Some Aspect of Higher Dimensional Cosmic String Model

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Abstract : -*Cosmic strings are important in the early stages of evolutions of the universe before the particle creation and can explain galaxy formation. Hence it is interesting to study string cosmology in higher-dimensional space–time. In this paper, it is observed that five dimensional Kaluza-Klein space-time in the presence of cosmic string in the frame work of Seaz and Ballester scalar tensor theory of gravitation, the string cosmological models corresponding to geometric string, Takabayasi's string and Reddy string are remain same. Here we consider the five dimensional Kaluza-Klein metric and discuss a model universe with different situations, by solving the modified Einstein field equations within the frame work of Lyra geometry. The extra dimensions play a physical role and they are being too small are unobservable. The explicit solutions of the scale factor are found via the assumption $R = \sqrt[3]{Ay + M}$. We obtain many interesting realistic solutions governing the present day model of the universe. Also some physical and kinematical properties of the models are discussed.*

Keywords*:- Cosmology, Five dimensional cosmological models, cosmic string, geometric string, Reddy string.*

1. *Introduction*

Cosmic strings are important in the early stages of evolutions of the universe before the particle creation and can explain galaxy formation. Hence it is interesting to study string cosmology in higher-dimensional space–time. In this paper we present a string cosmology in higher-dimensional space–time. The higher dimensional cosmological models play an important role in the description of the universe in its early stages of evolution. Therefore the study of higher dimensional cosmological models in recent years is highly motivated the researchers mainly by the possibility of geometrical unification of the fundamental interaction of the universe. Five dimensional space-time is more attractive because 10D and 11D super gravities admit solutions, which spontaneously reduces to 5D. In these days many efforts have been made to construct alternative theories of gravitation. Einstein's idea of geometrizing gravitation in general theory of relativity motivated others to geometrize other physical things. Exact solutions of Einstein's field equation in higher dimensions are of great interest in several contexts in view of the modern Kaluza-Klein theory. The extra dimensions play a physical role and they are being too small are unobservable. Chodos and Detweiler (1980) constructed a cosmological model which shows the contraction of the extra dimensions as a consequence of cosmological evolutions Guth , Alvarez and Gavela (1983) observed that during contraction process extra dimensions produce large amount of entropy, which provides an alternative resolution to the flatness and horizon problems as compared to usual inflationary scenario. Further Freund (1982), Appelquist and Chodos (1983), Randjbar-Daemi et al. (1984), Rahaman et al. (2002), Singh et al. (2004) and Mohanty et al. (2006), Mete and Elker (2017) claimed through solutions of the field equations that there is an expansion of four dimensional space-time while fifth dimension contracts or remains constant. Very recently the study of string cosmological models in alternative theory of gravitation is gaining momentum. Sen (2000), Barros et al. (2001), Sen et al. (1997), Gundalach and Ortiz (1990), Barros and Romero (1995), Bhattacharjee and Baruah (2001), Rehaman et al. (2003) and Reddy (2005) have presented string cosmological models in alternative theories of gravitation. In particular, Reddy (2003a, 2006) and Reddy et al. (2006) have discussed some string cosmological models in Saez-Ballester scalar-tensor theory of gravitation in four dimensions.

In this paper, we have discussed Nambu-Takabayasi and Reddy's string cosmological models in five dimensional space-time in the frame work of scalar-tensor theory of gravitation.The resulting model can be considered as ananalog of Reddy string (2003a, 2003b and Reddy and Rao 2006a, 2006b) in higher dimensions. The explicit solutions of the scale factor are found via the assumption $Ay + M = R^3$. We obtain many interesting realistic solutions governing the present day model of the universe. Also some physical and kinematical properties of the models are discussed.

2. *Metric And Field Equations*

Consider the five dimensional Kaluza-Klein metric in the form

$$ds^2 = -dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin2\theta d\phi^2\right] + L^2\, t\, d\mu^2 \qquad (1)$$

Unlike Wesson (1983), the fifth co-ordinate is taken to be space-like and the metric coefficients are assumed to be functions of time only. Here the spatial curvature has been taken as zero (Gron 1988). The field equations given by Seaz and Ballester (1985) for the combined scalar and tensor fields are

$$G_{ij} - \omega\varphi^n\left(\varphi_{;i}\varphi_{;j} - \frac{1}{2}g_{ij}\varphi_{;k}\varphi^{;k}\right) = T_{ij} \qquad (2)$$

and the scalar field satisfies the equation

$$2\,\varphi^n\,\varphi^{i}_{;i} + n\,\varphi^{n-1}\;\varphi_{;k}\varphi^{;k} = 0 \qquad (3)$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$, is the Einstein tensor, ω and n are constants, T_{ij} is the energy tensor of the matter. Comma and semicolon denotes partial and covariant differentiation respectively. Also

$$T^{ij}_{;j} = 0 \qquad (4)$$

is a consequence of the field equations (2) and (3).

The energy-momentum tensor for cosmic strings is,

$$T_{ij} = \rho u^i u_i - \lambda x^i x_j \quad (5)$$

Here ρ is the rest energy density of the system of strings with massive particles attached to the strings and λ the tension density of the system of strings. As pointed out by Letelier (1983), λ may be positive or negative; u^i describes the system four velocities and x^i represents direction of anisotropy, i.e. the direction of strings which is taken to be along fifth dimension.

We have $$u^i u_i = 1, \quad x = -1 \text{ and } = 0 \quad (6)$$

We consider,

$$\boldsymbol{y} = \boldsymbol{0}\,; \quad \text{then we get} \quad \boldsymbol{R} = \sqrt[3]{\boldsymbol{M}} \quad (7)$$

$$\rho = \rho_p + \lambda \quad (8)$$

where ρ_p is the rest energy density of particles attached to the strings. Here, we consider φ, ρ and λ are functions of t only.

The field equations (2), (3), and (4) for the metric (1) with the help of (5) and (6) can, explicitly, be written as

$$\frac{3\dot{R}^2}{R^2} + \frac{3\dot{R}\dot{M}}{RM} + \frac{3k}{R^2} + \frac{\omega}{2}\varphi^n\dot{\varphi}^2 = \rho \quad (9)$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{2\dot{R}\dot{M}}{RM} + \frac{k}{R^2} + \frac{\ddot{M}}{M} - \frac{\omega}{2}\varphi^n\dot{\varphi}^2 = 0 \quad (10)$$

$$\frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} - \frac{\omega}{2}\varphi^n\dot{\varphi}^2 = \lambda \quad (11)$$

$$\ddot{\varphi} + \dot{\varphi}\left(\frac{3\dot{R}}{R} + \frac{\dot{M}}{M}\right) - \frac{n}{2}\frac{\dot{\varphi}^2}{\varphi} = 0 \quad (12)$$

$$\rho + (\rho - \lambda)\frac{\dot{M}}{M} + \frac{3\rho\dot{R}}{R} = 0 \quad (13)$$

3. *Solutions of the Cosmic String Models*

The field equation (9) to (13) are four independent equations in five unknowns R, M, φ, ρ and λ. Hence to get a determinate solution one has to assume a physical or mathematical condition. In the literature, we have equations of state for string model (Letelier 1983), are

$\rho = \lambda$ (geometric string or Nambu string)

$\rho = (1 + \omega)\lambda$ (p-string or Takabayasi string)

In addition to above, recently, Reddy (2003a, 2003b), Reddy and Rao (2006); Reddy and Naidu (2007) have obtained inflationary string cosmological models in Brans and Dicke (1961), Seaz and Ballester (1985) and Lyra (1951) scalar-tensor theories of gravitation assuming a relation,

$$\rho + \lambda = 0 \text{ (Reddy string)} \tag{14}$$

i.e. the sum of rest energy density and tension density for a cloud of strings vanishes. The relation (14) is analogous to $\rho + p = 0$ in general relativity with perfect fluid as source which represents false vacuum case.

Here we find string cosmological models corresponding to

(i) $\rho = \lambda$,

(ii) $\rho = (1 + \omega)\lambda$ and

(iii) $\rho + \lambda = 0$ in five dimensions in Saez-Ballester scalar-tensor theory.

Case (i): ***Geometric String Or Nambu String ($\rho = \lambda$)***

we assume **M= R³** because of the fact that field equations are highly non-linear.

Using this relation, the field equation (9 - 13) admits the exact solution

$$R = M_1 t + M_2 = T \tag{15}$$

$$\rho = \lambda = \frac{8M_1^2 + 2k}{T^2} \tag{16}$$

$$\varphi = \left[\frac{M_3}{10M_1} T^{-5} + M_4\right]^{-2} \tag{17}$$

After a suitable choice of co-ordinate and constants of integration, the model corresponding to the solution (15, 16, 17) can be written as

$$ds^2 = -dt^2 - T^2\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin2\theta d\phi^2\right] + T^6 d\mu^2 \tag{18}$$

- ***Some physical properties***

The model given by (18) represents an exact string cosmological model in five dimensions in the framework of Brans–Dicke theory of gravitation, when the sum of the tension density λ

and the rest energy density ρ of the cosmic string vanishes. The model has no initial singularity. In order to gain a further insight into the behaviour of the model, physical and kinematical variables and to have relatively simple picture of their explicit expressions are

Spatial volume (V^3): $$V^3 = \sqrt{-g} = \frac{T^6 r^2 \sin\theta}{\sqrt{1-kr^2}} \qquad (19)$$

Scalar expansion (θ): $$\theta = \frac{1}{3} u^i_{;i} = \frac{1}{3T} \qquad (20)$$

Deceleration parameter (q): $$q = \frac{-3}{\theta^2}\left[\theta_{i\alpha} u^\alpha + \frac{\theta^2}{3}\right] = 2 > 0 \qquad (21)$$

Shear Scalar (σ^2) : $$\sigma^2 = \sigma_{ij}\sigma^{ij} = \frac{1}{54T^2} \qquad (22)$$

Volume increases with time. Since q > 0, therefore the model is not inflationary. Shear decreases as time increases. Since, $lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, therefore the model is anisotropic for values of T. The energy conditions imply that $\rho > 0$ and $\rho_p > 0$ leaving the sign of the string tension density unrestricted. The rest energy density ρ, string tension density λ and particle density ρ_p tend to zero as time increases indefinitely and they possess singularities at $t = 0$. The model (18) describes an expanding string cosmological model. For this model the spatial volume V tends to infinity and the expansion scalar θ and shear scalar tend to zero as $t\to\infty$. The negative value of the deceleration parameter q shows that the model inflates. However, the scalar field in the model is free from initial singularity.

In Case (ii) : ***p-string or Takabayasi string i.e,[$\rho = (1 + \omega) \lambda$)]***

Also we assume **M= R^3** because of the fact that field equations are highly non-linear.

Using this relation, the field equation (9 - 13) admits the exact solution

$$R = M_1 t + M_2 = T \qquad (23)$$

$$\rho = \lambda = \frac{8M_1^2 + 2k}{T^2} \qquad (24)$$

$$\varphi = \left[\frac{M_3}{10M_1} T^{-5} + M_4\right]^{-2} \qquad (25)$$

After a suitable choice of co-ordinate and constants of integration, the model corresponding to the solution (23, 24, 25) can be written as

$$ds^2 = -dt^2 - T^2\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin2\theta d\phi^2\right] + T^6\, d\mu^2 \qquad (26)$$

➢ ***Some physical properties***

The model given by (26) represents an exact string cosmological model in five dimensions in the framework of Brans–Dicke theory of gravitation, when the sum of the tension density λ and the rest energy density ρ of the cosmic string vanish. The model has no initial singularity. In order to gain a further insight into the behaviour of the model, physical and kinematical variables and to have relatively simple picture of their explicit expressions are

Spatial volume (V^3) : $V^3 = \sqrt{-g} = \frac{T^6 r^2 \sin\theta}{\sqrt{1-kr^2}}$ (27)

Scalar expansion (θ): $\theta = \frac{1}{3} u_{;i}{}^{i} = \frac{1}{3T}$ (28)

Deceleration parameter (q): $q = \frac{-3}{\theta^2}\left[\theta_{i\alpha} u^\alpha + \frac{\theta^2}{3}\right] = 2 > 0$ (29)

Shear Scalar (σ^2): $\sigma^2 = \sigma_{ij}\sigma^{ij} = \frac{1}{54T^2}$ (30)

Volume increases with time. Since q > 0, therefore the model is not inflationary. Shear decreases as time increases. Since, $lim_{T\to\infty}\ \frac{\sigma}{\theta} \neq 0$, therefore the model is anisotropic for values of T. The energy conditions imply that $\rho > 0$ and $\rho_p > 0$ leaving the sign of the string tension density unrestricted. The rest energy density ρ, string tension density λ and particle density ρ_p tend to zero as time increases indefinitely and they possess singularities at $t = 0$. The model (26) describes an expanding string cosmological model. For this model the spatial volume V tends to infinity and the expansion scalar θ and shear scalar tend to zero as $t\to\infty$. The negative value of the deceleration parameter q shows that the model inflates. However, the scalar field in the model is free from initial singularity.

Case (iii): ***Reddy string ($\rho + \lambda = 0$)*** *- As above.*

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It is observed that the five dimensional model in **Case (ii)** and **Case (iii)** remain same as in **Case (i).**

4. *Conclusion*

For dimensional Kaluza-Klein space-time in the presence of cosmic string in the frame work of Seaz and Ballester scalar tensor theory of gravitation, it is observed that string cosmological models corresponding to geometric string, Takabayasi string and Reddy string remain same. Equation (15, 16, 17) represents some physical and kinematical properties of the models. The model (18) describes an expanding string cosmological model. For this model the spatial volume V tends to infinity and the expansion scalar θ and shear scalar tend to zero as $t \to \infty$. The negative value of the deceleration parameter q shows that the model inflates. However, the scalar field in the model is free from initial singularity. It can be easily seen that physical quantities like energy density, tension density of the string and the scalar field diverges as $T \to 0$. This behaviour is similar to that of the cosmic string model obtained by Reddy in four dimensional Lyra manifold.

5. *Future Prospects*

The investigation on this topic can be further taken up in different directions:

- This topic has been a proliferation of works on higher dimensional cosmological model in Lyra geometry with scalar tensor theory of gravitation.
- It is important in a natural way to make a search for exact solutions for Kaluza-Klein space-time in the presence of cosmic string with different types of distributions of matter and for different type of symmetries of space time.
- This also helpful to provide the idea about study of physical situation at the early stages of the formation of the universe.

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➢ *Nomenclature*

k Gaussian curvature or reduced circumference
p pressure
q deceleration parameter
R_{ij} Ricci tensor
R(t) length Scale
T_{ij} energy-momentum tensor
u^i N-dimensional velocity vector or describes the system four velocities
V^3 Spatial volume
x^i represents direction of anisotropy

➢ *Greek Symbols*

λ tension density of the system of strings
$d\mu^2$ line element
φ_i displacement vector
ρ energy density
ρ_p the rest energy
σ^2 Shear Scalar
Θ Scalar expansion