Robust Delay-Interval-Dependent Stability Criteria for Neural Networks with Successive Time-Varying Delay Components

[1] S. Rajavel, [1] K. Murugan, [1] B. Geetha

[1]Department of Mathematics, Adhiparasakthi College of Arts and Science, Vellore-632 506, India.

Abstract : In this paper, the problem of delay-interval-dependent stability criteria for uncertain neural networks with successive time-varying delay components are studied. We construct a Lyapunov-Krasovskii function with triple and four integral terms and then utilizing Jenson's inequality technique. Moreover, the proposed sufficient conditions can be simplified into the form of linear matrix inequalities (LMIs) using Matlab LMI toolbox. Finally, a numerical example is presented to illustrate the effectiveness of the proposed criteria.

Keywords: Delay-interval-dependent stability; Neural networks; Successive time-varying delay components; Linear matrix inequality; Lyapunov-krasovskii function.

I. INTRODUCTION

Neural networks have been extensively studied in the past few decades because of their practical importance and have found successful applications in many engineering and scientific areas such as signal processing, pattern recognition, static image processing, fault diagnosis, associative memories, fixed-point computations, combinatorial optimization. Since the key feature of the applications are greatly, dependent on the property, of the equilibrium point of neural networks. For example, when designing a neural networks to solve optimization problems, the neural networks must have one unique and globally stable equilibrium point. A large number of important results on the dynamical behaviors have been reported for delayed neural networks (see [1]-[5] and the references therein).

To analyze the stability of dynamical properties of neural networks, it is sometimes necessary to take account of time delays. In reality, time delay are inevitable in implementing an artificial neural networks as a result of the finite conduction velocity and switching speed of amplifier. Since the existence of the times delays is an important source of oscillation divergence, and instability in a system [6]-[8]. Therefore, the stability of neural networks with time delay has become an important topic in many field. Depending on whether the stability criterion itself contains the size of delay, criterion for delayed neural network can be classified into two types, namely, delay-independent criteria [9]-[11] and delay-dependent criteria [12]-[15]. It is generally known that delay-dependent stability criteria are usually less conservative than delay-independent stability criteria especially for small size delays. Note that the delay-dependent stability results mentioned are based on systems with one single delay in the state. It is also well known that parameter uncertainty which can be commonly encountered because of the inaccuracies and changes in the environment of the model will break the stability of the systems. Recently, the problem on stability analysis of uncertain neural networks with delays has been extensively investigated (see [16, 17] and the references therein).

Among most of the reported results on stability criteria for delayed neural networks, time delays have been in a singular or simple form in the state. Based on this, a new type of neural network model with additive time-varying delay components has been introduced in [18]. This model has a strong application background in remote control and control system. For example, we consider a state-feedback networked control. Because the physical plant, controller, sensor, and actuator are located at different places, signals are transmitted from one device to another. There are essentially two kinds of network-induced delays: one from sensor to con- troller and the other from controller to actuator. Then, the closed loop system will appear with two additive time delays in the state. Thus, in the network transmission settings, two delays are usually time-varying with dissimilar properties. Therefore, the problem of stability analysis of neural networks with two successive time-varying delays in the state has been received much attention in recent years [19]-[25]. In [19] some new delay dependent stability criteria for neural networks with two additive time-varying delay components have been studied by using convex polyhedron method. By constructing a new Lyapunov-Krasovskii functional by using reciprocally convex method and convex polyhedron method, a new approaches on stability criteria for neural networks with two additive time-varying delay components are derived in [20]. Tian et al. [21] proposed improved delay dependent stability criteria for neural networks with two additive time-varying delay components are less conservative because reciprocally convex approach and convex polyhedron approach are considered. Recently, [22, 23] improved stability criteria for generalized neural networks with additive time-varying delay components have been studied. By introducing some zero equations and using free-weighting matrix (FWM) based techniques and reciprocally convex combination based techniques. Very recently, the problem of stability criteria for a class of uncertain neural networks with additive time-varying delay have been derived in the work [24, 25]. Which is the main motivation of this paper.

Motivated by the above discussions, a new delay dependent stability criteria for uncertain neural networks with time-varying delays and leakage delay is proposed in the paper. It is noted that two successive time-varying delay components are taken in the state. By constructing a suitable LyapunovKrasovskii functional with triple and four integral terms and by using Jenson's lemma, a new delay-interval-dependent stability criterion is derived in terms of linear matrix inequalities to ensure the asymptotic stability of the equilibrium point of the considered neural networks. The derived criteria use the information of the upper delay bounds, which may lead to conservativeness.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following neural networks with successive time-varying delays:

$$\dot{x}(t) = -Cx(t) + Ag\big(x(t)\big) + Bg\Big(x\big(t-\tau(t)\big)\Big) + D\int_{t-\sigma(t)}^{t} g\big(x(s)\big)ds + u \qquad (1)$$

Where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathbb{R}^n$ is the state at time; $g\big(x(t)\big) = [g_1\big(x_1(t)\big), g_2\big(x_2(t)\big), \dots, g_n\big(x_n(t)\big)]^T \in \mathbb{R}^n$ denotes the neuron activation function and $u = [u_1, u_2, \dots, u_n]^T \in \mathbb{R}^n$ is constant input vector. $C = diag(c_1, c_2, , \dots, c_n)$ is a diagonal matrix with $c_i > 0$, $i = 1, 2, , \dots, n$ and A, B and D the connection weight matrix, the discrete delayed connection weight matrix and the distributively delayed connection weight matrix: $\tau_1(t)$, $\tau_2(t)$ are continuous time-varying functions that represent the two delay components in the state, $\sigma(t)$ is distributed delay, which satisfy

$$0 \le \tau_1(t) \le \tau_1,\ 0 \le \tau_2(t) \le \tau_2,\ 0 \le \sigma(t) \le \sigma, \quad \dot{\tau}_1(t) \le \mu_1, \quad \dot{\tau}_2(t) \le \mu_2. \qquad (2)$$

where τ_1, τ_2, σ, μ_1 and μ_2 are constants. We denote $\tau(t) = \tau_1(t) + \tau_2(t)$, $\tau = \tau_1 + \tau_2$ and $\mu = \mu_1 + \mu_2$.

In addition, it is assumed that each neuron activation function in (1), $y_i(\ )$, $i = 1, 2, \dots, n$ satisfies the following condition:

$$0 \le \frac{g_i(x) - g_i(y)}{x - y} \le k_i, x, y \in \mathbb{R}, x \ne y, \qquad i = 0,1,2,\dots,n \qquad (3)$$

where k_i, $i = 1, 2, \dots, n$ are positive constants.

In the following, the equilibrium point $x^* = (x_1^* x_2^* \dots x_n^*)^T$ of (1) is shifted to the origin by the transformation $z(\cdot) = x(\cdot) - x^*$ which converts the system to the following form

$$\dot{z}(t) = -Cz(t) + Af\big(z(t)\big) + Bf\Big(z\big(t-\tau(t)\big)\Big) + D\int_{t-\sigma(t)}^{t} f\big(z(s)\big)ds \qquad (4)$$

where $z(\cdot) = [z_1(\cdot), z_2(\cdot), \dots, z_n(\cdot)]^T$ is the state vector of the transformed system, $f(z(\cdot)) = [f_1(z_1(\cdot)), f_2(z_2(\cdot)), \dots, f_n(z_n(\cdot))]^T$ and $f_i(z_i(\cdot)) = g_i(z_i(\cdot) + z_i^*) - g_i(z_i^*)$, $i = 1, 2, \dots, n$. Note that the functions $f_i(\cdot)$, $i = 1, 2, \dots, n$ satisfy the following condition.

$$0 \le \frac{f_i(z_i)}{z_i} \le k_i\, f_i(0)\ \ z_i \ne 0, \qquad i = 0,1,2,\dots,n \qquad (5)$$

which is equivalent to

$$f_i(z_i)[f_i(z_i) - k_i z_i] \le 0, \quad f_i(0) = 0, \quad i = 1, 2, \dots, n. \qquad (6)$$

Lemma 2.1 [26] For a positive matrix M, scalars $h_U > h_L > 0$ such that the following integration are well defined, then it holds that:

$$-(h_U - h_L)\int_{t-h_L}^{t-h_U} x^T(s)Mx(s)ds \le -\left(\int_{t-h_L}^{t-h_U} x(s)ds\right)^T M\left(\int_{t-h_L}^{t-h_U} x(s)ds\right)$$

$$-\frac{h_U^2-h_L^2}{2}\int_{t-h_L}^{t-h_U}\int_s^t x^T(u)Mx(u)duds \le -\left(\int_{t-h_L}^{t-h_U}\int_s^t x(u)duds\right)^T M\left(\int_{t-h_L}^{t-h_U}\int_s^t x(u)duds\right)$$

$$-\frac{h_U^3-h_L^3}{6}\int_{t-h_L}^{t-h_U}\int_s^t\int_u^t x^T(v)\,Mx(v)dvduds \le -\left(\int_{t-h_L}^{t-h_U}\int_s^t\int_u^t x(v)dvduds\right)^T M\left(\int_{t-h_L}^{t-h_U}\int_s^t\int_u^t x(v)dvduds\right)$$

Lemma 2.2 [27] Let H, E, and F(t) be real matrices of appropriate dimensions with F(t) satisfying $F^T(t)F(t) \le I$. Then, for any scalar $\epsilon > 0$.

$$HF(t)E + (HF(t)E)^T \le \epsilon^{-1}HH^T + \epsilon E^T E.$$

Lemma 2.3 [28] Given constant matrices Z_1, Z_2, Z_3 where $Z_1 = Z_1^T$ and $Z_2 = Z_2^T > 0$. Then $Z_1 + Z_3^T Z_2^{-1} Z_3 < 0$ if and only if

$$\begin{bmatrix} Z_1 & Z_3^T \\ Z_3 & -Z_2 \end{bmatrix} > 0 \ (or) \ \begin{bmatrix} -Z_2 & Z_3 \\ Z_3^{-1} & Z_1 \end{bmatrix} > 0.$$

III. MAIN RESULT

In this section, we investigated the stability criterion for neural networks with successive time-varying delay components.

Theorem 3.1 For given scalars τ_1, τ_2, σ, μ_1 and μ_2 then system (4) and time-varying delay satisfying condition (2) is asymptotically stable if there exist $P = P^T > 0, Q_L = Q_L^T \ge 0, (L = 1,2,\dots,6)$ $R_i = R_i^T \ge 0, (i = 1,2,3)$ $S_j = S_j^T \ge 0, (j = 1,2,3)$ $Z = Z^T \ge 0, T_k = T_k^T \ge 0, (k = 1,2), U_m = U_m^T \ge 0, (m = 1,2)$, $\Lambda = diag(\lambda_1, \lambda_2, \dots, \lambda_n) \le 0$, $\Lambda_j = diag(\lambda_{1j}, \lambda_{2j}, \dots, \lambda_{nj}) \le 0$, $(j = 1, 2)$ such that the linear matrix inequality (LMI):

$$\Omega_{22\times 22} < 0, \tag{7}$$

where

$\Omega_{1,1} = Q_1 + Q_3 + Q_5 + R_1 + R_3 + R_5 - S_1 - \tau_1^2 T_1 + (\tau_2-\tau_1)^2 T_2 - \left(\frac{\tau_1^2}{2}\right)^2 U_1 - \left(\frac{\tau_2^2-\tau_1^2}{2}\right)^2 U_2 - F_1 C - CF_1^T$,

$\Omega_{1,5} = S_1$, $\Omega_{1,8} = P + F_1 - F_2^T C$, $\Omega_{1,9} = K\Lambda_1 + F_1 A$, $\Omega_{1,12} = F_1 B$, $\Omega_{1,16} = \tau_1 T_1$, $\Omega_{1,17} = (\tau_2-\tau_1)T_2$,

$\Omega_{1,18} = (\tau_2-\tau_1)T_2$, $\Omega_{1,19} = F_1 D$, $\Omega_{1,20} = -U_1\frac{\tau_1^2}{2}$, $\Omega_{1,21} = \left(\frac{\tau_2^2-\tau_1^2}{2}\right)U_2$, $\Omega_{1,22} = \left(\frac{\tau_2^2-\tau_1^2}{2}\right)U_2$, $\Omega_{2,2} = -(1-\mu)Q_1$

$\Omega_{3,3} = -(1-\mu)Q_3$, $\Omega_{4,4} = -(1-\mu)Q_5$, $\Omega_{4,12} = K\Lambda_2$, $\Omega_{5,5} = -R_1 - S_1 - S_2$, $\Omega_{5,6} = S_2$, $\Omega_{6,6} = -R_3 - S_2$,

$\Omega_{7,7} = -R_5$, $\Omega_{8,8} = \tau_1^2 S_1 + (\tau_2-\tau_1)^2 S_2 - \left(\frac{\tau_1^2}{2}\right)^2 T_1 - \left(\frac{\tau_2^2-\tau_1^2}{2}\right)^2 T_2 + F_2 + F_2^T + -F_2 C - CF_2^T$, $\Omega_{8,9} = F_2 A + \Lambda$,

$\Omega_{8,12} = F_2 B$, $\Omega_{8,19} = F_2 D$, $\Omega_{9,9} = Q_2 + Q_4 + Q_6 - \sigma^2 S_3 - \Lambda_1 - \Lambda_1^T + R_2 + R_4 + R_6$, $\Omega_{10,10} = -(1-\mu_1)Q_2$,

$\Omega_{11,11} = -(1-\mu_2)Q_4$, $\Omega_{12,12} = -(1-\mu)Q_6 - \Lambda_2 - \Lambda_2^T$, $\Omega_{13,13} = -R_2$, $\Omega_{14,14} = -R_4$, $\Omega_{15,15} = -R_6$,

$\Omega_{16,16} = -T_1$, $\Omega_{17,17} = -T_2$, $\Omega_{17,18} = -T_2$, $\Omega_{18,18} = -T_2$, $\Omega_{19,19} = -S_3$, $\Omega_{20,20} = -U_1$, $\Omega_{21,21} = -U_2$

$\Omega_{21,22} = -U_2, \Omega_{22,22} = -U_2.$

Proof. We choose the following Lyapunov-Krasovskii function:

$$V(z(t)) = \sum_{i=1}^{6} V_i(z(t)) \qquad (8)$$

where

$V_1(z(t)) = z^T(t)Pz(t) + 2\sum_{i=1}^{n} \lambda_i \int_0^{z_i} f_i(s)ds,$

$$V_2(z(t)) = \int_{t-\tau_1(t)}^{t} [z^T(s)Q_1 z(s) + f^T(z(s))Q_2 f(z(s))]ds + \int_{t-\tau_2(t)}^{t} [z^T(s)Q_3 z(s) + f^T(z(s))Q_4 f(z(s))]ds$$

$$+ \int_{t-\tau(t)}^{t} [z^T(s)Q_5 z(s) + f^T(z(s))Q_6 f(z(s))]ds,$$

$$V_3(z(t)) = \int_{t-\tau_1}^{t} [z^T(s)R_1 z(s) + f^T(z(s))R_2 f(z(s))]ds + \int_{t-\tau_2}^{t} [z^T(s)R_3 z(s) + f^T(z(s))R_4 f(z(s))]ds$$

$$+ \int_{t-\tau}^{t} [z^T(s)R_5 z(s) + f^T(z(s))R_6 f(z(s))]ds,$$

$$V_4(z(t)) = \tau_1 \int_{-\tau_1}^{0} \int_{t+\theta}^{t} \dot{z}^T(s)S_1\dot{z}(s)ds + (\tau_2 - \tau_1) \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^{t} \dot{z}^T(s)S_2\dot{z}(s)ds + \sigma \int_{\sigma}^{0} \int_{t+\theta}^{t} f^T(z(s))S_3 f(z(s))ds$$

$$V_5(z(t)) = \frac{{\tau_1}^2}{2} \int_{-\tau_1}^{0} \int_{\theta}^{t} \int_{t+\lambda}^{t} \dot{z}^T(s)\, T_1\dot{z}(s)dsd\lambda d\theta + \frac{({\tau_2}^2 - {\tau_1}^2)}{2} \int_{-\tau_2}^{-\tau_1} \int_{\theta}^{t} \int_{t+\lambda}^{t} \dot{z}^T(s)\, T_2\dot{z}(s)dsd\lambda d\theta$$

$$V_6(z(t)) = \frac{{\tau_1}^3}{6} \int_{-\tau_1}^{0} \int_{\theta}^{0} \int_{\lambda}^{0} \int_{t+\kappa}^{t} \dot{z}^T(s)\, U_1\, \dot{z}(s)dsd\kappa d\lambda d\theta + \frac{({\tau_2}^3 - {\tau_1}^3)}{6} \int_{-\tau_2}^{-\tau_1} \int_{\theta}^{0} \int_{\lambda}^{0} \int_{t+\kappa}^{t} \dot{z}^T(s)\, U_2\, \dot{z}(s)dsd\kappa d\lambda d\theta$$

Calculating the derivatives of $V_i(z(t))$, $(i = 1, 2, \ldots, 6)$ defined in (8) along the trajectories of

$$\dot{V}(z(t)) = \sum_{i=1}^{6} \dot{V}_i(z(t)) \qquad (9)$$

where

$$\dot{V}_1(z(t)) = 2z^T(t)P\dot{z}(t) + 2\sum_{i=1}^{n} \lambda_i \int_0^{z_i} f_i(s)ds$$
$$\dot{V}_1(z(t)) = 2z^T(t)P\dot{z}(t) + 2\lambda\, f^T(z(t))\dot{z}(t) \qquad (10)$$

$$\dot{V}_2(z(t)) = z^T(t)[Q_1 + Q_3 + Q_5]z(t) + f^T(z(t))[Q_2 + Q_4 + Q_6]f(z(t))$$

$$-(1-\mu_1)\left[z^T(t-\tau_1(t))Q_1z(t-\tau_1(t))+f^T\left(z(t-\tau_1(t))\right)Q_2f\left(z(t-\tau_1(t))\right)\right]$$

$$-(1-\mu_2)\left[z^T(t-\tau_2(t))Q_3z(t-\tau_2(t))+f^T\left(z(t-\tau_2(t))\right)Q_4f\left(z(t-\tau_2(t))\right)\right]$$

$$-(1-\mu)\left[z^T(t-\tau(t))Q_5z(t-\tau(t))+f^T\left(z(t-\tau(t))\right)Q_6f\left(z(t-\tau(t))\right)\right] \quad (11)$$

$$\dot V_3(z(t))=z^T(t)[R_1+R_3+R_5]z(t)-z^T(t-\tau_1)R_1z(t-\tau_1)-z^T(t-\tau_2)R_3z(t-\tau_2)-z^T(t-\tau_3)R_5z(t-\tau_3)+f^T(z(t))[R_2+R_4+R_6]f(z(t))-f^T(z(t-\tau_1))R_2f(z(t-\tau_1))-f^T(z(t-\tau_2))R_4f(z(t-\tau_2))-f^T(z(t-\tau))R_6f(z(t-\tau)) \quad (12)$$

$$\dot V_4(z(t))=\tau_1{}^2[\dot z^T(t)S_1\dot z(t)]-\tau_1\int_{t-\tau_1(t)}^{t}\dot z^T(t)S_1\dot z(t)ds+(\tau_2-\tau_1)^2[\dot z^T(t)S_1\dot z(t)]-(\tau_2-\tau_1)\int_{t-\tau_1}^{t-\tau_2}\dot z^T(s)S_2\dot z(s)ds+\sigma^2[f^T(z(t))S_3f(z(t))]-\sigma\int_{t-\sigma}^{t}f^T(z(s))S_3f(z(s))ds \quad (13)$$

By applying lemma 2.1, we have

$$\dot V_4(z(t))=\tau_1{}^2[\dot z^T(t)S_1\dot z(t)]-[z(t)-z(t-\tau_1)]^TS_1[z(t)-z(t-\tau_1)]+(\tau_2-\tau_1)^2[\dot z^T(t)S_2\dot z(t)]$$

$$-[z(t-\tau_1)-z(t-\tau_2)]^TS_2[z(t-\tau_1)-z(t-\tau_2)]+\sigma^2[f^T(z(t))S_3f(z(t))]$$

$$-\left(\int_{t-\sigma}^{t}f(z(s))ds\right)^TS_3\left(\int_{t-\sigma}^{t}f(z(s))ds\right) \quad (14)$$

$$\dot V_5(z(t))=\frac{\tau_1{}^2}{2}[\dot z^T(t)T_1\dot z(t)]\left[\frac{-\tau_1{}^2}{2}\right]-\frac{\tau_1{}^2}{2}\int_{-\tau_1}^{0}\int_{t+\theta}^{t}[\dot z^T(s)T_1\dot z(s)]dsd\theta+\left(\frac{\tau_2{}^2-\tau_1{}^2}{2}\right)^2[\dot z^T(t)T_2\dot z(t)]$$

$$-\left(\frac{\tau_2{}^2-\tau_1{}^2}{2}\right)\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}[\dot z^T(s)T_2\dot z(s)]dsd\theta \quad (15)$$

By applying lemma 2.1, we have

$$\dot V_5(z(t))=\left(\frac{\tau_1{}^2}{2}\right)^2[\dot z^T(t)T_1\dot z(t)]-\left(\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot z(s)dsd\theta\right)^TT_1\left(\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot z(s)dsd\theta\right)$$

$$+\left(\frac{\tau_2{}^2-\tau_1{}^2}{2}\right)^2[\dot z^T(t)T_2\dot z(t)]-\left(\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}\dot z(s)dsd\theta\right)^TT_2\left(\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}\dot z(s)dsd\theta\right)$$

$$\dot V_5(z(t))=\left(\frac{\tau_1{}^2}{2}\right)^2[\dot z^T(t)T_1\dot z(t)]-\left(\tau_1z(t)-\int_{t-\tau_1}^{t}z(s)ds\right)^TT_1\left(\tau_1z(t)-\int_{t-\tau_1}^{t}z(s)ds\right)$$

$$+\left(\frac{{\tau_2}^2-{\tau_1}^2}{2}\right)^2[\dot{z}^T(t)T_2\dot{z}(t)]-\left[(\tau_2-\tau_1)z(t)-\int\limits_{t-\tau(t)}^{t-\tau_1}z(s)ds-\left(\int\limits_{t-\tau_2}^{t-\tau(t)}z(s)ds\right)\right]^T T_2$$
$$\times\left[(\tau_2-\tau_1)z(t)-\int\limits_{t-\tau(t)}^{t-\tau_1}z(s)ds-\left(\int\limits_{t-\tau_2}^{t-\tau(t)}z(s)ds\right)\right] \qquad (16)$$

$$\dot{V}_6(z(t))=\left(\frac{{\tau_1}^3}{6}\right)^2[\dot{z}^T(t)U_1\dot{z}(t)]-\left(\int_{-\tau_1}^0\int_\theta^0\int_{t+\theta}^t\dot{z}(s)\,dsd\lambda d\theta\right)^T U_1\left(\int_{-\tau_1}^0\int_\theta^0\int_{t+\theta}^t\dot{z}(s)\,dsd\lambda d\theta\right)$$

$$\left(\frac{({\tau_2}^3-{\tau_1}^3)}{6}\right)^2[\dot{z}^T(t)U_2\dot{z}(t)]-\left(\int_{-\tau_2}^{-\tau_1}\int_\theta^0\int_{t+\lambda}^t\dot{z}(s)\,dsd\lambda d\theta\right)^T U_2\times\left(\int_{-\tau_2}^{-\tau_1}\int_\theta^0\int_{t+\lambda}^t\dot{z}(s)\,dsd\lambda d\theta\right)$$

$$=\left(\frac{{\tau_1}^3}{6}\right)^2[\dot{z}^T(t)U_1\dot{z}(t)]-\left(\frac{{\tau_1}^2}{2}z^T(t)-\int\limits_{-\tau_1}^{0}\int\limits_{t+\theta}^{t}z^T(t)dsd\theta\right)U_1\left(\frac{{\tau_1}^2}{2}z^T(t)-\int\limits_{-\tau_1}^{0}\int\limits_{t+\theta}^{t}z^T(t)dsd\theta\right)$$
$$+\left[\frac{({\tau_2}^2-{\tau_1}^2)}{2}z(t)-\int\limits_{-\tau(t)}^{-\tau_1}\int\limits_{t+\theta}^{t}z(s)dsd\theta-\int\limits_{-\tau_2}^{-\tau(t)}\int\limits_{t+\theta}^{t}z(s)dsd\theta\right]^T U_2$$
$$\times\left[\frac{({\tau_2}^2-{\tau_1}^2)}{2}z(t)-\int\limits_{-\tau(t)}^{-\tau_1}\int\limits_{t+\theta}^{t}z(s)dsd\theta-\int\limits_{-\tau_2}^{-\tau(t)}\int\limits_{t+\theta}^{t}z(s)dsd\theta\right]$$

On the other hand, it is clear from (6) that

$$f_i(z_i(t))[f_i(z_i(t))-K_i(z_i(t))]\le 0\,, i=1,2,\ldots.,n$$

And

$$f_i\left(z_i(t-\tau(t))\right)\left[f_i\left(z_i(t-\tau(t))\right)-K_i\left(z_i(t-\tau(t))\right)\right]\le 0\,, i=1,2,\ldots.,n$$

Thus, for any $\Lambda_j=diag(\lambda_{1j},\lambda_{2j},\ldots.,\lambda_{nj})\ge 0, j=1,2.$

$$0\le -2\sum_{i=1}^n t_{i1}\,f_i(z_i(t))[f_i(z_i(t))-K_i(z_i(t))]$$
$$-2\sum_{i=1}^n t_{i2}f_i\left(z_i(t-\tau(t))\right)\left[f_i\left(z_i(t-\tau(t))\right)-K_i\left(z_i(t-\tau(t))\right)\right]$$

$$=2z^T(t)KT_1f(z(t))-2f^T(z(t))T_1f(z(t))+2z^T(t-\tau(t))KT_2f\left(z(t-\tau(t))\right)$$
$$-2f^T\left(z(t-\tau(t))\right)T_1f\left(z(t-\tau(t))\right) \qquad (17)$$

On the other hand for any matrices $U1$ and $U2$ with appropriate dimensions, it is true that

$$0=2[z^T(t)U_1+\dot{z}^T(t)U_2]\left[\dot{z}(t)-z(t)+Af(z(t))+Bf\left(z(t-\tau(t))\right)+D\int_{t-\sigma(t)}^t f(z(s))ds\right]. \qquad (18)$$

Substituting (10)-(18) into (9), we have

$$\dot{V}\big(z(t)\big) = \xi^T(t)\Omega\xi(t) \qquad (19)$$

where

$$\xi^T(t) = [z^T(t)z^T\big(t-\tau_1(t)\big)z^T\big(t-\tau_2(t)\big)z^T\big(t-\tau(t)\big)z^T(t-\tau_1)z^T(t-\tau_2)$$

$$z^T(t-\tau)\dot{z}^T(t)f^T\big(z(t)\big)f^T\Big(z\big(t-\tau_1(t)\big)\Big)f^T\Big(z\big(t-\tau_2(t)\big)\Big)$$

$$f^T\Big(z\big(t-\tau(t)\big)\Big)f^T\big(z(t-\tau_1)\big)f^T\big(z(t-\tau_2)\big)f^T\big(z(t-\tau)\big)\int_{t-\tau_1}^{t} z^T(s)ds$$

$$\int_{t-\tau(t)}^{t-\tau_1} z^T(s)ds \int_{t-\tau_2}^{t-\tau(t)} z^T(s)ds \int_{t-\sigma}^{t} f^T\big(z(s)\big)ds \int_{-\tau_1}^{0}\int_{t+\theta}^{t} z^T(t)dsd\theta$$

$$\int_{-\tau(t)}^{-\tau_1}\int_{t+\theta}^{t} z^T(s)dsd\theta \int_{-\tau_2}^{-\tau(t)}\int_{t+\theta}^{t} z^T(s)dsd\theta\ ],$$

According to (7), we have $\dot{V}\big(z(t)\big) < 0$. Therefore, we obtain $\dot{V}\big(z(t)\big) < -\varepsilon\|z(t)\|^2$ for a sufficient small $\varepsilon > 0$, which means the system in (4) is asymptotically stable and the proof is completed.

IV STABILITY CRITERIA FOR UNCERTAIN SYSTEM

In this section, based on Theorem 3.1, we are now ready to develop delay-interval-dependent stability criterion for the neural networks with time-varying parameters uncertainties. Now, we consider the following uncertain neural networks as:

$$\dot{z}(t) = -\big(C+\Delta(t)\big)z(t) + \big(A+\Delta A(t)\big)f\big(z(t)\big) + \big(B+\Delta B(t)\big)f\Big(z\big(t-\tau(t)\big)\Big)$$

$$+\big(D+\Delta D(t)\big)\int_{t-\sigma(t)}^{t} f\big(z(s)\big)ds \qquad (20)$$

Where $\Delta C(t)$, $\Delta A(t)$, $\Delta B(t)$ and $\Delta D(t)$ are the time-varying parameters uncertainties. Which are assumed to be of the form

$$[\Delta A(t)\Delta B(t)\Delta C(t)\Delta D(t)] = HF(t)[E_1E_2E_3E_4]$$

where H and Ei, i=1,2,3,4 are known real constant matrices, and $F(\cdot)$ is an unknown time varying matrix function satisfying $F(t)T\,F(t) \le I$. Based on Theorem 3.1, the following criterion can be readily derived.

Theorem 4.1 For given scalars τ_1, τ_2, σ, μ_1 and μ_2 then system (4) and time-varying delay satisfying condition (2) is asymptotically stable if there exist $P = P^T > 0, Q_L = Q_L^T \ge 0, (L = 1,2,\ldots,6)$ $R_i = R_i^T \ge 0, (i = 1,2,3)$ $S_j = S_j^T \ge 0, (j = 1,2,3)$ $Z = Z^T \ge 0, T_k = T_k^T \ge 0, (k = 1,2), U_m = U_m^T \ge 0, (m = 1,2)$, $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_n) \le 0$, $\Lambda_j = diag(\lambda_{1j}, \lambda_{2j}, \ldots, \lambda_{nj}) \le 0$, $(j = 1, 2)$ here exists scalar ϵ such that the linear matrix inequality(LMI).

$$\begin{bmatrix} \Omega + \epsilon\Theta_2^T & \Theta_1^T \\ \Theta_1 & -\epsilon I \end{bmatrix} < 0, \qquad (21)$$

Where

$$\Theta_1 = \Big[\mathrm{F}_1 H \quad \underbrace{0^{14times}}\Big]^T$$

$$\Big[-\mathrm{E}_1 \quad \underbrace{0^{7times}} \quad \mathrm{E}_2 \quad 0 \quad 0 \quad \mathrm{E}_3 \quad \underbrace{0^{9times}} \quad \mathrm{E}_4 \quad 0 \quad 0 \quad 0\Big]^T$$

and Ω is defined in Theorem 3.1.

Proof: Replacing A, W, $W1$, $W2$, $W3$, in LMI (8) with $C + HF(t)E1$, $A + HF(t)E2$, $B + HF(t)E3$ and $D + HF(t)E4$, yields

$$\Omega + \Theta_1^T F(t)\Theta_2 + \Theta_2^T F(t)\Theta_1 < 0 \qquad (22)$$

Applying Lemma 2.2, it can be deduced that for $\varepsilon > 0$

$$\Omega + \varepsilon^{-1}\Theta_1^T \Theta_1 + \varepsilon\Theta_2^T F(t)\Theta_1 < 0 \qquad (23)$$

which is equivalent to (21) in the sense of the Schur complement [28]. The proof is completed.

V. Numerical Example

In this section, we list some illustrative example to demonstrate the less conservatism of our result and the effectiveness of the proposed method.

Example 4.1 Consider system (4) with the following parameters:

$$C = \begin{bmatrix} 1.9 & 0 \\ 0 & 1.2 \end{bmatrix}, A = \begin{bmatrix} 0.8 & -0.2 \\ 0.1 & 0.3 \end{bmatrix}, B = \begin{bmatrix} 0.5 & 0.2 \\ -0.2 & -0.1 \end{bmatrix}, D = \begin{bmatrix} 0.05 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}$$

we assume $\tau 1 = 1.8$, $\tau 2 = 2.5$, $\sigma = 1.5$, $\mu 1 = 0.2$ and $\mu 2 = 0.5$ by using Matlab LMI toolbox, it is found that LMI (7) is feasible. The simulation results for the above-mentioned delay values also ensure the asymptotic stability of the model (4). Fig. 1

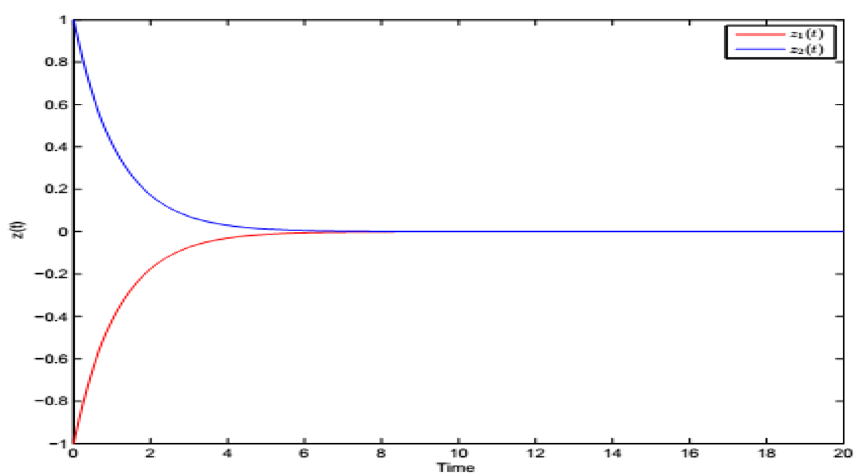

Figure 1: State trajectory of the system (4) in Example 4.1

IV. Conclusion

In this paper, delay-interval-dependent stability criteria for neural networks with successive time-varying delay components as well as generalized activation functions. By employing a combination of Lyapunov functional, the Jenson's inequality technique, several delay-interval-dependent criteria for checking the stability criteria of the addressed neural networks have been established in terms of linear matrix inequalities. Finally, a numerical example is presented to illustrate the effectiveness of the proposed criteria.

References

[1] L.O. Chua, L. Yang, Cellular neural networks: applications, IEEE Trans. Circuits Syst. 35 (1988) 1273- 1290.

[2] A. Cichocki, R. Unbehauen, Neural Networks for Optimization and Signal Processing, Wiley, Hoboken, NJ, 1993.

[3] G. Joya, M.A. Atencia, F. Sandoval, Hopfield neural networks for optimization: study of the different dynamics, Neurocomputing 43 (2002) 219-237.

[4] W.J. Li, T. Lee, Hopfield neural networks for affine invariant matching, IEEE Trans. Neural Netw. 12 (2001) 1400-1410.

[5] W. Xie, Q. Zhu, F. Jiang, Exponential stability of stochastic neural networks with leakage delays and expectations in the coefficients, Neurocomputing 173 (2016) 1268-1275.

[6] S.S. Chen, Chaotic simulated annealing by a neural network with avariable delay: design and application, IEEE Trans. Neural Netw. 22 (2011) 1557-1565.

[7] P. Baldi, A.F. Atiya, How delays affect neural dynamics and learning, IEEE Trans. Neural Netw. 5 (1994) 612-621.

[8] J.H. Park, A delay-dependent asymptotic stability criterion of cellular neural networks with time-varying discrete and distributed delays, Chaos Solitons Fract. 33 (2007) 436-442.

[9] S. Arik, Ananalysis of global asymptotic stability of delayed cellular neural networks, IEEE Trans. Neural Netw. 13 (2002) 1239-1242.

[10] T. Chen, L. Rong, Delay-independent stability analysis of Cohen-Grossberg neural networks, Phys. Lett. A 317 (2003) 436-499.

[11] T.L. Liao, F.C. Wang, Global stability for cellular neural networks with time delay, IEEE Trans. Neural Netw. 11 (2000) 1481-1484.

[12] W. Zhang, X. Cai, Z. Han, Robust stability criteria for systems with interval time-varying delay and nonlinear perturbations, J. Comput. Appl. Math. 234 (2010) 174-180.

[13] W. Qian, S. Cong, Y. Sun, S. Fei, Novel robust stability criteria for uncertain systems with time-varying delay, Appl. Math. Comput. 215 (2009) 866-872.

[14] Y. He, M. Wu, J.H. She, Delay-dependent exponential stability of delayed neural networks with time- varying delay, IEEE Trans. Circuits and Syst. I - Express Briefs, 53 (2006) 553-557.

[15] Q.F. Yang, Q.H. Ren, X. Xie, New delay dependent stability criteria for recurrent neural networks with interval time-varying delay, ISA Trans. 53 (2014) 994-999.

[16] J. Li, M. Hu, L. Guo, Y. Yang, Y. Jin, Stability of uncertain impulsive stochastic genetic regulatory net- works with time-varying delay in the leakage term, Abstr. Appl. Anal. (2014) 1-15.

[17] R. Raja, R. Samidurai, New delay dependent robust asymptotic stability for uncertain stochastic recurrent neural networks with multiple time varying delays, J. Frankl. Inst. 349 (2012) 2108-2123.

[18] Y. Zhao, H. Gao, S. Mou, Asymptotic stability analysis of neural networks with successive time delay components, Neurocomputing 71 (2008) 2848-2856.

[19] H. Shao, Q.L. Han, New delay-dependent stability criteria for neural networks with two additive time- varying delay components, IEEE Trans. Neural Netw. 22 (2011) 812-818.

[20] N. Xiao, Y. Jia, New approaches on stability criteria for neural networks with two additive time-varying delay components, Neurocomputing 118 (2013) 150-156.

[21] J. Tian, S. Zhong, Improved delay-dependent stability criteria for neural networks with two additive time- varying delay components, Neurocomputing 77 (2012) 114-119.

[22] C. Zhang, Y. He, L. Jiang, Q.H. Wu, M. Wu, Delay-dependent stability criteria for generalized neural networks with

two delay components, IEEE Trans. Neural Netw. Learn. Syst. 25 (2014) 1263-1276.

[23] R. Rakkiyappan, R. Sivasamy, J.H. Park, T.H. Lee, An improved stability criterion for generalized neural networks with additive time-varying delays, DOI:10.1016/j.neucom.2015.07.004.

[24] J. Li, M. Hu, L. Guo, Y. Yang, Y. Jin, Stability of uncertain impulsive stochastic fuzzy neural networks with two additive time delays in the leakage term, Neural Comput. Appl. 26 (2014) 417-427.

[25] Y. Liu, S. M. Lee, H. G. Lee, Robust delay-depent stability criteria for uncertain neural networks with two additive time-varying delay components, Neurocomputing 151 (2015) 770-775.

[26] O.M. Kwon, J.H. Park, S.M. Lee, E.J. Cha, New augmented Lyapunov-Krasovskii functional approach to stability analysis of neural networks with time-varying delays, Nonlinear Dynam. 76 (2014) 221-236.

[27] L. Xie, Output feedback H_∞ control of systems with parameter uncertainty, Int. J. Control 63 (1996) 741-750.

[28] S. Boyd, L. Ghaoui, E. Feron, V. Balakrishnan, Linear Matrix Inequalities in Systemand Control Theory, SIAM, Philadelphia, 1994.