Interval Valued Vague Li – Ideals of Lattice Implication Algebras

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Abstract : We introduce the concept of Interval valued vague LI – ideals and Interval valued vague lattice ideals of lattice implication algebra by linking the Interval valued vague set and LI-ideal theory of Lattice implication algebras. We discuss the relation between these ideals.

IndexTerms **- Lattice implication algebras, Li – ideals, Interval valued vague LI – ideals.**

I. Introduction

In order to research the logical system whose proportional value is given lattice, Y. XU [5] proposed the concept of lattice implication algebras, and discussed their some properties. Y.XU, Y.B. Jun and E.H. Roh [4] introduced the notion of LI – ideals of a lattice implication algebras, and discussed their some properties.

Vague set theory was first introduced by Gau and Buehrer[3] in1993. The vague set is an extension of fuzzy set. A vague set H in the universal of discourse U is characterized by a truth membership function tA and a false membership function fA. Actually, vague sets can realistically reflect the actual problem. But more often, the truth-membership and false-membership are in a range. For this reason, the notion of interval valued vague sets was presented by Atanassov in 1989 [1]. And it is regarded as an extension of the theory of vague sets. In this theory, the truth-membership function and false-membership function are a subinterval on [0,1]. Anitha.t, Amarendra.V [2] introducd the notion of vague LI – ideals of lattice implication algebras L.

The object of this paper is to make a study of Properties of Interval valued vague LI – ideals and Interval valued vague lattice ideals of lattice implication algebras L.

II. Preliminaries

Definition 2.1: [5] Let (L ,∨, ∧, ', 0, I) be a complemented lattice with the universal bounds 0, I and →: L × L → L be a mapping. (L, ∨, ∧, →, ', 0, I) is called a lattice implication algebra, if the following axioms hold, ∀ x, y, z ϵ L:

(I_1) $x \to (y \to z) = y \to (x \to z)$;
(I_2) $x \to x = I$;
(I_3) $x \to y = y' \to x'$;
(I_4) $x \to y = y \to x = I$ implies $x = y$;
(I_5) $(x \to y) \to y = (y \to x) \to x$;
(L_1) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$;
(L_2) $(x \wedge y) \to z = (x \to z) \vee (y \to z)$.

Theorem 2.2:[5] Let L be a lattice implication algebra. Then for any x, y, z ϵ L, the following conclusions hold:

(1) If $I \to x = I$ then $x = I$.
(2) $I \to x = x$ and $x \to 0 = x$.
(3) $0 \to x = I$ and $x \to I = I$.
(4) $x \leq y$ if and only if $x \to y = I$.
(5) $x \to y \geq x' \vee y$.

Definition 2.3: [4] Let A be a subset of a lattice implication algebra L. A is said to be an LI - ideal of L if it satisfies the following conditions:

(1) 0 ϵ A ;
(2) ∀ x, y ϵ *L*, $(x \to y)'$ ϵ A and y ϵ A imply x ϵ A.

Definition 2.4:[4] Let A be a subset of a lattice implication algebra L. A is said to be an lattice ideal of L if it satisfies the following conditions:

(1) x ϵ A, y ϵ L and $y \leq x$ imply that y ϵ A:
(2) ∀ x, y ϵ A imply $x \vee y$ϵ A.

Definition2.5: [3] A vague set A in the universal of discourse X is characterized by two membership functions given by:

(1) A truth membership function $t_A : X \to [0,1]$ and
(2) A false membership function $f_A : X \to [0,1]$,

Where $t_A(x)$ is a lower bound of the grade of membership of x derived from the "evidence for x", and $f_A(x)$ is a lower bound on the negation of x derived from the "evidence against x" and $t_A(x) + f_A(x) \leq 1$.Thus the grade of membership of x in the vague set A is bounded by subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. The vague set A is written as

$A =\{\langle x, [t_A(x), f_A(x)] \rangle / x \in X \}$.

Where the interval $[t_A(x), 1 - f_A(x)]$ is called the value of x in the vague set A and denoted by $V_A(x)$.

Notation [8]: Let I [0, 1] denote the family of all closed subintervals of [0, 1]. If $I_1 = [a_1, b_1]$, $I_2 = [a_2, b_2]$ are two elements of I [0, 1], we call $I_1 \geq I_2$ if $a_1 \geq a_2$ and $b_1 \geq b_2$. We define the term imax to mean the maximum of two interval as
$\text{imax}[I_1, I_2] = [\max\{a_1, a_2\}, \max\{b_1, b_2\}]$.
Similarly, we can define the term imin of any two intervals.

Definition 2.6[1]: An interval valued vague set A in the universe of discourse *U* is characterized by a truth-membership function T_G and false membership function F_G given by

$T_A : U \rightarrow I[0, 1]$, $F_A : U \rightarrow I[0, 1]$

Where T_A and F_A are set-valued functions on the interval [0,1], respectively. $T_A(z) = [T_A^-(z), T_A^+(z)]$, $T_A^-(z)$ *and* $T_A^+(z)$denote the lower and upper bound on the grade of membership of z derived from "the evidence for z", respectively.

Similarly, $F_A(z) = [F_A^-(z), F_A^+(z)]$, $F_A^-(z)$ *and* $F_A^+(z)$denote, respectively, the lower and upper bound on the negation of z derived from "the evidence against z", and $T_A^+(z) + F_A^+(z) \leq 1$.

The interval valued vague set *G* is denoted by $A = \{< z, T_A(z), F_A(z) > / z \in U\}$.

Definition 2.7: [2]Let A be a vague set of a lattice implication algebra L. A is said to be a vague LI - ideal of L if it satisfies the following conditions:

(1) $\forall x \in L$, $V_A(0) \geq V_A(x)$,
(2) $\forall x, y \in L$, $V_A(x) \geq \text{imin}\{V_A((x \rightarrow y)'), V_A(y)\}$.

III. Interval Valued Vague LI– Ideals

Definition 3.1: Let A be a Interval valued vague set of a lattice implication algebra L. A is said to be a Interval valued vague LI - ideal (briefly IVVLI) of L if it satisfies the following conditions:

$T_A(x) \geq \min\{T_A((x \rightarrow y)'), T_A(y)\}$ and $1 - F_A(x) \geq \min\{1 - F_A((x \rightarrow y)'), 1 - F_A(y)\}$, for all x,y $\in L$.
That is $T_A^+(x) \geq \min\{T_A^+((x \rightarrow y)'), T_A^+(y)\}$, $T_A^-(x) \geq \min\{T_A^-((x \rightarrow y)'), T_A^-(y)\}$ and
$1 - F_A^+(z) \geq \min\{1 - F_A^+(z)((x \rightarrow y)'), 1 - F_A^+(z)(y)\}$, $1 - F_A^-(x) \geq \min\{1 - F_A^-((x \rightarrow y)'), 1 - F_A^-(y)\}$, for all x,y $\in L$.

Example 3.2: Let L = {0, a, b, c, d, I} be a set with Cayley table as follows:

→	0	a	b	c	d	I
0	I	I	I	I	I	I
A	c	I	b	c	b	I
B	d	a	I	b	a	I
C	a	a	I	I	a	I
d	b	I	I	b	I	I
I	0	a	b	c	d	I

Define $'$, $\vee$ *and* $\wedge$ −operations on L as follows:

$x' = x \rightarrow 0$, $x \vee y = (x \rightarrow y) \rightarrow y$, $x \wedge y = ((x' \rightarrow y') \rightarrow y')'$, for all x, y $\in L$.
Then $(L, \vee, \wedge, \rightarrow, ', 0, I)$ is a lattice implication algebra [13]. Let $A = \{< x, T_A(x), F_A(x) > / x \in L\}$ be a the intetval valued vague set as follows:

	T_A^+	T_A^-	F_A^+	F_A^-
0	0.7	0.65	0.2	0.18
a	0.5	0.45	0.31	0.22
b	0.5	0.45	0.31	0.22
c	0.7	0.65	0.2	0.18
d	0.5	0.45	0.31	0.22
I	0.5	0.45	0.31	0.22

Then A is a IVVLI-ideal of L.

Theorem 3.3: Every IVVLI- ideal A of a lattice implication algebra L is order reversing.

Proof: Let A be a IVVLI – ideal of L.
If x, y ϵ L and $x \leq y$, then$(x \to y)' = I' = 0$, and so
$T_A^+(x) \geq \min\{T_A^+((x \to y)'), T_A^+(y)\} = \min\{T_A^+(0), T_A^+(y)\} = T_A^+(y)$,
$T_A^-(x) \geq \min\{T_A^-((x \to y)'), T_A^-(y)\} = \min\{T_A^-(0), T_A^-(y)\} = T_A^-(y)$
And $1 - F_A^+(z) \geq \min\{1 - F_A^+(z)((x \to y)'), 1 - F_A^+(z)(y)\}$
$= \min\{1 - F_A^+(z)(0), 1 - F_A^+(z)(y)\}$
$= 1 - F_A^+(z)(y)$
$1 - F_A^-(x) \geq \min\{1 - F_A^-((x \to y)'), 1 - F_A^-(y)\}$
$= \min\{1 - F_A^-(0), 1 - F_A^-(y)\}$
$= 1 - F_A^-(y)$
So $T_A(x) \geq T_A(y)$, $F_A(x) \geq F_A(y)$.
This shows that A is order reversing.

Definition 3.4: Let A be a interval valued vague set of a lattice implication algebra L. A is said to be a interval valued vague lattice ideal of L if it satisfies the following conditions:
(1) $y \leq x$ then $T_A(x) \geq T_A(y)$, $1 - F_A(x) \geq 1 - F_A(y)$,
(2) $T_A(x \vee y) \geq \text{imin}\{T_A(x), T_A(y)\}$,
$1-F_A(x \vee y) \geq \text{imin}\{1-F_A(x), 1-F_A(y)\}$ for x, y ϵ L.

Example 3.5: Let $(L\vee, \wedge, \to, ', 0, I)$ is a lattice implication algebra in example 3.3 . Define a vague set A of L as follows:

	T_A^+	T_A^-	F_A^+	F_A^-
0	0.7	0.65	0.2	0.18
a	0.5	0.45	0.31	0.22
b	0.5	0.45	0.31	0.22
c	0.5	0.45	0.31	0.22
d	0.7	0.65	0.2	0.18
I	0.5	0.45	0.31	0.22

Then clearly A is a vague lattice ideal of L.

Theorem 3.6: Every IVLI – ideal of a lattice implication algebra L is a interval valued vague lattice ideal of L.

Proof: Let A be a VILI – ideal of a lattice implication algebra L.
Let x, y ϵ L such that $x \leq y$.
Then clearly $T_A(x) \geq T_A(y)$, $F_A(x) \geq F_A(y)$.
By $((x \vee y) \to y)' = ((x \to y) \wedge (y \to y))'$ (by L_1)
$= (x \to y)'$
$\leq (x' \vee y)'$
$= (x')' \wedge y'$

$= x \wedge y'$

$\leq x,$

We get $T_A(((x \vee y) \to y)') \geq T_A(x)$, $1- F_A(((x \vee y) \to y)') \geq 1- F_A(x)$.

From the definition of IVLI – ideal , we have

$$T_A(x \vee y) \geq \text{imin}\{T_A(((x \vee y) \to y)'), T_A(y)\}$$
$$\geq \text{imin}\{T_A(x), T_A(y)\}$$

And $1-F_A(x \vee y) \geq \text{imin}\{1- F_A(((x \vee y) \to y)'), 1-F_A(y)\}$

$$\geq \text{imin}\{1-F_A(x), 1-F_A(y)\}$$

Hence every vVILI – ideal of L is a interval valued vague lattice ideal of L.

Remark 3.7: Converse of the above theorem need not to be true. For example the interval valued vague set A defined in example 3.7 is a interval valued vague lattice ideal of L, but not a IVVLI- ideal since $T_A(a) \ngeq \text{imin}\{T_A((a \to d)'), V_A(d)\}$.

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IV. Conclusion

The concept of Interval valued vague LI – ideals and Interval valued vague lattice ideals of lattice implication algebra by linking the Interval valued vague set and LI-ideal theory of Lattice implication algebras were introduced and discussed with the relation between these ideals.

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