

# INTUITIONISTIC FUZZY $\omega$ SUPRA CLOSED SETS IN IFST SPACES

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**Abstract:** Necla Turanl introduced the concept of Intuitionistic fuzzy supra topological space which is a special case of Intuitionistic fuzzy topological space. Aim of this paper is we introduced in Intuitionistic fuzzy  $\omega$  supra closed sets in Intuitionistic fuzzy supra topological space.and also we studied about properties and characterization of Intuitionistic fuzzy  $\omega$  supra open sets and Intuitionistic fuzzy  $\omega$  supra closed sets in Intuitionistic fuzzy supra topological spaces

**Key Words:** Intuitionistic fuzzy semi supra open set, Intuitionistic fuzzy semi supra closed set, Intuitionistic fuzzy  $\omega$  supra open sets, Intuitionistic fuzzy  $\omega$  supra closed sets, Intuitionistic fuzzy supra topological spaces

## 1. INTRODUCTION

Topology is a classical subjects, as a generalization topological spaces many type of topological spaces introduced over the year. C.L. Chang [4] was introduced and developed fuzzy topological space by using L.A. Zadeh's[21] fuzzy sets. Coker[5] introduced the notion of Intuitionistic fuzzy topological spaces by using Atanassov's[2] Intuitionistic fuzzy set

A.S. Mashhour et al. introduced the supra topological spaces and studied in the year 1983. M. E. AbdEl-Monsef et al. [1] introduced the fuzzy supra topological spaces and studied fuzzy supra-continuous functions and obtained some properties and characterizations.In 2003 Necla Turanl [20] introduced the concept of Intuitionistic fuzzy supra topological space.

In 2015 M. Parimala[12] et al. introduced Intuitionistic fuzzy semi supra open sets in Intuitionistic fuzzy supra topological spaces. Aim of this paper is we introduced and studied about Intuitionistic fuzzy  $\omega$  supra closed sets in Intuitionistic fuzzy supra topological spaces and its properties are discussed details.

## 2. PRELIMINARIES

### Definition 2.1: [3]

An Intuitionistic fuzzy set (IF for short) A is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$$

where the functions  $\mu_A: X \rightarrow [0,1]$  and  $\nu_A: X \rightarrow [0,1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set A respectively, and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in X$ .

Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form

$$A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X \}.$$

### Definition 2.2: [3]

Let A and B be two Intuitionistic fuzzy sets of the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \} \text{ and } B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}.$$

(i)  $A \subseteq B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$ ,

(ii)  $A = B$  if and only if  $A \subseteq B$  and  $A \supseteq B$ ,

$$(iii) A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \},$$

$$(iv) A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X \},$$

$$(v) A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X \}.$$

$$(vi) [A] = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle, x \in X \};$$

$$(vii) \langle A \rangle = \{ \langle x, 1 - \nu_A(x), \nu_A(x) \rangle, x \in X \};$$

The Intuitionistic fuzzy sets  $0 \sim = \langle x, 0, 1 \rangle$  and  $1 \sim = \langle x, 1, 0 \rangle$  are respectively the empty set and the whole set of X

### Definition 2.3. [3]

Let  $\{A_i : i \in J\}$  be an arbitrary family of Intuitionistic fuzzy sets in X. Then

$$(i) \bigcap A_i = \{ \langle x, \bigwedge \mu_{A_i}(x), \bigvee \nu_{A_i}(x) \rangle : x \in X \};$$

$$(ii) \bigcup A_i = \{ \langle x, \bigvee \mu_{A_i}(x), \bigwedge \nu_{A_i}(x) \rangle : x \in X \}.$$

### Definition 2.4. [3]

Since our main purpose is to construct the tools for developing Intuitionistic fuzzy topological spaces, we must introduce the Intuitionistic fuzzy sets  $0 \sim$  and  $1 \sim$  in X as follows:

$$0 \sim = \{ \langle x, 0, 1 \rangle : x \in X \} \text{ and } 1 \sim = \{ \langle x, 1, 0 \rangle : x \in X \}.$$

### Definition 2.5:[3]

Let A,B,C are Intuitionistic fuzzy sets in X. Then

$$(i) A \subseteq B \text{ and } C \subseteq D \Rightarrow A \cup C \subseteq B \cup D \text{ and } A \cap C \subseteq B \cap D,$$

$$(ii) A \subseteq B \text{ and } A \subseteq C \Rightarrow A \subseteq B \cap C,$$

$$(iii) A \subseteq C \text{ and } B \subseteq C \Rightarrow A \cup B \subseteq C,$$

$$(iv) A \subseteq B \text{ and } B \subseteq C \Rightarrow A \subseteq C,$$

$$(v) \overline{A \cup B} = \bar{A} \cap \bar{B}$$

$$(vi) \overline{A \cap B} = \bar{A} \cup \bar{B},$$

$$(vii) A \subseteq B \Rightarrow \bar{B} \subseteq \bar{A},$$

(viii)  $\overline{\overline{A}} = A$ ,

(ix)  $\overline{1\sim} = 0\sim$ ,

(x)  $\overline{0\sim} = 1\sim$ .

**Definition 2.6.[4]**

Let  $f$  be a mapping from an ordinary set  $X$  into an ordinary set  $Y$ ,

If  $B = \{ \langle y, \mu_B(y), \nu_B(y) \rangle : y \in Y \}$  is an Intuitionistic fuzzy set in  $Y$ , then the inverse image of  $B$  under

$f$  is an Intuitionistic fuzzy set defined by  $f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) \rangle : x \in X \}$

The image of Intuitionistic fuzzy set  $A = \{ \langle y, \mu_A(y), \nu_A(y) \rangle : y \in Y \}$  under  $f$  is an Intuitionistic fuzzy set defined by

$$f(A) = \{ \langle y, f(\mu_A)(y), f(\nu_A)(y) \rangle : y \in Y \}.$$

**Definition 2.7[4]**

Let  $A, A_i$  ( $i \in J$ ) be Intuitionistic fuzzy sets in  $X$ ,  $B, B_i$  ( $i \in K$ ) be Intuitionistic fuzzy sets in  $Y$  and  $f : X \rightarrow Y$  is a function. Then

(i)  $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$ ,

(ii)  $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$ ,

(iii)  $A \subseteq f^{-1}(f(A))$  { If  $f$  is injective, then  $A = f^{-1}(f(A))$  },

(iv)  $f(f^{-1}(B)) \subseteq B$  { If  $f$  is surjective, then  $f(f^{-1}(B)) = B$  },

(v)  $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

(vi)  $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

(vii)  $f(\cup B_j) = \cup f(B_j)$

(viii)  $f(\cap B_j) \subseteq \cap f(B_j)$  { If  $f$  is injective, then  $f(\cap B_j) = \cap f(B_j)$  }

(ix)  $f^{-1}(1\sim) = 1\sim$ ,

(x)  $f^{-1}(0\sim) = 0\sim$ ,

(xi)  $f(1\sim) = 1\sim$ , if  $f$  is surjective

(xii)  $f(0\sim) = 0\sim$ ,

(xiii)  $\overline{f(A)} \subseteq f(\overline{A})$ , if  $f$  is surjective,

(xiv)  $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$ .

**Definition 2.8[21]**

A family  $\tau_\mu$  Intuitionistic fuzzy sets on  $X$  is called an Intuitionistic fuzzy supra topology (in short, IFST) on  $X$

if  $0\sim \in \tau_\mu, 1\sim \in \tau_\mu$  and  $\tau_\mu$  is closed under arbitrary suprema.

Then we call the pair  $(X, \tau_\mu)$  an Intuitionistic fuzzy supra topological space (in short, IFSTS).

Each member of  $\tau_\mu$  is called an Intuitionistic fuzzy supra open set and the complement of an Intuitionistic fuzzy supra open set is called an Intuitionistic fuzzy supra closed set.

**Definition 2.9[21]**

The Intuitionistic fuzzy supra closure of a set  $A$  is denoted by  $S\text{-cl}(A)$  and is defined as

$$S\text{-cl}(A) = \cap \{ B : B \text{ is Intuitionistic fuzzy supra closed and } A \subseteq B \}.$$

The Intuitionistic fuzzy supra interior of a set  $A$  is denoted by  $S\text{-int}(A)$  and is defined as

$$S\text{-int}(A) = \cup \{ B : B \text{ is Intuitionistic fuzzy supra open and } A \supseteq B \}$$

**Definition 2.10 [21]**

(i).  $\neg(A \supseteq B) \Leftrightarrow A \subseteq B^c$ .

(ii).  $A$  is an Intuitionistic fuzzy supra closed set in  $X \Leftrightarrow S\text{-cl}(A) = A$ .

(iii).  $A$  is an Intuitionistic fuzzy supra open set in  $X \Leftrightarrow S\text{-int}(A) = A$ .

(iv).  $S\text{-cl}(A^c) = (S\text{-int}(A))^c$ .

(v).  $S\text{-int}(A^c) = (S\text{-cl}(A))^c$ .

(vi).  $A \subseteq B \Rightarrow S\text{-int}(A) \subseteq S\text{-int}(B)$ .

(vii).  $A \subseteq B \Rightarrow S\text{-cl}(A) \subseteq S\text{-cl}(B)$ .

(viii).  $S\text{-cl}(A \cup B) = S\text{-cl}(A) \cup S\text{-cl}(B)$ .

(ix).  $S\text{-int}(A \cap B) = S\text{-int}(A) \cap S\text{-int}(B)$ .

**Definition 2.11**

Let  $(X, \tau_\mu)$  be an Intuitionistic fuzzy supra topological space. An IFS  $A \in IF(X)$  is called

Intuitionistic fuzzy semi-supra open [12] iff  $A \subseteq S\text{-cl}(S\text{-int}(A))$ ,

**Definition 2.12: [21]**

Let  $(X, \tau_\mu)$  and  $(Y, \sigma_\mu)$  be two Intuitionistic fuzzy supra topological spaces and let  $f : X \rightarrow Y$  be a function. Then  $f$  is said to be

(i) Intuitionistic fuzzy supra continuous if the pre image of each Intuitionistic fuzzy supra open set of  $Y$  is an Intuitionistic fuzzy supra open set in  $X$ .

(ii) Intuitionistic fuzzy supra closed if the image of each Intuitionistic fuzzy supra closed set in  $X$  is an Intuitionistic fuzzy supra closed set in  $Y$ .

(iii) Intuitionistic fuzzy supra open if the image of each Intuitionistic fuzzy supra open set in  $X$  is an Intuitionistic fuzzy supra open set in  $Y$ .

**Definition 2.13. [12]**

Let  $(X, \tau_\mu)$  be a Intuitionistic fuzzy supra topological space. Then  $A$  is called Intuitionistic fuzzy supra semi open set (IFS supra open set for short) if  $A \subseteq S\text{-cl}(S\text{-int}(A))$ .

**Definition 2.14. [12]**

Let  $(X, \tau_\mu)$  be a Intuitionistic fuzzy supra topological space. Then  $A$  is called Intuitionistic fuzzy semi supra closed set (IFS supra-closed set for short) if  $S\text{-int}(S\text{-cl}(A)) \subseteq A$ .

**Definition 2.15. [12]**

Let  $A$  be a Intuitionistic fuzzy set of a Intuitionistic fuzzy supra topological space  $(X, \tau_\mu)$ . Then,

- i). The Intuitionistic fuzzy supra semi closure of  $A$  is defined as  

$$\text{IFS supra-cl}(A) = \bigcap \{K: K \text{ is a IFS-supra closed in } X \text{ and } A \subseteq K\}$$
- ii). The Intuitionistic fuzzy supra semi interior of  $A$  is defined as  

$$\text{IFS-int}(A) = \bigcup \{G: G \text{ is a IFS-supra open in } X \text{ and } G \subseteq A\}$$

### 3. INTUITIONISTIC FUZZY $\omega$ SUPRA CLOSED SETS

In this section, we introduce the concept of Intuitionistic fuzzy  $\omega$  supra closed sets (Shortly IF $\omega$ -supra closed set) and some of their properties. Throughout this paper  $(X, \tau_\mu)$  represent a Intuitionistic fuzzy supra topological spaces.

#### Definition 3.1.

Let  $(X, \tau_\mu)$  be a Intuitionistic fuzzy supra topological space. Then  $A$  is called Intuitionistic fuzzy supra  $\omega$  closed set (IF $\omega$ -supra closed set for short) if  $S\text{-cl}(A) \subseteq G$  whenever  $A \subseteq G$  and  $G$  is Intuitionistic fuzzy semi supra open set.

#### Theorem 3.2.

Every Intuitionistic fuzzy supra closed set is IF $\omega$ -supra closed set, but the converse may not be true.

#### Proof:

If  $A$  is any Intuitionistic fuzzy set in  $X$  and  $G$  is any IFS-supra open set containing  $A$ , then  $S\text{-cl}(A) \subseteq G$ . Hence  $A$  is IF $\omega$ -supra closed set. The converse of the above theorem need not be true as seen from the following example.

#### Example 3.3.

Let  $X = \{a, b\}$  and  $\tau_\mu = \{0\sim, 1\sim, A_1, A_2, A_1 \cup A_2\}$  be an Intuitionistic fuzzy supra topology on  $X$ , where  $A_1 = \{x, \langle 0.3, 0.5 \rangle, \langle 0.6, 0.7 \rangle\}$ ,  $A_2 = \{x, \langle 0.5, 0.3 \rangle, \langle 0.4, 0.5 \rangle\}$ .

Then the Intuitionistic fuzzy set  $A = \{x, \langle 0.3, 0.3 \rangle, \langle 0.3, 0.3 \rangle\}$  is Intuitionistic fuzzy  $\omega$  supra closed but it is not Intuitionistic fuzzy supra closed.

#### Proposition 3.4.

The concepts of IF $\omega$ -supra closed sets and IFS-supra closed sets are independent.

#### Example 3.5.

Let  $X = \{a, b\}$  and  $\tau_\mu = \{0\sim, 1\sim, A_1, A_2, A_1 \cup A_2\}$  is a Intuitionistic fuzzy supra topology and  $(X, \tau_\mu)$  is a Intuitionistic fuzzy supra topological spaces.

where  $A_1 = \{x, \langle 0.2, 0.4 \rangle, \langle 0.5, 0.6 \rangle\}$ ,  $A_2 = \{x, \langle 0.4, 0.2 \rangle, \langle 0.3, 0.4 \rangle\}$ .

Then the Intuitionistic fuzzy set  $A = \{x, \langle 0.2, 0.2 \rangle, \langle 0.2, 0.2 \rangle\}$  Then the set  $A$  is IF $\omega$ -supra closed set but  $A$  is not a IFS-supra closed set.

#### Example 3.6.

Let  $X = \{a, b\}$  and  $\tau_\mu = \{0\sim, 1\sim, A_1, A_2, A_1 \cup A_2\}$  is a Intuitionistic fuzzy supra topology and  $(X, \tau_\mu)$  is a Intuitionistic fuzzy supra topological spaces.

where  $A_1 = \{x, \langle 0.2, 0.4 \rangle, \langle 0.5, 0.6 \rangle\}$ ,  $A_2 = \{x, \langle 0.4, 0.2 \rangle, \langle 0.3, 0.4 \rangle\}$ .

Then the Intuitionistic fuzzy set  $A = \{x, \langle 0.4, 0.4 \rangle, \langle 0.4, 0.4 \rangle\}$  Then the set  $A$  is IFS-supra closed set but  $A$  is not a IF $\omega$ -supra closed.

#### Theorem 3.7.

If  $A$  and  $B$  are IF $\omega$ -supra closed sets, then  $A \cup B$  is IF $\omega$ -supra closed set.

#### Proof:

If  $A \cup B \subseteq G$  and  $G$  is IFS-supra open set, then  $A \subseteq G$  and  $B \subseteq G$ . Since  $A$  and  $B$  are IF $\omega$ -supra closed sets,  $S\text{-cl}(A) \subseteq G$  and  $S\text{-cl}(B) \subseteq G$  and hence  $S\text{-cl}(A) \cup S\text{-cl}(B) \subseteq G$ . This implies  $S\text{-cl}(A \cup B) \subseteq G$ . Thus  $A \cup B$  is IF $\omega$ -supra closed set in  $X$ .

#### Theorem 3.8.

A Intuitionistic fuzzy set  $A$  is IF $\omega$ -supra closed set then  $S\text{-cl}(A) - A$  does not contain any nonempty Intuitionistic fuzzy supra closed sets.

#### Proof:

Suppose that  $A$  is IF $\omega$ -supra closed set. Let  $F$  be a Intuitionistic fuzzy supra closed subset of  $S\text{-cl}(A) - A$ . Then  $A \subseteq F^c$ . But  $A$  is IF $\omega$ -supra closed set. Therefore  $S\text{-cl}(A) \subseteq F^c$ . Consequently  $F \subseteq (S\text{-cl}(A))^c$ . We have  $F \subseteq S\text{-cl}(A)$ . Thus  $F \subseteq S\text{-cl}(A) \cap (S\text{-cl}(A))^c = \phi$ . Hence  $F$  is empty.

#### Theorem 3.9.

A Intuitionistic fuzzy set  $A$  is IF $\omega$ -supra closed set if and only if  $S\text{-cl}(A) - A$  contains no non-empty IFS-supra closed set.

#### Proof:

Suppose that  $A$  is IF $\omega$ -supra closed set. Let  $H$  be a IFS-supra closed subset of  $S\text{-cl}(A) - A$ . Then  $A \subseteq H^c$ . Since  $A$  is IF $\omega$ -supra closed set, we have  $S\text{-cl}(A) \subseteq H^c$ . Consequently  $H \subseteq (S\text{-cl}(A))^c$ . Hence  $H \subseteq S\text{-cl}(A) \cap (S\text{-cl}(A))^c = \phi$ . Therefore  $H$  is empty.

Conversely, suppose that  $S\text{-cl}(A) - A$  contains no nonempty IFS-supra closed set. Let  $A \subseteq G$  and that  $G$  be IFS-supra open. If  $S\text{-cl}(A) \not\subseteq G$ , then  $S\text{-cl}(A) \cap G^c$  is a non-empty IFS-supra closed subset of  $S\text{-cl}(A) - A$ . Hence  $A$  is IF $\omega$ -supra closed set.

#### Corollary 3.10.

A IF $\omega$ -supra closed set  $A$  is IFS-supra closed if and only if IFS supra -cl( $A$ ) -  $A$  is IFS-supra closed.

#### Proof:

Let  $A$  be any IF $\omega$ -supra closed set. If  $A$  is IFS-supra closed set, then IFS supra -cl( $A$ ) -  $A = \phi$ . Therefore IFS supra -cl( $A$ ) -  $A$  is IFS-supra closed set.

Conversely, suppose that  $S\text{-cl}(A) - A$  is IFS-supra closed set. But  $A$  is IF $\omega$ -supra closed set and  $S\text{-cl}(A) - A$  contains IFS-supra closed. By theorem 3.9, IFS supra -cl( $A$ ) -  $A = \phi$ . Therefore IFS supra -cl( $A$ ) =  $A$ . Hence  $A$  is IFS-supra closed set.

#### Theorem 3.11.

Suppose that  $B \subseteq A \subseteq X$ ,  $B$  is a IF $\omega$ -supra closed set relative to  $A$  and that  $A$  is IF $\omega$ -supra closed set in  $X$ . Then  $B$  is IF $\omega$ -supra closed set in  $X$ .

#### Proof:

Let  $B \subseteq G$ , where  $G$  is IFS-supra open in  $X$ . We have  $B \subseteq A \cap G$  and  $A \cap G$  is IFS-supra open in  $A$ . But  $B$  is a IF $\omega$ -supra closed set relative to  $A$ . Hence  $S\text{-cl}_A(B) \subseteq A \cap G$ . Since  $S\text{-cl}_A(B) = A \cap S\text{-cl}(B)$ . We have  $A \cap S\text{-cl}(B) \subseteq A \cap G$ . It implies  $A \subseteq G \cup (S\text{-cl}(B))^c$  and  $G \cup (S\text{-cl}(B))^c$  is a IFS-supra open set in  $X$ . Since  $A$  is IF $\omega$ -supra closed in  $X$ , we have  $S\text{-cl}(A) \subseteq G \cup (S\text{-cl}(B))^c$ . Hence  $S\text{-cl}(B) \subseteq G \cup (S\text{-cl}(B))^c$  and  $S\text{-cl}(B) \subseteq G$ . Therefore  $B$  is IF $\omega$ -supra closed set relative to  $X$ .

**Theorem 3.12.**

If  $A$  is  $IF\omega$ -supra closed and  $A \subseteq B \subseteq S\text{-cl}(A)$ , then  $B$  is  $IF\omega$ -supra closed.

**Proof:**

Since  $B \subseteq S\text{-cl}(A)$ , we have  $S\text{-cl}(B) \subseteq S\text{-cl}(A)$  and  $S\text{-cl}(B) - B \subseteq S\text{-cl}(A) - A$ . But  $A$  is  $IF\omega$ -supra closed. Hence  $S\text{-cl}(A) - A$  has no non-empty IFS-supra closed subsets, neither does  $S\text{-cl}(B) - B$ . By theorem 3.9,  $B$  is  $IF\omega$ -supra closed.

**Theorem 3.13.**

Let  $A \subseteq Y \subseteq X$  and suppose that  $A$  is  $IF\omega$ -supra closed in  $X$ . Then  $A$  is  $IF\omega$ -supra closed relative to  $Y$ .

**Proof:**

Let  $A \subseteq Y \cap G$  where  $G$  is IFS-supra open in  $X$ . Then  $A \subseteq G$  and hence  $S\text{-cl}(A) \subseteq G$ . This implies,  $Y \cap S\text{-cl}(A) \subseteq Y \cap G$ . Thus  $A$  is  $IF\omega$ -supra closed relative to  $Y$ .

**Theorem 3.14.**

If  $A$  is IFS-supra open and  $IF\omega$ -supra closed, then  $A$  is Intuitionistic fuzzy supra closed set.

**Proof:**

Since  $A$  is IFS-supra open and  $IF\omega$ -supra closed, then  $S\text{-cl}(A) \subseteq A$ . Therefore  $S\text{-cl}(A) = A$ . Hence  $A$  is Intuitionistic fuzzy supra closed.

**4. INTUITIONISTIC FUZZY SUPRA  $\omega$  OPEN SETS**

In this section, we introduce and study about Intuitionistic fuzzy supra  $\omega$  open sets and some of their properties.

**Definition 4.1.**

A Intuitionistic fuzzy set  $A$  in  $X$  is called  $IF\omega$ -supra open in  $X$  if  $A^C$  is  $IF\omega$ -supra closed in  $X$ .

**Theorem 4.2.**

A Intuitionistic fuzzy set  $A$  is  $IF\omega$ -supra open if and only if  $F \subseteq S\text{-int}(A)$  where  $F$  is IFS-supra closed and  $F \subseteq A$ .

**Proof:**

Suppose that  $F \subseteq S\text{-int}(A)$  where  $F$  is IFS-supra closed and  $F \subseteq A$ . Let  $A^C \subseteq G$  where  $G$  is IFS-supra open. Then  $G^C \subseteq A$  and  $G^C$  is IFS-supra closed. Therefore  $G^C \subseteq S\text{-int}(A)$ . Since  $G^C \subseteq S\text{-int}(A)$ , we have  $(S\text{-int}(A))^C \subseteq G$ . This implies  $S\text{-cl}((A)^C) \subseteq G$ . Thus  $A^C$  is  $IF\omega$ -supra closed. Hence  $A$  is  $IF\omega$ -supra open.

Conversely, suppose that  $A$  is  $IF\omega$ -supra open,  $F \subseteq A$  and  $F$  is IFS supra -closed. Then  $F^C$  is IFS-supra open and  $A^C \subseteq F^C$ . Therefore  $S\text{-cl}((A)^C) \subseteq F^C$ . But  $S\text{-cl}((A)^C) = (S\text{-int}(A))^C$ . Hence  $F \subseteq S\text{-int}(A)$ .

**Theorem 4.3.**

A Intuitionistic fuzzy set  $A$  is  $IF\omega$ -supra open in  $X$  if and only if  $G = X$  whenever  $G$  is IFS-supra open and  $(S\text{-int}(A) \cup A^C) \subseteq G$ .

**Proof:**

Let  $A$  be a  $IF\omega$ -supra open,  $G$  be IFS-supra open and  $(S\text{-int}(A) \cup A^C) \subseteq G$ . This implies  $G^C \subseteq (S\text{-int}(A))^C \cap ((A)^C)^C = (S\text{-int}(A))^C - A^C = S\text{-cl}((A)^C) - A^C$ . Since  $A^C$  is  $IF\omega$ -supra closed and  $G^C$  is IFS-supra closed, By theorem 3.12, it follows that  $G^C = \phi$ . Therefore  $X = G$ .

Conversely, suppose that  $F$  is IFS-supra closed and  $F \subseteq A$ . Then  $S\text{-int}(A) \cup A^C \subseteq S\text{-int}(A) \cup F^C$ . This implies  $S\text{-int}(A) \cup F^C = X$  and hence  $F \subseteq S\text{-int}(A)$ . Therefore  $A$  is  $IF\omega$ -supra open.

**Theorem 4.4.**

If  $S\text{-int}(A) \subseteq B \subseteq A$  and if  $A$  is  $IF\omega$ -supra open, then  $B$  is  $IF\omega$ -supra open.

**Proof:**

Suppose that  $S\text{-int}(A) \subseteq B \subseteq A$  and  $A$  is  $IF\omega$ -supra open. Then  $A^C \subseteq B^C \subseteq S\text{-cl}(A^C)$  and since  $A^C$  is  $IF\omega$ -supra closed. We have By theorem 3.9,  $B^C$  is  $IF\omega$ -supra closed. Hence  $B$  is  $IF\omega$ -supra open.

**Theorem 4.5.**

A Intuitionistic fuzzy set  $A$  is  $IF\omega$ -supra closed, if and only if  $S\text{-cl}(A) - A$  is  $IF\omega$ -supra open.

**Proof:**

Suppose that  $A$  is  $IF\omega$ -supra closed set. Let  $H$  be a IFS supra -closed subset of  $S\text{-cl}(A) - A$ . Then  $A \subseteq H^C$ . Since  $A$  is  $IF\omega$ -supra closed set, we have  $S\text{-cl}(A) \subseteq H^C$ . Consequently  $H \subseteq (S\text{-cl}(A))^C$ . Hence  $S \subseteq S\text{-cl}(A) \cap (S\text{-cl}(A))^C = \phi$ . Therefore  $H$  is empty.

Conversely, suppose that  $S\text{-cl}(A) - A$  contains no nonempty IFS-supra closed set. Let  $A \subseteq G$  and that  $G$  be IFS-supra open. If  $S\text{-cl}(A) \not\subseteq G$ , then  $S\text{-cl}(A) \cap G^C$  is a non-empty IFS-supra closed subset of  $S\text{-cl}(A) - A$ . Hence  $A$  is  $IF\omega$ -supra closed set.

**Theorem 4.6.**

A Intuitionistic fuzzy set  $A$  is  $IF\omega$ -supra closed, if and only if  $S\text{-cl}(A) - A$  is  $IF\omega$ -supra open.

**Proof:**

Suppose that  $A$  is  $IF\omega$ -supra closed. Let  $F \subseteq S\text{-cl}(A) - A$  Where  $F$  is IFS supra -closed. By theorem 3.9,  $F = \phi$ . Therefore  $F \subseteq S\text{-int}((S\text{-cl}(A) - A))$  and by Theorem 4.2, we have  $S\text{-cl}(A) - A$  is  $IF\omega$ -supra open. Conversely, let  $A \subseteq G$  where  $G$  is a IFS supra -open set. Then  $S\text{-cl}(A) \cap G^C \subseteq S\text{-cl}(A) \cap A^C = S\text{-cl}(A) - A$ . Since  $S\text{-cl}(A) \subseteq G^C$  is IFS supra -closed and  $S\text{-cl}(A) - A$  is  $IF\omega$ -supra open. By Theorem 4.2, we have  $S\text{-cl}(A) \cap G^C \subseteq S\text{-int}(S\text{-cl}(A) - A) = \phi$ . Hence  $A$  is  $IF\omega$ -supra closed.

**Theorem 4.7.**

For a subset  $A \subseteq X$  the following are equivalent:

- (i)  $A$  is  $IF\omega$ -supra closed.
- (ii)  $S\text{-cl}(A) - A$  contains no non-empty IFS supra closed set.
- (iii)  $S\text{-cl}(A) - A$  is  $IF\omega$ -supra open set.

**Proof:**

Follows from theorem 3.9 and Theorem 4.5.

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