CHARACTERIZATION OF FUZZY SOFT INTERSECTION USING NEAR RINGS

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Abstract In this paper, we first define fuzzy soft intersection near ring by using the intersection operation of sets. This new notion can be regarded as a connection among fuzzy soft set theory, soft set theory, set theory and near ring theory. Further, We have also discussed about basic properties and we analog the applications of fuzzy soft intersection near ring to near ring theory.

Key words Near ring, Fuzzy soft set, Fuzzy soft near rings, Soft intersection near rings, Fuzzy soft intersection near rings, Image, and Pre-image.

I. INTRODUCTION

Most of our traditional tools for formal modelling, reasoning and computing are crisp, deterministic, and precise in character. However there are many complicated problems in economics, medical science, etc.., The soft set theory was introduced by Molodstov [7] in 1999. Soft set theory has a rich potential for applications in several directions. The fuzzy soft set theory was initiated by L.A.Zadeh[11] in 1965. These set have a broad utility for expressing the gradual transition from membership to non membership and conversely. By fuzzy set theory we can express vague concepts into natural language.

The notion of near ring was first introduced by Dickson and Leonard in 1905. The primary step towards near rings was an axiomatic research done by Dickson. It is a generalization of a ring. If in a ring we ignore commutativity of addition and one distributive law then we get a near ring. G.Pilz [8], J.D.P.Meldrum [6] and many other researchers have contributed and are contributing the near ring theory

In this paper, we analyzed the fuzzy soft intersection near ring by using the intersection operation of sets. This new notion can be regarded as a connection among fuzzy soft set theory, soft set theory, set theory and near ring theory. Finally, We have also discoursed about basic properties and we analog the applications of fuzzy soft intersection near ring to near ring theory with respect to the image and pre image.

II. PRELIMINARIES

In this section we first all recall the basic definitions related to near rings, fuzzy soft intersection near ring, image and pre image which would be used in the sequel.

2.1 Definition

Let U be an initial universal set, E be the of parameters. Let A be a subset of E. Let P(U) denote the power set of U. A pair (F, A) is called a **Soft Set** over U, where F is a mapping given by $F: A \to P(U)$.

2.2 Definition

Let X be the collection of objects denoted generally by x then a **Fuzzy Set** A in x is defined as, $A = \{< x, \mu_A(x) > x \in X\}$
Where,

$\mu_A(x)$ is called the membership value of x in A and $0 \leq \mu_A(x) \leq 1$

2.3 Definition

Let U be an initial universe set and E be the parameters. Let A be a subset of E .A pair (F, A) is called a **Fuzzy Soft Set** over U, where F is a mapping given by $F: A \to I^U$,
Where, I^U denotes the collection of all fuzzy subsets of U.
i.e. For each $a \in A$, $F(a) = F_a: U \to I$ is a fuzzy set on U.

2.4 Definition

A non empty set R with two binary operations '$+$' and '$\cdot$' satisfying the following axioms :

i. $(R, +)$ is a group
ii. $(R, \cdot)$ is a semi- group
iii. $x \cdot (y + z) = x \cdot y + x \cdot z$ for all $x, y, z \in R$.

It is a **Left Near - Ring**, because it satisfies the left distributive law.

iv $(x + y).z = x \cdot z + y \cdot z$ for all $x, y, z \in R$.

It is a **Right Near - Ring**, because it satisfies the right distributive law.

Example

Let $R = \{0, 1, 2, 3\}$ be a non empty set with two binary operations '+' and '.' Defined as follows:
Then $(R, +, \cdot)$ is a near ring.

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	1	0
3	3	2	0	1

.	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	0	0	0
3	0	1	2	3

2.5 Definition

Let $(R,+,\cdot)$ be a near ring and E be the set of parameters and $A \subset E$. Let (F,A) be a non null soft set over R. Then (F,A) is called a **Soft Near Ring** over R if and only if for each $a \in supp(F,A)$ and $F(a) = F_a$ is a sub near ring of R.

i. $x,y \in F_a \Rightarrow x-y \in F_a$

ii. $x,y \in F_a \Rightarrow xy \in F_a$

2.6 Definition

Let $(R,+,\cdot)$ be a near ring and E be the set of parameters and $A \subset E$. Let F be a mapping given by $F: A \to [0,1]^R$ where $[0,1]^R$ is the collection of all fuzzy subsets of R, then (F,A) is called a **Fuzzy Soft Near Ring** over R if and only if for each $a \in A$, the corresponding fuzzy subset F_a of R is a fuzzy sub near ring of R. i.e.,

i. $\mathrm{F_a(x+y) \geq min(F_a(x), F_a(y))}$

ii. $\mathrm{F_a(-x) \geq min(F_a(x))}$

iii. $\mathrm{F_a(xy) \geq min(F_a(x), F_a(y))}$ for all $x,y \in R$.

2.7 Definition

Let (F,A) be a fuzzy soft set then the set $supp(F,A) = \{x \in A \,/\, F(x) = F_x \neq \emptyset\}$ is called the **Support of The Fuzzy Soft Set** (F,A). A fuzzy soft set is called **Non – Null** if its support is not equal to the empty set.

2.8 Definition

Let $(F,\ A)$ be a fuzzy soft near ring over R then $(F,\ A)$ is called a fuzzy soft intersection near ring over R if it satisfies the following properties for all $a \in supp(F,\ A)$

i. $\mathrm{F_a(x+y) \geq F_a(x) \cap F_a(y)}$

ii. $\mathrm{F_a(-x) = (F_a(x))}$

iii. $\mathrm{F_a(xy) \geq F_a(x) \cap F_a(y)}$ for all $x,y \in R$.

Example

Let $R = \{0,1,2,3\}$ be a right near ring. Assume that A is the set of parameters and $U = \left\{\begin{bmatrix} x & x \\ y & y \end{bmatrix} / x,y \in z_4\right\}$, 2×2 matrices with z_4 terms is the universal set. We construct a fuzzy soft set $\mathrm{F_a}$ over U by

$$\mathrm{F_a(0)} = \left\{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 3 & 3 \end{bmatrix}\right\}$$

$$\mathrm{F_a(1)} = \left\{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}\right\}$$

$$\mathrm{F_a(2)} = \left\{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}\right\}$$

$$\mathrm{F_a(3)} = \left\{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}\right\}$$

Hence the fuzzy soft set $\mathrm{F_a}$ is a fuzzy soft intersection near ring over R.

2.9 Definition

Let (F,A) and (F,B) be fuzzy soft set over R then $\wedge$ product of (F,A) and (F,B) denoted by $F_a \wedge F_b$ is defined as $F_a \wedge F_b = F_{a \wedge b}$ Where, $F_{a\wedge b}(x,y) = F_a(x) \cap F_b(y)$ for all $(x,y) \in E \times E$.

2.10 Definition

Let (F,A) and (G,B) be fuzzy soft intersection near ring over R then the product of fuzzy soft intersection near rings (F,A) and (G,B) is defined as $F_a \times G_b = H_{a \times b}$

Where,

$H_{a\times b}(x,y) = F_a(x) \times G_b(y)$ for all $(a,b) = A \times B$.

2.11 Definition

Let (F,A) and (F,B) be fuzzy soft set over R and Φ be a function from A to B then fuzzy soft image of F_a under Φ denoted by $\Phi(F_a)$ is a fuzzy soft set over R by

$$\left(\Phi(F_a)\right)(y) = \begin{cases} \cup\{F_a(x) / x \in A \ and \ \Phi(x) = y\} & if \ \Phi^{-1}(y) \neq \emptyset \\ \emptyset & otherwise \end{cases}$$

For all $y \in B$, and

fuzzy soft pre image (or fuzzy soft inverse image) of F_b under Φ denoted by $\Phi^{-1}(F_b)$ is a fuzzy soft set over R by

$\Phi^{-1}(F_b)(x) = (F_b)(\Phi(x))$ for all $x \in A$.

III. FUZZY SOFT INTERSECTION NEAR RINGS

3.1 Theorem

Let (F,A) be a fuzzy soft intersection near ring over R then $F_a(0) \geq F_a(x)$ for all $x \in R$.

Proof

Assume that (F,A) be a fuzzy soft intersection near ring over R then for all $x \in R$

$$F_a(0) = F_a(x-x)$$
$$\geq F_a(x) \cap F_a(-x)$$

$$\geq F_a(x) \cap F_a(x)$$
$$\geq F_a(x)$$
$$F_a(0) \geq F_a(x)$$

Hence $F_a(0) \geq F_a(x)$ for all $x \in R$.

3.2 Theorem

Let (F, A) be a fuzzy soft intersection near ring over R and $x \in R$ then $F_a(x) = F_a(0) \Leftrightarrow F_a(x+y) = F_a(y+x) = F_a(y)$ for all $x \in R$.

Proof

Suppose that $F_a(x+y) = F_a(y+x) = F_a(y)$ for all $x \in R$

Then by choosing $y = 0 \Rightarrow F_a(x+0) = F_a(0+x) = F_a(0)$

We obtain that $F_a(x) = F_a(0)$

Conversely,

Assume that $F_a(x) = F_a(0)$ then by using above theorem $F_a(0) = F_a(x) \geq F_a(y)$ for all $y \in R$

Let (F, A) be a fuzzy soft intersection near ring over R then

$$F_a(x+y) \geq \mathrm{F_a(x) \cap F_a(y)}$$
$$\geq \mathrm{F_a(0) \cap F_a(y)}$$
$$= \mathrm{F_a(y)} \text{ for all } y \in R$$

More over for all $y \in R$

$$F_a(y) = F_a((-x+x)+y)$$
$$F_a(y) = F_a(-x+(x+y))$$
$$\geq \mathrm{F_a(-x) \cap F_a(x+y)}$$
$$\geq \mathrm{F_a(x) \cap F_a(x+y)}$$
$$= F_a(x+y)$$

Since for all $y \in R$ $F_a(x) \geq \mathrm{F_a(y)}$ hence $xy \in R$ implies that $x+y \in R$

Therefore $F_a(x) \geq \mathrm{F_a(x+y)}$ for all $y \in R$

$$F_a(y+x) = F_a(y+x+(y-y))$$
$$F_a(y+x) = F_a(y+(x+y)-y)$$
$$\geq \mathrm{F_a(y) \cap F_a(x+y)} \cap F_a(-y)$$
$$\geq \mathrm{F_a(y) \cap F_a(x+y)} \cap F_a(y)$$
$$\geq \mathrm{F_a(x) \cap F_a(x+y)}$$
$$= F_a(y)$$

Since $F_a(x+y) = F_a(y)$ furthermore $y \in R$

$$F_a(y) = F_a(y+(-x+x))$$
$$= F_a\big((x+y)-x\big)$$
$$\geq \mathrm{F_a(x+y) \cap F_a(-x)}$$
$$\geq \mathrm{F_a(x+y) \cap F_a(x)}$$
$$= F_a(x+y)$$

It follows that $F_a(y+x) = F_a(y)$ and so $F_a(x+y) = F_a(y+x) = F_a(y)$ for all $x \in R$

3.3 Theorem

If (F, A) and (F, B) be a fuzzy soft intersection near ring over R then so is $(F, A) \wedge (F, B)$ over R.

Proof

By using the definition

Let $F_a \wedge F_b = F_{a\wedge b}$

Where,

$$F_{a\wedge b}(x, y) = F_a(x) \cap F_b(y) \text{ for all } (x, y) \in A \times B.$$

Let $(x_1, y_1), (x_2, y_2) \in A \times B$ then

$$F_{a\wedge b}\big((x_1, y_1) - (x_2, y_2)\big) = F_{a\wedge b}(x_1 - x_2,\ y_1 - y_2)$$
$$= F_a(x_1 - x_2) \cap F_b(\ y_1 - y_2)$$
$$F_{a\wedge b}\big((x_1, y_1) - (x_2, y_2)\big) \geq (F_a(x_1) \cap F_a(-x_2)) \cap\ (F_b(\ y_1) \cap F_b(-\ y_2))$$
$$\geq (F_a(x_1) \cap F_a(x_2)) \cap\ (F_b(\ y_1) \cap F_b(\ y_2))$$
$$F_{a\wedge b}\big((x_1, y_1) - (x_2, y_2)\big) \geq (F_a(x_1) \cap F_b(y_1)) \cap\ (F_a(\ x_2) \cap F_b(\ y_2))$$

Hence

$$F_{a\wedge b}\big((x_1, y_1) - (x_2, y_2)\big) \geq F_{a\wedge b}(x_1 - y_1) \cap F_{a\wedge b}(x_2 - y_2)$$

And,

$$F_{a\wedge b}\big((x_1, y_1)(x_2, y_2)\big) = F_{a\wedge b}(x_1x_2,\ y_1y_2)$$
$$= F_a(x_1x_2) \cap F_b(\ y_1y_2)$$
$$\geq (F_a(x_1) \cap F_a(x_2)) \cap\ (F_b(\ y_1) \cap F_b(\ y_2))$$
$$F_{a\wedge b}\big((x_1, y_1)(x_2, y_2)\big) \geq (F_a(x_1) \cap F_b(y_1)) \cap\ (F_a(\ x_2) \cap F_b(\ y_2))$$

Hence

$$F_{a\wedge b}\big((x_1, y_1)(x_2, y_2)\big) \geq F_{a\wedge b}(x_1y_1) \cap F_{a\wedge b}(x_2y_2)$$

Thus

$F_a \wedge F_b$ is a fuzzy soft intersection near ring over R.

3.4 Theorem

If (F, A) and (G, B) be a fuzzy soft intersection near rings over R then the product of $(F, A)$and (G, B) is also fuzzy soft intersection near rings over $A \times B$.

Proof

Assume that (F,A) and (G,B) be a fuzzy soft intersection near rings over R.

Let $F_a \times G_b = H_{a\times b}$

Where,

$H_{a\times b}(x,y) = F_a(x) \times G_b(y)$ for all $(x,y) \in A \times B$.

Let $(x_1,y_1),(x_2,y_2) \in A \times B$ then

i. $H_{a\times b}\big((x_1,y_1)-(x_2,y_2)\big) \geq H_{a\times b}(x_1-y_1) \cap H_{a\times b}(x_2-y_2)$

Consider,

$$H_{a\times b}\big((x_1,y_1)-(x_2,y_2)\big) = F_a(x_1-x_2) \times G_b(y_1-y_2)$$
$$\geq (F_a(x_1) \cap F_a(-x_2)) \times (G_b(y_1) \cap G_b(-y_2))$$
$$\geq (F_a(x_1) \cap F_a(x_2)) \times (G_b(y_1) \cap G_b(y_2))$$
$$H_{a\times b}\big((x_1,y_1)-(x_2,y_2)\big) \geq (F_a(x_1) \times G_b(y_1)) \cap (F_a(x_2) \times G_b(y_2))$$

Hence

$$H_{a\times b}\big((x_1,y_1)-(x_2,y_2)\big) \geq H_{a\times b}(x_1-y_1) \cap H_{a\times b}(x_2-y_2)$$

And

ii. $H_{a\times b}\big((x_1,y_1)(x_2,y_2)\big) \geq H_{a\times b}(x_1y_1) \cap H_{a\times b}(x_2y_2)$

Consider

$$H_{a\times b}\big((x_1,y_1)(x_2,y_2)\big) = F_a(x_1x_2) \times G_b(y_1y_2)$$
$$\geq (F_a(x_1) \cap F_a(x_2)) \times (G_b(y_1) \cap G_b(y_2))$$
$$H_{a\times b}\big((x_1,y_1)(x_2,y_2)\big) \geq (F_a(x_1) \times G_b(y_1)) \cap (F_a(x_2) \times G_b(y_2))$$

Hence

$$H_{a\times b}\big((x_1,y_1)(x_2,y_2)\big) \geq H_{a\times b}(x_1y_1) \cap H_{a\times b}(x_2y_2)$$

Thus $F_a \times G_b = H_{a\times b}$ is a fuzzy soft intersection near ring over R.

IV. APPLICATIONS OF FUZZY SOFT INTERSECTION NEAR RINGS

In this section we give the applications of fuzzy soft image and fuzzy soft pre image to near ring theory with respect to fuzzy soft intersection near rings of a near ring.

4.1 Theorem

Let (F,A) and (G,B) be a fuzzy soft intersection near rings over R and Φ be a near ring isomorphism from A to B. If (F,A) is a fuzzy soft intersection of A over R then $\Phi(F_a)$ is a fuzzy soft intersection of B over R.

Proof

Let $y_1,y_2,y_3 \in B$. Since Φ is surjective, then there exist $x_1,x_2,x_3 \in A$ such that $\Phi(x_1) = y_1$, $\Phi(x_2) = y_2$, and $\Phi(x_3) = y_3$. Then

1. $\big(\Phi(F_a)\big)(y_1-y_2) \geq \big(\Phi(F_a)\big)(y_1) \cap \big(\Phi(F_a)\big)(y_2)$

Consider $\big(\Phi(F_a)\big)(y_1-y_2) = \cup\{F_a(x): x \in A, \Phi(x) = y_1-y_2\}$

$$= \cup\{F_a(x): x \in A, \text{x} = \Phi^{-1}(y_1-y_2)\}$$
$$= \cup\{F_a(x): x \in A, \text{x} = \Phi^{-1}(\Phi(x_1-x_2)) = x_1-x_2\}$$
$$= \cup\{F_a(x_1-x_2): x_i \in A, \Phi(x_i) = y_i\ i = 1,2\}$$
$$\geq \cup\{F_a(x_1) \cap F_a(x_2): x_i \in A, \Phi(x_i) = y_i\ i = 1,2\}$$
$$= \cup\{F_a(x_1): x_1 \in A, \Phi(x_1) = y_1\} \cap \cup\{F_a(x_2): x_2 \in A, \Phi(x_2) = y_2\}$$

Hence $\big(\Phi(F_a)\big)(y_1-y_2) \geq \big(\Phi(F_a)\big)(y_1) \cap \big(\Phi(F_a)\big)(y_2)$

2. similarly you can prove that $\big(\Phi(F_a)\big)(y_1y_2) \geq \big(\Phi(F_a)\big)(y_1) \cap \big(\Phi(F_a)\big)(y_2)$

Consider

$$\big(\Phi(F_a)\big)(y_1y_2) = \cup\{F_a(x): x \in A, \Phi(x) = y_1y_2\}$$
$$= \cup\{F_a(x): x \in A, \text{x} = \Phi^{-1}(y_1y_2)\}$$
$$= \cup\{F_a(x): x \in A, \text{x} = \Phi^{-1}(\Phi(x_1x_2)) = x_1x_2\}$$
$$= \cup\{F_a(x_1x_2): x_i \in A, \Phi(x_i) = y_i\ i = 1,2\}$$
$$\geq \cup\{F_a(x_1) \cap F_a(x_2): x_i \in A, \Phi(x_i) = y_i\ i = 1,2\}$$
$$= (\cup\{F_a(x_1): x_1 \in A, \Phi(x_1) = y_1\}) \cap (\cup\{F_a(x_2): x_2 \in A, \Phi(x_2) = y_2\})$$

Hence

$$\big(\Phi(F_a)\big)(y_1y_2) \geq \big(\Phi(F_a)\big)(y_1) \cap \big(\Phi(F_a)\big)(y_2)$$

Therefore

$\Phi(F_a)$ is a fuzzy soft intersection near ring of B over R.

4.2 Theorem

Let (F,A) and (G,B) be a fuzzy soft intersection near rings over R and Φ be a near ring homomorphism from A to B. If (F,B) is a fuzzy soft intersection of B over R then $\Phi^{-1}(F_b)$ is a fuzzy soft intersection near ring of A over R.

Proof

Let $x_1,x_2,x_3 \in A$. Then

1. $\big(\Phi^{-1}(F_b)\big)(x_1-x_2) \geq \big(\Phi^{-1}(F_b)\big)(x_1) \cap \big(\Phi^{-1}(F_b)\big)(x_2)$

Consider

$$\big(\Phi^{-1}(F_b)\big)(x_1-x_2) = F_b(\Phi(x_1-x_2))$$
$$\big(\Phi^{-1}(F_b)\big)(x_1-x_2) = F_b(\Phi(x_1) - \Phi(x_2))$$
$$\geq F_b(\Phi(x_1) \cap F_b(\Phi(x_2)$$

$$= (\Phi^{-1}(F_b))(x_1) \cap (\Phi^{-1}(F_b))(x_2)$$

Hence

$$(\Phi^{-1}(F_b))(x_1 - x_2) \geq (\Phi^{-1}(F_b))(x_1) \cap (\Phi^{-1}(F_b))(x_2)$$

2. Similarly one can show that

$$(\Phi^{-1}(F_b))(x_1 x_2) \geq (\Phi^{-1}(F_b))(x_1) \cap (\Phi^{-1}(F_b))(x_2)$$

Consider

$$(\Phi^{-1}(F_b))(x_1 x_2) = F_b(\Phi(x_1 x_2))$$

$$(\Phi^{-1}(F_b))(x_1 x_2) = F_b(\Phi(x_1)\,\Phi(x_2))$$

$$(\Phi^{-1}(F_b))(x_1 x_2) \geq F_b(\Phi(x_1) \cap F_b(\Phi(x_2)$$

$$= (\Phi^{-1}(F_b))(x_1) \cap (\Phi^{-1}(F_b))(x_2)$$

Hence

$$(\Phi^{-1}(F_b))(x_1 x_2) \geq (\Phi^{-1}(F_b))(x_1) \cap (\Phi^{-1}(F_b))(x_2)$$

Therefore

$\Phi^{-1}(F_b)$ is a fuzzy soft intersection near ring of A over R.

V. CONCLUSION

In this paper, studied the fuzzy soft intersection near ring by using the intersection operation of sets. We have also discussed about basic properties and we have introduced the new concept of the applications of fuzzy soft intersection near ring to near ring theory with respect to fuzzy soft image and fuzzy soft pre image . This new notion can be regarded as a connection among fuzzy soft set theory, soft set theory, set theory and near ring theory.

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