chromatic number $\chi(G)$ of a graph is the smallest number of colors needed to color the vertices so that no two adjacent vertices sharing the same color [skiena 1990, P210].i.e) The smallest value of possible to obtain a k-coloring and is often denoted $\chi(G)$.

A subset of vertices assigned to the same color is called a color class, every such class forms an independent set. The color class corresponding to color I is denoted by Vi.

Examples:


$P_3$ is 3 colourable | $P_3$ is 2 colourable

Therefore, $\chi(G) = 2$

**SOME BASIC FACTS:**

* If G has n vertices, then $\chi(G) \leq n$.

* $\chi(G) = 1 \Leftrightarrow$ G has no edges.

* $\chi(C_{2n}) = 2$ and $\chi(C_{2n+1}) = 3$.

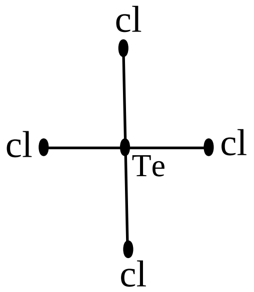
* $\chi(k_n) = n$.

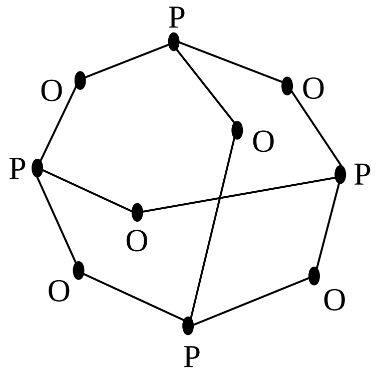
* If H is a subgraph ogf g then $\chi(G) \geq \chi(H)$.

**Definition 1.7**

A graph is said to be a Neighbourly Irregular Chemical Graph(NIC Graph) if molecular structure of corresponding element of the atoms has different valency bond in its adjacent atoms.

Examples:




$TeCl_4$ ( Tellurium Chloride ) $P_4O_6$ ( Tetra phosphorus Hexaoxide )

## 2. DOMINATOR CHROMATIC NUMBER OF CENTRAL NIC GRAPH

Dominator chromatic of central NIC graph of various classes is obtained in this section.

### Theorem 2.1

For cycle graph $c_n$ of order $n \geq 3$,

$$\chi_d [C(c_n)] = \begin{cases} [2n/3]+2 & \text{when } n=3 \\ [2n/3]+1 & \text{otherwise.} \end{cases}$$

### Proof:

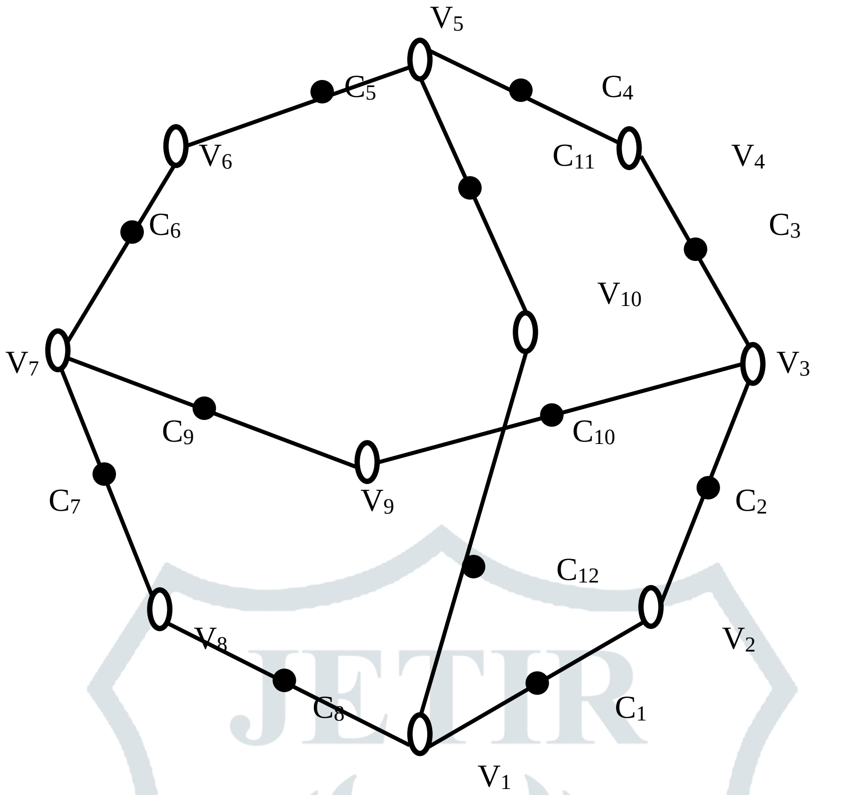
Let $c_n$ be the Neighbourly Irregular Graph of $P_4O_6$ which is order of $n \geq 3$ and

Let $V(c_n) = \{ v_1, v_2, \ldots, v_n \}$ and $E(c_n) = \{ e_1, e_2, e_3, \ldots, e_n \}$

where $e_i = v_i v_{i+1}$, $1 \leq I \leq n-1$ and $e_n = v_n v_1$.

By the definition of Central NIC Graph, $C(c_n)$ is obtained by subdividing each covalent bond of $v_i v_{i+1}$, $1 \leq i \leq n-1$ of $C_n$ exactly once by newly added atom of $C_i$, and subdividing $v_n v_1$ by $C_n$ and joining $v_i$ with v, $1 \leq i, j \leq n$, $I \neq j$ and

$v_i v_j \neq E(C_n)$.




Central graph of NIC on $P_4O_6$( Tetra Phosphorus Hexaoxide )

Let $V_1 = \{ v_1,v_2,v_3,\dots,v_n\}$ and $V_2 = \{ c_1,c_2,c_3,\dots,c_n \}$ then $V(C(c_n)) = V_1 \cup V_2$.

This above procedure gives a dominator coloring of $C(c_n)$. Therefore all the newly added atoms $c_i$, $1\leq i \leq n$ in $C(c_n)$ forms an independent set, color these atom of the molecular graph by color 1.

When n = 3k, k ≥ 2, the atoms $v_i$ and $v_{i+1}$, i = 1,4,7,…,(n-2) are colored by color 2[i/3] and the remaining atoms $v_j$, j = 3,6,9,…,n are colored by 1+(2j/3) when n = 3k-1, k ≥ 2, the atoms $v_i$ and $v_{i+1}$, i = 1,4,7,…,(n-4) are colored by color 2[i/3] and the remaining atoms $v_j$, j = 3,6,9,…,(n-2) are colored by color i+(2j/3) and $v_{n-1}$ and $v_n$ are colored by [ 2n/3] and [ 2n/3] + 1.

When n = 3k+1, k ≥ 2, the atoms of the molecular $v_i$ and $v_{i+1}$, i = 1,4,7,…,(n-3) are colored by color 2[i/3] and the remaining atoms $v_j$, j = 3,6,9,….,n-1are colored by 1+(2j/3) for and $v_n$ is colored by [ 2n/3]+1. When n =. When n = 3., the atoms $v_i$ is colored by color i+1, $1\leq i \leq 3$. When n = 4, the atoms of $v_i$ are colored by the color sequence( 2,2,3,4)`

The vertices of the atom $v_i$ and $v_{i+1}$, i = 1,4,7,…, dominates one of the color classes of $v_j$,

j = 3,6,9,… the atoms $v_j$, j = 3,6,9,… dominates itself. The centre atoms $c_i$, i = 1,4,7,… dominates the color class of $v_i$. The centre atoms $c_i$ and $c_{i+1}$, i = 2,5,8,… dominates the color class of $v_{i+1}$, as they are adjacent to $v_{i+1}$.

When n = 3, it is easy to see that $\chi_d$ [C(C3)] = [2n/3]+2 and when n=4, it is seen that $\chi_d$ [C(c4)] = [2n/3]+1.

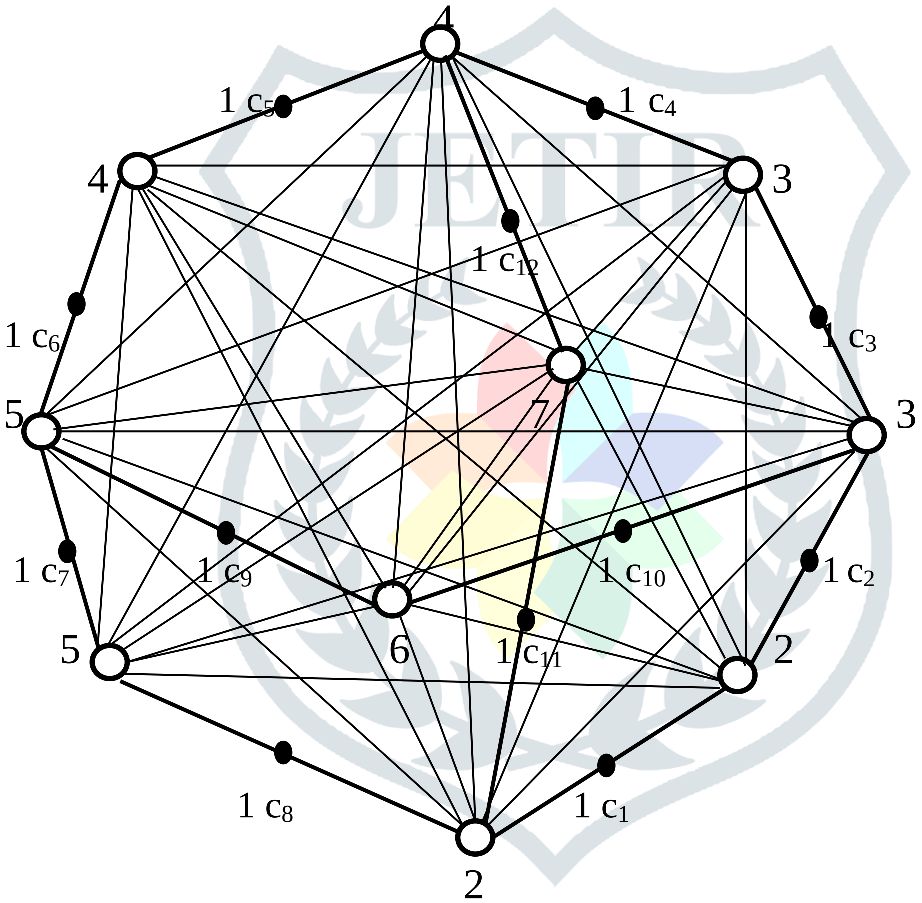
Hence,

$$\chi_d[\, C\,(C_n)\,] = \begin{cases} [2n/3]+2 & \text{when } n=3 \\ [2n/3]+1 & \text{otherwise} \end{cases}$$

The following illustration of the example of Central Neighbourly Irregular Chemical Graph would discussed in the above result.

Example: 2.1

In this figure Central of Neighbourly Irregular Chemical Graph of $P_4O_6$( Tetra Phosphorus Hexaoxide ) $C_{10}$ is depicted with a dominator colouring.




Chromatic Number of NIC on $P_4O_6$( Tetra Phosphorus Hexaoxide )

The color classes of $c_7$ are

$V_1 = \{ c_1,c_2,c_3,c_4,c_5,c_6,c_7,c_8,c_9,c_{10},c_{11},c_{12} \}$,

$V_2 = \{ v_1,v_2 \}$, $V_3 = \{ v_3,v_4 \}$, $V_4 = \{ v_5,v_6 \}$,

$V_5 = \{ v_7,v_8 \}$, $V_6 = \{ v_9,v_{10}\}$, $V_7 = \{ v_{10} \}$.

Therefore, the dominator chromatic number is $\chi_d[\, C(\, c_{10})] = 7$.

## Theorem: 2.2

For Neighbourly Irregular Chemical Path Graph Pn of order n≥ 2,

$$\chi_d[ C (p_n) ] = \begin{cases} [ n\backslash 2 ] + 1 \text{ when n is odd.} \\ [ n\backslash 2 ] + 2 \text{ when n is even.} \end{cases}$$

Proof:

Let $P_n$ be a path of order n≥3 and l

Let V( $P_n$ ) = { $v_1$, $v_2$, $v_3$,…, $v_n$ } the central NIC graph C($P_n$) is obtained by subdividing each covalent bond of $v_iv_{i+1}$, $1\leq i \leq n-1$ of $P_n$ exactly one by adding new atom $c_i$ in C( $P_n$ ) and joining each atom $v_j$, $1\leq i \leq n-2$ with each atom $v_k$, j+2≤ $k \leq n$.

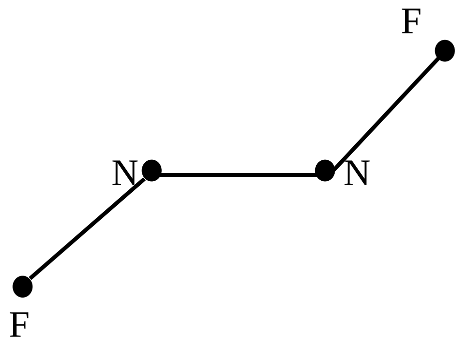
Let $V_1$ = { $v_1$, $v_2$, $v_3$,…, $v_n$} and $V_2$ = { $c_1$, $c_2$, $c_3$,… $c_{n-1}$ }. Then V(C($P_n$)) = $V_1 \cup V_2$ Relable the atom of C($P_n$) by $u_1 = v_1$ , $u_2 = c_1$, $u_3 = v_2$,…,$u_{2n-1}$ alternatively.

For the following procedure gives a Dominator Coloring of C($P_n$). When

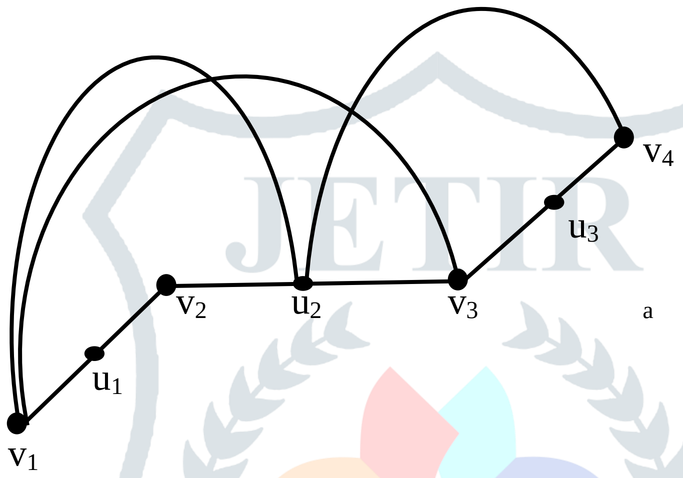
n ≥ 4, the atom $u_1$ is colored by color 1, atom $u_2$ is colored by a color 2 and the vertex $u_i$ i = 4, 6, 8,…, 2n-2 is colored by color 1.

If n is odd, the atom $u_i$,i = 5, 9, 13,…, 2n-1is colored by color 2 and the vertices $u_j$, j = 3, 7, 11,…, 2n-3 are respectively colored by invidual color 3, 4, 5, …, [ n/2 ] +1. If n is even, the atom $u_i$, i = 5, 9, 13,…, 2n-3 is colored by color 2 and the atoms $u_j$, j = 3, 7, 11,…, 2n-1 are colored respectively by individual colors 3, 4, 5, …, [n/2]+2. When n = 3, 4, or 5, the atoms of C($P_n$) are colored by the color sequences (1, 2, 1, 3, 2) (1, 2, 3, 1, 2, 4, 2, 1) in order to get a dominator coloring.When n ≥ 4, the atom $u_1$ dominates the color class of $u_7$.

If n is odd, atom $u_i$ and $u_{i+2}$, i = 2, 6, 10, 14, …, 2n-3 dominate the color class of $u_{i+1}$. The atom $u_j$, for j = 3, 7, 11, …,2n-3 dominates itself. If n is even, atom $u_i$ and $u_{i+2}$, i = 2, 6, 10, 14, …, 2n-6 dominate the color class of $u_{i+1}$ and $u_{2n-2}$ dominates the color class of $u_{2n-1}$. The atom $u_j$ for j = 3, 7, 11, …, 2n-3 dominates itself. The atom $u_j$, for j = 5, 9, 13, …, 2n-1 dominates either $u_{j+6}$, $u_{j-6}$ or both.




$N_2F_2$( Nitrogen Floride )




Molecular of the Central graph of $P_4$ is depicted with dominator coloring

The color classes of $c(P_4)$ are

$V_1 = \{ u_1, u_2, u_3 \}$, $V_2 = \{ v_1, v_2 \}$, $V_3 = \{ v_3 \}$, $V_4 = \{ v_4 \}$

The dominator chromatic number,

$\chi_d\, [C(p_4)] = [\ n/2\ ] + 2$ when n is even.

$= [\ 4/2\ ] + 2$

$= 2 + 2$

$= 4$

Hence,

$\chi_d\, [C(p_n)] =$ $[n/2]+1$when n is odd

$[n/2]+2$ when n is even.

## Some Structural Properties of $C(P_n)$ has

i) $(n+1)$ vertices of degree n.

ii) n vertices of degree 2.

iii) The number of Vertices in $P_c(P_n) = (2n+1)$.

iv) The number of edges in $q_c(P_n) = \frac{n^2+3n}{2}$.

## CONCLUSION

In this paper, We consider the Neighbourly Irregular Chemical Graph only among the p – block elements and further we constructing Dominator Chromatic Number of above graph.

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