A Boreholed Dipole Antenna Transfer Function

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ABSTRACT: In this work a semi-empirical model is presented to characterize the transfer function and scattering parameters of a vertical dipole antenna, in the 164–174 MHz bandwidth. The antenna is either radiating in free space, or it is inserted in an air-filled borehole in a salt mine. The model uses the signal flow graph for modeling of the antenna coupling with the environment and the effects of radio wave propagation in the layered medium. Analytical results are fitted to measurements performed with a vector network analyzer, and the transfer functions are obtained by applying a least mean square algorithm. Experimental data acquired in free space allows also determination of the intrinsic parameters of the antennas, providing a stand-alone method.

INDEX TERMS: boreholed antenna, layered medium, transfer function.

1. INTRODUCTION

The behavior of antennas in different borehole environments is not well understood. Antenna's characterization is necessary in inverse problems to correctly identify the properties of natural dielectrics. Such analysis is performed before the construction of radio cosmic neutrino detectors in ice [1], salt [2], to detect buried objects, and to characterize the subsurface structure and properties in a wide variety of applications (e.g. archaeology, civil engineering, forensics, and geology [3]). In borehole systems the antenna acts as a filter so a transfer function must be associated with it [4]. The problem is not trivial because propagation of radio waves in heterogeneous media is a subject not fully resolved [5] and the effect of antenna coupling with the medium cannot be isolated from the propagation model. The problem of antennas radiating in the proximity of a dielectric medium was studied by several authors. Gentili and Spagnolini [6] addressed the problem of an antenna radiating above a layered medium, but the multiple reflections between antenna and the medium were not accounted for. Other papers presented results for antennas in the 30-60 kHz range situated above the ground, at far field distances [7]. Lambot and Andre [8] improved the model for a near field scenario. Unfortunately the algorithm is tested for a medium formed by 2 homogeneous layers (air and water) of a few centimeters each, thus the simulation complexity and time are small. For antennas boreholed in dielectrics the analysis is usually done using numerical methods to solve an integral equation for the current distribution. King and Smith [9] considered an approximate solution of this equation. The validation of King's expression through experimental measurements was done in [10]. Other attempts to model antennas in dielectrics were performed in [11]. The Chen's more general model [12] gives unphysical input impedances of the antenna (negative input resistance) when the ambient medium is of low conductivity. In the work addressed here a semi-empirical model is used to characterize the transfer function and the scattering parameters of a vertical dipole boreholed in a dielectric medium, in the 164-174 MHz bandwidth. This bandwidth was chosen as a compromise between the radio wave penetration depth in the medium and the physical dimension of the antennas: lower frequencies allow larger propagation distances, but require large antennas. Nonetheless the dimensions of the antennas are limited by the size of the borehole, which in turn are constrained by the available drilling instrument. Even though measurements are performed in a bandwidth specific to a peculiar commercial antenna available on the market, the method can be applied to characterize any vertical dipole antenna boreholed in any dielectric medium. The semi-empirical model presented avoids a full analytical characterization of the antenna together with the numerical evaluation of Somerfield type integrals that account for propagation in layered media. Instead, the signal flow graph is used to describe the antenna coupling with the environment, together with the effects of radio wave propagation, while the antenna is described by a 2-port network. The analytical model fits data measured with a vector network analyzer (VNA) to determine the unknown parameters, among which the transfer function of the antenna.

2. MODEL FORMULATION

The modeling of an antenna as a two-port network is not new. A valid two-port representation of an antenna must include the antenna's input impedance as well as its realized transfer function. Any antenna can be regarded as a 2-port [13] as indicated in figure 1. The radiation resistance Rrad is described by a transformer. Port two is terminated with the intrinsic impedance of the medium where the antenna radiates (e.g. $Z0 = 120\pi\Omega$ for free space) and the characteristic impedance at port 1 is Zc (50 Ω). The impedance Ztot describes in a unitary form the intrinsic ohminc, dielectric and conduction losses of the antenna (Zint), together with near field associated losses. The antenna transfer function Ha(f) included as an amplifier (or attenuator) stage. The antenna transfer function concept is used in literature in situations where the overall emitting–receiving system transfer function is necessary, especially when the propagation channel requires modeling [14], [15]. For simplicity the antennas used should be identical and perfectly aligned such as to have identical transfer functions. Following the approach of [16], the transfer function is an intrinsic attribute of the antenna, defined independently on the distance between the transmitter and receiver (and this is can be

observed in figure 1). The transfer function describes antenna behavior in frequency, and it might incorporate other factors also (i.e. polarization, gain etc.).



Fig.1: Antenna as a two-port network

Referring to figure 1, the S11a parameter represents the reflection coefficient at the antenna input (the index a will be used to refer to a parameter of the antenna from now on). All the parameters of the scattering matrix can be determined analytically from the 2port model once the transfer function Ha is known. Regardless the type of antenna the intrinsic impedance Zint can only be measured experimentally. For that I have used an emission (Tx)-reception (Rx) system, and the measurements have been performed in free space (a detailed description of the measurements is presented in section III-A). The antennas were identical, such as the scattering parameters are the same, and perfectly aligned to exclude polarization losses. A particularly useful technique that can be used to study the physical networks is the signal flow graph. The technique provides a solution to the network equations using a systematic procedure [17]. The signal flow diagram of the Tx–Rx antenna system together with the effect of the propagation medium in between is presented in figure 2. Port 1 represents the transmitting port, and port 2-the receiving one (this convention will be used from now on throughout this paper). The free-space factor is [14]:



Fig.2: The signal flow graph for a transmitting-receiving antenna system

The scattering parameters of the overall system can be determined from the graph flow, using Masons' rules [17]. The graph has only one closed loop, whose transmittance is $m2.S^{2}_{22}a$. Following the direction of the arrows (that indicate the propagation direction), the reflection coefficient at port 1 and the transmission coefficient from port 1 to port 2 can be calculated analytically:

$$\begin{split} S_{11,sys} &= S_{11,a} + \frac{S_{21a} \cdot S_{12a} \cdot m^2}{1 - m^2 \cdot S_{22a}^2} \\ S_{21,sys} &= \frac{S_{21a} \cdot S_{12a} \cdot m}{1 - m^2 \cdot S_{22a}^2} \end{split}$$

and can be measured directly with a VNA. If the medium where the antennas radiate is more complex (such as a salt dome) the signal flow graph should be modified to characterize all the phenomena that occur at propagation, and this is the subject of the next sub-section.

A. The heterogeneous propagation environment

The salt deposit where the measurements for this work were carried out is the result of layers of dried solutes from ancient seas that suffered deformation through tectonic and buoyant forces (i.e. a salt diaper). Each layer has different thickness and dielectric characteristics according to the nature of the deposited sediments, which are not normally known. This is one of the main reasons why the problem addressed in this work is difficult to solve, as commercial analysis software for antenna and propagation description require as input exactly the mentioned unknown quantities. Some recent work using detailed Finite Difference Time Domain models [18] presented simulations for lossy heterogeneous environments, but there the medium was continuous, which already excludes the main source of power losses, reflections at interfaces of separation [19].

In this work it was considered that the propagation medium is formed by nearly parallel non-dispersive layers of sediments in the 164–174 MHz bandwidth (this layer distribution was observed at the site of the experiment). The radio waves were transmitted

and received by vertical dipole antennas placed in boreholes in salt filled with air, parallel to the layers of sedimentary salt, and separated by a distance d of 10 m. The waves emitted by the Tx antenna propagate first in the air in the borehole, thus a first reflection will occur at the interface between air and the first layer of salt. The reflection coefficient at this first interface is R1, and depends on the complex permittivity of the first layer of salt-1. Afterwards each layer of salt will produce a new reflection, Ri; i > 2. The model also includes multiple reflection within each layer. The attenuation between interfaces i - 1 and I is F(i-1)i, and it depends on: the thickness of the layer, the permittivity of the layer etc. The dependence on the incidence angle hasn't been explicitly written because it is beyond the purpose of this work: here all reflections and attenuations (or expressions that contain combinations of them) are extracted from measurements. To conclude, when a layered medium of propagation is considered equations (2) and (3) should be modified as:

$$S_{11,sys} = S_{11,as} - \frac{S_p}{S_{22,as}} + T$$
$$S_{21,sys} = \frac{S_p \cdot F_{12} \cdot F_{23} \cdot ... \cdot F_{nn+1}A_{n+1}}{\Delta}$$

where

$$S_p = S_{12,as} \cdot S_{21,as}$$

and:

$$T = 2S_p R_1 \left(1 + \frac{S_{22,as} \cdot R_1 - 1}{\Delta} \right) + \frac{S_p}{S_{22,as} \cdot \Delta}.$$
$$\Delta = 1 - S_{22,as} \left[R_1 + R_2 \cdot F_{12}^2 + R_3 \cdot F_{12}^2 \cdot F_{23}^2 + .. \right]$$

In equations (4) - (6) the index s has been introduced to refer to the scattering parameters of an antenna when placed in a borehole in salt The derivation of last equations is based on Masons' rules [17]. Each reflection at separation borders creates a new transmission loop, easily observed in equation (6); there are no second or higher order loops in the system. However, each of the new created loops touches a forward path, either from port 1 to port 1, or from port 1 to port 2, preventing it to appear in the expression for S11;sys or S21;sys. The only exception for a reflection coefficient is S22;as R1, and this is the reason why R1 is displayed in formula (4). The transmission from port 1 to port 2 consists of only one possible path, shown in equation (5).

The scattering parameters of the antenna when it radiates in salt can be deduced from the 2-port scheme of the antenna, adapted after figure 1. The scattering parameters depend on the medium where the antenna radiates (medium defined by its far field dielectric characteristics), but not on the distance between emitter and receiver. The impedance Ztot; includes the intrinsic properties of antenna (Zint, that depends only on the antenna and not on the medium when the antenna radiates), and the loading caused by the natural dielectric. The loading itself can be quantified in a complex impedance at the antenna feed point [23]. The radiation resistance in salt is marked by

Rrad;s, and the antenna transfer function by Ha;s. With the exception of Zint all parameters are different when antennas radiate in air, compared to salt, and should be determined experimentally. The input impedance of an antenna in an air-filled borehole in a dielectric medium was determined in [24]:

$$Z_{in} = \frac{Z_0 \Psi(k)}{2\pi \sqrt{\epsilon_r}} \left[1 + \frac{1}{jkh} - 2\frac{\Phi'(x)}{\Phi(x)} \right]$$

$$\Psi(k) = 2 \left[\sinh^{-1} \left(\frac{h}{a} \right) - C_i(2ka, 2kh) - jS_i(2ka, 2kh) \right]_{(8)} + A$$
with
$$A = j \frac{1 - \exp(-2jkh)}{kh}.$$

3. MEASUREMENTS AND RESULTS

All measurements were performed using a Vector Network Analyzer (VNA), produced by Deviser, model NA7300A, that recorded the S-scattering parameters between the input and output ports (both with a characteristic impedance of 50 Ω). The pair of omnidirectional dipoles used for measurements was produced by Kathrein, model K552628 (164-174 MHz). Antenna's total length was 993 mm, the radius equal to 3.509 cm, it had a vertical polarization, and gain of 2 dBi. According to the manufacturer the antenna exhibits an omnidirectional pattern in the horizontal plane, while in the vertical plane it has a 3 dB angle of 78 degrees, centered on the direction normal to the antenna. The input impedance of the antenna is 50 Ω . For connections to the ports of the VNA, 50 Ω N type coaxial cables (EC400 +) were used. Other 2 coaxial cables (RG 58) of 1 m length each were used to interconnect the SMA input/output of an amplifier with the Rx antenna. The effects of the RG 58 cables were excluded in the data post processing.

The amplifier used was produced by Aaronia, model UBBV1, with a gain of about 40 dB in the 50 MHz-1 GHz band. The exact values of the gain of the amplifier at each frequency were extracted from the data sheet provided by the manufacturer. The

frequency-dependent complex ratios S11(f) and S21(f) between the returned signal/transmitted signal and the emitted signal (corresponding to equations (2) and (3) for measurements in air, and equations (4) and (5)-for measurements in salt) were measured sequentially at the maximum number allowed by the VNA, 1601 evenly stepped operating frequencies in the nominal frequency band. The sweep time was set to 2 seconds to increase the accuracy of the measurements, and an averaging factor of 10 was chosen to improve the signal-to noise ratio. The system had a constant transmission power of 10 dBm. To reduce the systematic errors, the measurement setup was kept the same for measurements in free space, and also when the antennas were placed in boreholes in salt. The reflection losses caused by connection of most cables (together with changes in the measurement reference planes) were removed by a null calibration of the VNA.

Measurements in free space

The main purpose of measurements in free space was to determine the intrinsic properties of the commercial antennas (e.g. Zint). All the measurements were performed in the main chamber of the salt mine"Unirea" (Slanic Prahova, Romania). The main chamber has a height of 50 m and a surface larger than 5000 m2 [23]. It is a radio quiet zone because it is situated at 208 m depth and the soil above (more than 50 m thick) absorbs all man made or natural radio frequencies. However, the reflections from the floor of the chamber and the measuring instrumentation could not be avoided. The noise level inside the chamber was measured with the VNA, and a mean value of -97 dB was found in the 164–174 MHz band.



Fig.3: The impedance of the antenna: the solid line shows the resistance loss and the interrupted line- the reactance of the antenna

In the measurement setup the receiving and transmitting antennas were connected to the ports of the VNA. The distance between antennas (i.e. the propagation distance) measured accurately with a laser tape was r0 = 11:7 m. The reflection and transmission coefficients between the two

ports of the analyzer were measured, and by using eqs. (2) and (3) together with the 2-port model of the antenna, the intrinsic parameters of the antenna were determined. For the radiation resistance and the losses associated with near field effects the standard formulas for a sleeve dipole were used [22]. The variation with frequency of the complex intrinsic impedance Zint was determined and it is represented in figure.3. The modulus and phase of the antenna transfer function H_a were also determined, and are presented in figure 4.



Fig.4: The antenna transfer function in air: the left y-axis shows the modulus of the transfer function (continuous line); the right y-axis shows the phase of the transfer function (interrupted line)

Measurements in salt

The measurements with antennas positioned in boreholes were also performed in the main chamber on"Unirea" salt mine. The mine itself is positioned as such the thickness of the salt walls is larger than 100 m, and there are more than 100 m of salt below and above the holes. To measure the attenuation of radio waves in natural salt, 2 cylindrical holes separated by 10 m were drilled in a wall. The holes have been drilled parallel to each other, at the same height from the chamber floor, each hole having a total depth of 1.2 m, and a diameter of 75 mm. The emitter and receiver antennas were inserted into the borehole and connected to the ports of the VNA (figure 5). The Rx antenna was followed by an amplifier. Again S11 (f) and S21 (f) were measured in 1601

frequency points in the nominal band. The distance of 10 m between the boreholes was chosen to meet the far field measurement criteria. In an a priori calculus the permittivity of the medium has been considered in the 5–7 range for determining the far field limit. A safety margin was also added. The analytical model represented by equations (4) - (8),

Combined with the scattering parameters of the antennas Sij;as determined from the 2-port model of the antennas in salt should fit the measured data in order to determine the unknown quantities. Even though the goal of the paper is to determine the transfer function Ha;s (that is included in the Sij;as scattering parameters), the indefinite system of equations requires estimation of all variables simultaneously. All parameters involved are complex numbers, which doubles the number of unknowns. This is the main reason why a classical Levenberg-Marquardt fitting algorithm is unusable. The approach in this is work is outlined in the next paragraphs.



Fig.5: The measurement principle. In the lower part of the picture a schematic shows that the Tx an Rx antennas are connected to the two ports of the VNA (the antenna at reception is followed by an amplifier). The upper part of the picture presents the actual instrumentation used. Each antenna is inserted into a hole drilled in the wall of the mine chamber, as shown in a detail in the

right hand side of the figure. The distance between the Tx-Rx antennas is 10m.

Based on the result obtained for the permittivity the antenna transfer function was extracted from the 3D matrix [Ha;s]. The function includes yet another effect that has not been mathematically modeled: the effect of scattering on impurities. Even though the overall effect in terms of amplitudes is small, each scattering will produce a phase shift [5]. A2 test was performed, constraint by the fact that the transfer function should remain a continuous function. The modulus of the transfer function was determined and it is represented in figure 6. The dashed curve represents the best fit. The types of impurities and their concentration within each layer vary and are unknown, thus the effect on the phases cannot be estimated nor reconstructed.



Fig.6: The modulus of the transfer function when antenna is inserted in a borehole in salt

Once the transfer function |Ha;s| was determined with the help of measurements, the scattering parameters of the antenna Sij;as can be determined from the 2-port model of the antenna. For illustrative purposes figure 7 shows in a unitary form the reconstructed scattering parameters of the antenna in a fixed bandwidth, when the antenna radiates in air (continuous line) and when the antenna is in a borehole in salt (dashed line).



Fig.7: The determined scattering parameters of the antenna when it radiates in air (continuous line), and salt (dashed line)

Another aspect that should be discussed is the resonance frequency of the antenna. The resonance frequency, as a standard definition, it the frequency for which the reactance of the antenna equals zero or is near zero. When an antenna radiates in a dielectric medium the presence of the medium affects the current distribution on the antenna. The current velocity along the antenna varies, thus changing (lowering) the resonance frequency [3]. More to this, the electrical impedance of an antenna is lower in a dielectric medium and this reflects in a drop in the central frequency of the antenna. If the antenna radiates in the proximity of a dielectric medium, the wave velocity will have a value between the velocity of an electromagnetic wave in the air and in the medium [21]. In it is proven in a very elegant way that in a half–spaces problem the drop in frequency will scale with $n3=(1+n)^2$ (in the direction of interest for this work), n being the refraction index of the dielectric medium.

4. CONCLUSIONS

In this work the magnitudes of the transfer function and the characteristic scattering parameters of a vertical dipole boreholed in natural rock salt were determined with the help of a semi-empirical model. The scattering parameters allow a full characterization of the functionality of commercial antennas in a certain frequency bandwidth, which can be directly used in inverse problems. The dipole antenna was chosen because it radiates a symmetric radiation pattern relative to the horizontal plane at their feed point that helps the detection of a multilayered structure.

The typical method for dealing with layered media requires numerical evaluation of Somerfield type integrals. If the layers have a thickness of a few centimeters, and the overall propagation length is at the order of tens of meters, the method is inapplicable from the simulation time perspective. The large number of unknowns could even make the method non-convergent. The model addressed here eliminates this inconvenient. Moreover, the empirical results estimate all the near field phenomena (associated with losses) that no longer have to be analytically modeled or simulated with specific software.

The internal impedance of the antenna was determined by measurements in free space. Although properties of antennas are usually measured in anechoic chambers, for the 164–174 MHz bandwidth there are just few such facilities, and none available for the public in Romania. This is the reason why measurements were performed in the low noise main chamber of the mine, in an area where no reflecting objects were present. However, the reflections from the floor of the chamber and the measuring instrumentation could not be avoided.

Another aspect that required investigation was that due to the limitations from the drilling tool (that dictated the dimensions of the borehole, including its height) the dipoles were situated very close to a reflecting plane perpendicular to the antennas (the interface between the salt wall and air). This had mainly two effects: a complex loading at the input of the antenna (that was considered in the model), and secondary lobes that result in power loss.

A complete analysis of complex image theory in no perfect media is presented in [25]. Due to the small diameter of the boreholes relative to the wavelength, propagation of energy along the borehole is small and limited to very high frequencies [22].

The small losses were confirmed by simulations in [23] that shown that a dipole antenna in an air-filled borehole in salt, with one end close to the ground plane, would produce one secondary lobe with an amplitude less than a quarter of that of the main lobe. The direction of the maximum of the main lobe remained the same but the half power beam width angle decreased to about 60 degrees (compared to 78 degrees in free space).

The effect of an off-centered antenna has been estimated by repeating the measurements in several campaigns. The antennas were introduced in the boreholes, parallel and centered. The magnitude of the recorded S21 varied by less than 0.5 dB, while the phase with less than 2 degrees, in all bandwidth. It was concluded that small, unintentional un-alignment does not influence the final results.

The only effect that cannot be completely modeled and removed in data post-processing is the scattering on impurities. Not only the concentration of impurities within each layer is unknown, but also the type of impurities within each layer. This results in the impossibility to reconstruct the phases of the transfer function and scattering parameters, as the uncertainty is too high to allow a significant fitting.

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