AN INTUITIONISTIC FUZZY IDEAL IN FIELD

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Abstract: In this paper, We study intuitionistic fuzzy field and introduce intuitionistic fuzzy subfield and anti intuitionistic fuzzy ideal of field. And investigate some related theorems.

Index Terms: Fuzzy set, Intuitionistic fuzzy set, fuzzy field, intuitionistic fuzzy field, intuitionistic fuzzy sub-field, intuitionistic fuzzy ideal, anti intuitionistic fuzzy ideal.

I. INTRODUCTION:

The concept of Fuzzy set (IF was short) was introduced Zadeh.L.A [12] in 1965 has showed meaningful applications in many field of studies and the theory of intuitionistic fuzzy set (IFS for short) as an extension of fuzzy set is introduced Atanassov.K [2] in 1983. The notion of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by Biswas.R [6,7]. In this paper, we introduce some new kind of intuitionistic fuzzy subfield and intuitionistic fuzzy ideal (IFSF and IFI for short) of a field. Intuitionistic fuzzy fields which are regarded as a generalization of field have been found useful in solving problems in different areas of mathematics and Engineering.

1.1 Definition:

Let X be a non-empty set. A fuzzy set A drawn from X is defined as

\[ A = \{x, \mu_A(x) : x \in X\} \]

Where, \( \mu_A(x) : x \rightarrow [0,1] \) is the membership function of the fuzzy set A. fuzzy set is a collection of object with graded membership. i.e having degrees of membership.

1.2 Definition:

An intuitionistic fuzzy set A of a non-empty set F is a object of the form

\[ A = \{x, \mu_A(x), \nu_A(x) : x \in F\} \]

Where, \( \mu_A(x) \rightarrow F[0,1] \) and \( \nu_A(x) \rightarrow F[0,1] \) are membership and non-membership functions such that for each \( x \in F \), we have, \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \).

1.3 Remark:

i) When \( \mu_A(x) = \nu_A(x) = 1 \) then A is called fuzzy set.

ii) We denote the intuitionistic fuzzy set, 
\[ A = \{x, \mu_A(x), \nu_A(x) : x \in F\} \]
By \( A = (\mu_A, \nu_A) \).

1.4 Definition:

Let F be a field and A is a fuzzy set of F if the following conditions hold,

i) \( \mu_A(x + y) \geq \min(\mu_A(x), \mu_A(y)) \)

ii) \( \mu_A(-x) \geq \mu_A(x) \)

iii) \( \mu_A(xy) \geq \min(\mu_A(x), \mu_A(y)) \)

iv) \( \mu_A(x^{-1}) \geq \mu_A(x) \)

v) \( V_A(x + y) \leq \max[V_A(x), V_A(y)] \)

vi) \( V_A(-x) \leq V_A(x) \)

vii) \( V_A(xy) \leq \max[V_A(x), V_A(y)] \)

viii) \( V_A(x^{-1}) \leq V_A(x) \)

1.5 Definition:

Let F be a field and \( A \in IFS(F) \). If for all \( x, y \in F \) the following conditions are valid then A is called an intuitionistic fuzzy field over F. shortly IFF(F).

i) \( \mu_A(x + y) \geq \min(\mu_A(x), \mu_A(y)) \)

ii) \( \mu_A(-x) \geq \mu_A(x) \)

iii) \( \mu_A(xy) \geq \min(\mu_A(x), \mu_A(y)) \)

iv) \( \mu_A(x^{-1}) \geq \mu_A(x) \)

v) \( V_A(x + y) \leq \max[V_A(x), V_A(y)] \)

vi) \( V_A(-x) \leq V_A(x) \)

vii) \( V_A(xy) \leq \max[V_A(x), V_A(y)] \)

viii) \( V_A(x^{-1}) \leq V_A(x) \)

1.6 Definition:

Let F be a field. An intuitionistic fuzzy , 
\[ A = \{< x, \mu_A(x), V_A(x) : x \in F\} \]
Of F is said to be intuitionistic fuzzy subfield (in short IFSF) of F if,

i) \( \mu_A(x + y) \geq \min(\mu_A(x), \mu_A(y)) \)


1.7 Definition:
An intuitionistic fuzzy set \( A = \{ x, \mu_A(x), V_A(x) : x \in F \} \) of a field \( F \) is said to be intuitionistic fuzzy ideal (in short IFI) of \( F \) if,
\[
\begin{align*}
\text{i)} & \quad \mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\} \\
\text{ii)} & \quad \mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \\
\text{iii)} & \quad V_A(x+y) \leq \max\{V_A(x), V_A(y)\} \\
\text{iv)} & \quad V_A(-x) \leq V_A(x) \\
\text{v)} & \quad V_A(xy) \leq \max\{V_A(x), V_A(y)\} \\
\text{vi)} & \quad V_A(x^{-1}) \leq V_A(x)
\end{align*}
\]

1.8 Definition:
An intuitionistic fuzzy set \( A = \{ x, \mu_A(x), V_A(x) : x \in F \} \) of a field \( F \) is said to be anti intuitionistic fuzzy ideal (in short AIFI) of \( F \) if,
\[
\begin{align*}
\text{i)} & \quad \mu_A(x+y) \leq \max\{\mu_A(x), \mu_A(y)\} \\
\text{ii)} & \quad \mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\} \\
\text{iii)} & \quad V_A(x+y) \geq \min\{V_A(x), V_A(y)\} \\
\text{iv)} & \quad V_A(xy) \geq \min\{V_A(x), V_A(y)\}
\end{align*}
\]

II. INTUITIONISTIC FUZZY SUBFIELD AND INTUITIONISTIC FUZZY IDEAL

2.1 Theorem:
Let \( A = (\mu_A(x), V_A(x)) \) be intuitionistic fuzzy subfield \( F \) then,
\[
\begin{align*}
\text{i)} & \quad \mu_A(x+y) = \min\{\mu_A(x), \mu_A(y)\} \\
\text{ii)} & \quad \mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\} \\
\text{iii)} & \quad V_A(x+y) = \min\{V_A(x), V_A(y)\} \\
\text{iv)} & \quad V_A(xy) = \max\{V_A(x), V_A(y)\}
\end{align*}
\]

Proof:
\[
\begin{align*}
\text{i)} & \quad \mu_A(x+y) = \min\{\mu_A(x), \mu_A(y)\} \\
\text{ii)} & \quad \mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\} \\
\text{iii)} & \quad V_A(x+y) = \min\{V_A(x), V_A(y)\} \\
\text{iv)} & \quad V_A(xy) = \max\{V_A(x), V_A(y)\}
\end{align*}
\]

2.2 Corollary:
Let \( A \) be an IFS of a field \( F \), then
\[
\begin{align*}
\text{i)} & \quad \mu_A(x+y) = \min\{\mu_A(x), \mu_A(y)\} \\
\text{ii)} & \quad \mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\}
\end{align*}
\]

Proof:
\[
\begin{align*}
\text{i)} & \quad \mu_A(x+y) = \min\{\mu_A(x), \mu_A(y)\} \\
\text{ii)} & \quad \mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\}
\end{align*}
\]
We have,
\[ V_A(x + y) = \max\{V_A(x), V_A(y)\} \]
suppose, \( V_A(x) < V_A(y) \)
then, \( V_A(x + y) = V_A(y) \)

2.3 Theorem:
Let \( A \) be IFS of a filed then \( A \) is IFSF of \( F \) if and only if \( A^c \) is IFSF of \( F \).

Proof:
Let, \( A \) be IFSF of a filed \( F \) then,

i) \( \mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\} \)
ii) \( V_A(x + y) \leq \max\{V_A(x), V_A(y)\} \)
iii) \( \mu_A(-x) \geq \mu_A(x) \)
iv) \( V_A(-x) \leq V_A(x) \)
v) \( \mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \)
vi) \( V_A(xy) \leq \max\{V_A(x), V_A(y)\} \)
vii) \( \mu_A(x^{-1}) \geq \mu_A(x) \)
viii) \( V_A(x^{-1}) \leq V_A(x) \)

To show that, \( A^c = (\mu_A^c, V_A^c) = (\mu_A, V_A) \) is IFSF of \( F \).
From (i),
We have,
\[ \mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\} \]
\[ \text{i.e.} \quad V_A^c(x + y) \geq \min\{V_A^c(x), V_A^c(y)\} \]
\[ \Rightarrow \min\{V_A^c(x), V_A^c(y)\} \leq V_A^c(x + y) \]
\[ \text{i.e.} \quad \min\{1 - \mu_A^c(x), 1 - \mu_A^c(y)\} \leq 1 - \mu_A^c(x + y) \]
\[ \Rightarrow 1 - \max\{\mu_A^c(x), \mu_A^c(y)\} \leq 1 - \mu_A^c(x + y) \]
\[ \text{i.e.} \quad \mu_A^c(x + y) \leq \max\{\mu_A^c(x), \mu_A^c(y)\} \]

and, we can show that, (ii)
\[ V_A(x + y) \leq \max\{V_A(x), V_A(y)\} \]
\[ \text{i.e.} \quad V_A^c(x + y) \leq \max\{V_A^c(x), V_A^c(y)\} \]
\[ \Rightarrow \max\{V_A^c(x), V_A^c(y)\} \geq V_A^c(x + y) \]
\[ \text{i.e.} \quad \max\{1 - V_A^c(x), 1 - V_A^c(y)\} \geq 1 - V_A^c(x + y) \]
\[ \Rightarrow 1 - \min\{V_A^c(x), V_A^c(x)\} \geq 1 - V_A^c(x + y) \]
\[ \text{i.e.} \quad V_A^c(x + y) \geq \min\{V_A^c(x), V_A^c(y)\} \]

from (iii)
We have,
\[ \mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \]
\[ \text{i.e.} \quad V_A^c(xy) \geq \min\{V_A^c(x), V_A^c(y)\} \]
\[ \Rightarrow \min\{V_A^c(x), V_A^c(y)\} \leq V_A^c(xy) \]
\[ \text{i.e.} \quad \min\{1 - V_A^c(x), 1 - V_A^c(y)\} \leq 1 - V_A^c(xy) \]
\[ \Rightarrow 1 - \max\{\mu_A^c(x), \mu_A^c(y)\} \leq 1 - \mu_A^c(xy) \]
\[ \text{i.e.} \quad \max\{\mu_A^c(x), \mu_A^c(y)\} \geq \mu_A^c(xy) \]
\[ \mu_A^c(xy) \leq \max\{\mu_A^c(x), \mu_A^c(y)\} \]

and, we can show that, (iv)
\[ V_A(xy) \leq \max\{V_A(x), V_A(y)\} \]
\[ \text{i.e.} \quad V_A^c(xy) \leq \max\{V_A^c(x), V_A^c(y)\} \]
\[ \Rightarrow \max\{V_A^c(x), V_A^c(y)\} \geq V_A^c(xy) \]
\[ \text{i.e.} \quad \max\{1 - V_A^c(x), 1 - V_A^c(y)\} \geq 1 - V_A^c(xy) \]
\[ \Rightarrow 1 - \min\{V_A^c(x), V_A^c(x)\} \geq 1 - V_A^c(xy) \]
\[ \text{i.e.} \quad \min\{V_A^c(x), V_A^c(x)\} \leq V_A^c(xy) \]
\[ \text{i.e.} \quad V_A^c(xy) \geq \min\{V_A^c(x), V_A^c(y)\} \]

from (v):
we have,
\[ \mu_A(-x) \geq \mu_A(x) \]
\[ \text{i.e.} \quad V_A^c(-x) \geq V_A^c(x) \]

thus, \( V_A^c(x) \leq V_A^c(-x) \)
\[ \text{i.e.} \quad 1 - \mu_A^c(x) \leq 1 - \mu_A^c(-x) \]
\[ \mu_A^c(x) \geq \mu_A^c(-x) \]
\[ \mu_A^c(-x) \leq \mu_A^c(x) \]
And also, we can show that (vi),
we have,
From (vii), we have,

\[ \mu_A(x)^{-1} \geq \mu_A(x) \]

\[ \therefore \] \[ V_A^c(x^{-1}) \geq V_A^c(x) \]

Thus, \[ V_A^c(x) \leq V_A^c(x^{-1}) \]

\[ \therefore 1 - \mu_A^c(x) \leq 1 - \mu_A^c(x^{-1}) \]

\[ \mu_A^c(x) \geq \mu_A^c(x^{-1}) \]

\[ \mu_A^c(x^{-1}) \leq \mu_A^c(x) \]

And also, we can show that (viii), we have,

\[ V_A(x^{-1}) \leq V_A(x) \]

\[ \therefore \] \[ \mu_A^c(x^{-1}) \leq \mu_A^c(x) \]

Thus, \[ \mu_A^c(x) \geq \mu_A^c(x^{-1}) \]

\[ \therefore 1 - V_A^c(x) \geq 1 - V_A^c(x^{-1}) \]

\[ V_A^c(x) \leq V_A^c(x^{-1}) \]

\[ V_A^c(x^{-1}) \geq V_A^c(x) \]

Therefore, A is IFS of F then \( A^c \) is IFS of F.

Similarly, when \( A^c \) is IFS of F we can show that A is IFS of F.

Hence the proof.

2.4 Theorem:

Let A be intuitionistic fuzzy set of a field F, then A is intuitionistic fuzzy ideal of F if and only if \( A^c \) is intuitionistic fuzzy ideal of F.

Proof:

Firstly, let A be intuitionistic fuzzy ideal of F, then

i) \[ \mu_A(x + y) \geq \min[\mu_A(x), \mu_A(y)] \]

ii) \[ \mu_A(xy) \geq \min[\mu_A(x), \mu_A(y)] \]

iii) \[ V_A(x + y) \leq \max[V_A(x), V_A(y)] \]

iv) \[ V_A(xy) \leq \max[V_A(x), V_A(y)] \]

To show that \( A^c = (\mu_A^c, V_A^c) = (A, \mu_A) \) is intuitionistic fuzzy ideal of F.

From (i), we have,

\[ \mu_A(x + y) \geq \min[\mu_A(x), \mu_A(y)] \]

\[ \therefore \] \[ V_A^c(x + y) \geq \min[V_A^c(x), V_A^c(y)] \]

\[ \Rightarrow \] \[ \min[V_A^c(x), V_A^c(y)] \leq V_A^c(x + y) \]

\[ \therefore \] \[ V_A^c(x + y) \leq \max[\mu_A^c(x), \mu_A^c(y)] \]

And, we can show that, (ii)

\[ V_A(x + y) \leq \max[V_A(x), V_A(y)] \]

\[ \therefore \] \[ \mu_A^c(x + y) \leq \max[\mu_A^c(x), \mu_A^c(y)] \]

\[ \Rightarrow \] \[ \max[\mu_A^c(x), \mu_A^c(y)] \geq \mu_A^c(x + y) \]

\[ \therefore \] \[ \mu_A^c(x + y) \leq \max[\mu_A^c(x), \mu_A^c(y)] \]

From (iii), we have,

\[ \mu_A(xy) \geq \max[\mu_A(x), \mu_A(y)] \]

\[ \therefore \] \[ V_A^c(xy) \geq \max[V_A^c(x), V_A^c(y)] \]

\[ \Rightarrow \] \[ \max[V_A^c(x), V_A^c(y)] \leq V_A^c(xy) \]

\[ \therefore \] \[ V_A^c(xy) \leq \min[\mu_A^c(x), \mu_A^c(y)] \]

\[ \Rightarrow \] \[ \min[\mu_A^c(x), \mu_A^c(y)] \geq \mu_A^c(xy) \]

\[ \therefore \] \[ \mu_A^c(xy) \leq \min[\mu_A^c(x), \mu_A^c(y)] \]

And, we can show that, (iv)

\[ V_A(xy) \leq \min[V_A(x), V_A(y)] \]

\[ \therefore \] \[ \mu_A^c(xy) \leq \min[\mu_A^c(x), \mu_A^c(y)] \]

\[ \Rightarrow \] \[ \min[\mu_A^c(x), \mu_A^c(y)] \geq \mu_A^c(xy) \]
i.e) \( \min(1 - V_A(x), 1 - V_A(y)) \geq 1 - V_A(xy) \)
\[ \Rightarrow 1 - \max(V_A(x), V_A(y)) \geq 1 - V_A(xy) \]
i.e) \( \max(V_A(x), V_A(y)) \leq V_A(xy) \)
i.e) \( V_A(xy) \geq \max(V_A(x), V_A(y)) \)

Therefore, A is IFSF of F then A is IFSF of F.
Similarly, when F is IFSF of F we can show that A is IFSF of F.

Hence the proof.

2.5 Theorem:
Union of two IFI of a field F is also IFI of F.

Proof:
Let, X and Y be any two intuitionistic fuzzy ideal of a field F and let, x and y in F.

Let,
\[ A = \{ x \mid \mu_A(x), V_A(x) : x \in X \} \text{ and} \]
\[ B = \{ y \mid \mu_B(y), V_B(y) : y \in Y \} \]

And also let, \( C = A \cup B \)
\[ C = \{ x \mid \mu_C(x), V_C(x) : x \in F \} \]

Where,
\[ \min(\mu_A(x), \mu_B(x)) = \mu_c(x) \text{ and } \max(V_A(x), V_B(x)) = V_c(x) \]

Now,
\[ \mu_c(x + y) = \min(\mu_A(x + y), \mu_B(x + y)) \]
\[ V_c(x + y) = \max(V_A(x + y), V_B(x + y)) \]

And,
\[ \mu_c(xy) = \max(\mu_A(xy), \mu_B(xy)) \]
\[ V_c(xy) = \min(V_A(xy), V_B(xy)) \]

Now,
\[ \mu_c(x + y) = \min(\mu_A(x + y), \mu_B(x + y)) \]
\[ \leq \min(\mu_A(x), \mu_B(y)) \]
\[ = \min(\mu_A(x), \mu_B(y)) \]
\[ \leq \min(\mu_A(x), \mu_B(y)) \]
\[ = \min(\mu_A(x), \mu_B(y)) \]

Therefore, \( \mu_c(x + y) \geq \min(\mu_c(x), \mu_c(y)) \) for all x and y in F.
And,
\[ \mu_c(xy) = \max(\mu_A(xy), \mu_B(xy)) \]
\[ \leq \max(\mu_A(x), \mu_B(y)) \]
\[ = \max(\mu_A(x), \mu_B(y)) \]
\[ \leq \max(\mu_A(x), \mu_B(y)) \]
\[ = \max(\mu_A(x), \mu_B(y)) \]

Therefore, \( \mu_c(xy) \geq \max(\mu_c(x), \mu_c(y)) \) for all x and y in F.

Also,
\[ V_c(x + y) = \max(V_A(x + y), V_B(x + y)) \]
\[ \leq \max(\mu_A(x), \mu_B(y)) \]
\[ = \max(\mu_A(x), \mu_B(y)) \]
\[ \leq \max(\mu_A(x), \mu_B(y)) \]
\[ = \max(\mu_A(x), \mu_B(y)) \]

Therefore, \( V_c(x + y) \leq \max(V_c(x), V_c(y)) \) for all x and y in F.
And,
\[ V_c(xy) = \min(V_A(xy), V_B(xy)) \]
\[ \leq \min(\mu_A(x), \mu_B(y)) \]
\[ = \min(\mu_A(x), \mu_B(y)) \]
\[ \leq \min(\mu_A(x), \mu_B(y)) \]
\[ = \min(\mu_A(x), \mu_B(y)) \]

Therefore, \( V_c(xy) \leq \min(V_c(x), V_c(y)) \) for all x and y in F.

Hence,
Union of any two intuitionistic fuzzy ideal of a field F is an intuitionistic fuzzy ideal of F.

2.6 Theorem:
Union of two anti intuitionistic fuzzy ideals of a field F is also anti intuitionistic fuzzy ideal of F.

Proof:
Let A and B be two anti intuitionistic fuzzy ideal of F \( \Rightarrow A^c \) and \( B^c \) are intuitionistic fuzzy ideal of F [by theorem 2.5]
\[ \Rightarrow A^c \cap B^c \text{ is intuitionistic fuzzy ideal of F } \Rightarrow (A \cup B)^c \text{ is intuitionistic fuzzy ideal of F } \Rightarrow A \cup B \text{ is anti intuitionistic fuzzy ideal of F.} \]

2.7 Corollary:
Union of an arbitrary collections of anti intuitionistic fuzzy ideal of a field F is also anti intuitionistic fuzzy ideal of F.

III. CONCLUSION:
Recently the fuzzification of algebraic structures plays an fundamental role in many branches of mathematics and engineering. In this present paper we have expound a new kind of intuitionistic fuzzy ideal of field and we also express intuitionistic fuzzy subfield and intuitionistic fuzzy ideal. These structural properties will be accommodating to the readers as well as researchers to compose and further supplement of intuitionistic fuzzy field.
REFERENCES: