FUZZY SOFT IDEALS OF A FUZZY SOFT LATTICE

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Abstract In this paper, we define the concept of fuzzy soft ideals and filters over a collection of fuzzy soft sets, study their related properties and illustrate them with some examples. We also define the maximum and minimum conditions in fuzzy soft lattice. In addition, we characterized fuzzy soft modularity and fuzzy soft distributivity of fuzzy soft lattices of fuzzy soft ideals.

Index Terms Fuzzy soft ideals and filters, Prime fuzzy soft ideals and filters, Principal fuzzy soft ideal and filter, Modular fuzzy soft lattices, Distributive fuzzy soft lattices.

I. INTRODUCTION

The theory of soft sets was firstly introduced by Molodtsov[11] in 1999 as a general Mathematical tool for dealing with uncertainty. At present, research works on soft set theory and its application are making progress rapidly. The theory of fuzzy set was introduced by L.A.Zadeh [14] in 1965. Fuzzy set is to used in many areas of daily life such as Engineering, Medicine, Meteorology. The theory of lattices was introduced by Richard Dedekind. Faruk Karaaslam and Naim Cagman [7] defined the concept of modular fuzzy soft lattice and distributive fuzzy soft lattice. In this paper we define the concept of fuzzy soft ideal and filter, prime fuzzy soft ideal and filter, principal fuzzy soft ideal and filter. Also, we prove that set of fuzzy soft ideals of a fuzzy soft lattice. Further, we prove fuzzy soft lattice is modular if and only if the fuzzy soft ideal lattice is distributive if and only if the fuzzy soft ideal lattice is distributive.

The readers are asked to refer for basic definitions and results on fuzzy soft set theory and [7, 12, 13] for results on fuzzy soft lattices. Throughout this work, X refers to the initial universe, P(X) is the power set of X, E is a set of parameters and A ⊆ E.F(X) denotes the set of all fuzzy soft sets over X.

II. FUZZY SOFT IDEALS AND FUZZY SOFT FILTERS

In this section we introduce the concept of fuzzy soft ideals and fuzzy soft filters with examples. We prove that every fuzzy soft ideal and fuzzy soft filter of a fuzzy soft lattice is a convex fuzzy soft sublattice of and conversely. We also study about prime fuzzy soft ideals and prime fuzzy soft filters. Throughout this work, the fuzzy soft lattice means the fuzzy soft lattice (f_L, ∧, ∨).

2.1 Definition

A non – empty fuzzy soft subset f_i of a fuzzy soft lattice f_L is said to be fuzzy soft ideal if

(f_1) f_i(x), f_i(y) ∈ f_i implies f_i(x ∨ f_i(y) ∈ f_i.

(f_2) f_i(x) ∈ f_i implies f_i(x ∧ f_i(a) ∈ f_i for every element f_i(a) of f_i, or equivalently f_i(x) ∈ f_i and f_i(a) ∈ f_i implies f_i(a) ∈ f_i.

2.2 Definition

A non – empty fuzzy soft subset f_F of a fuzzy soft lattice f_L is said to be fuzzy soft filter if

(f_1) f_F(x), f_F(y) ∈ f_F implies f_F(x ∧ f_F(y) ∈ f_F.

(f_2) f_F(x) ∈ f_F implies f_F(x ∨ f_F(a) ∈ f_F for every element f_F(a) of f_F, or equivalently f_F(x) ∈ f_F and f_F(a) ∈ f_F implies f_F(a) ∈ f_F.

2.3 Note

Every fuzzy soft ideal of a fuzzy soft lattice of f_L is a fuzzy soft sublattice of f_L.

2.4 Example

Let X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}, be the universe and E = \{e_1, e_2, e_3\}

be the set of parameters, P = \{e_1\}, Q = \{e_2\}, R = \{e_3\}, S = \{e_2, e_3\}, T = \{e_4, e_5\}, U = \{e_5, e_6\}, V = \{e_4, e_2, e_3\} where P, Q, R, S, T, U, V ⊆ E and f_L = \{f_L(0), f_L(P), f_L(Q), f_L(R), f_L(S), f_L(T), f_L(U), f_L(V)\} ⊆ F(X) with the operations ∨ and ∨.

Assume that, f_L(0) = ∅

f_L(P) = \{\{(e_1, \{x_1\}\}\}

f_L(Q) = \{\{(e_2, \{x_2\}\}\}

f_L(R) = \{\{(e_3, \{x_3\}\}\}

f_L(S) = \{\{(e_1, \{x_1, x_4\}\), \{(e_2, \{x_2, x_3\}\}\}

f_L(T) = \{\{(e_1, \{x_1, x_5\}\), \{(e_3, \{x_3, x_7\}\}\}

f_L(U) = \{\{(e_2, \{x_2, x_5, x_6\}\), \{(e_3, \{x_3, x_8\}\}\}

f_L(V) = \{\{(e_1, \{x_1, x_4, x_6\}\), \{(e_2, \{x_2, x_5, x_8\}\), \{(e_3, \{x_3, e_7\}\}\}


Then \( (f_L \cup \bar{\eta}) \) is a fuzzy soft lattice. The Hasse diagram of it appears in figure 1.

![Hasse diagram](image)

Figure 1

(a) Consider the fuzzy soft set \( f_1 = \{ f_\emptyset, f_I, f_Q, f_L \} \subseteq f_L \) clearly \( f_1 \neq f_\emptyset \). It also satisfies the properties \( f_{i_1} \) and \( f_{i_2} \). Hence \( f_1 \) is a fuzzy soft ideal of \( f_L \).

(b) Consider the fuzzy soft set \( f_1 = \{ f_\emptyset, f_I, f_Q, f_L \} \subseteq f_L \) clearly \( f_1 \neq f_\emptyset \). It also satisfies the properties \( f_{i_1} \) but \( f_1(T) \subseteq f_{i_1} \) and \( f_1(R) \subseteq f_{i_1} \) implies \( f_1(R) \) does not belong to \( f_1 \). Hence \( f_1 \) is not a fuzzy soft ideal of \( f_L \).

(c) Consider the fuzzy soft set \( f_F = \{ f_\emptyset, f_I, f_Q, f_L \} \subseteq f_L \) clearly \( f_F \neq f_\emptyset \). It also satisfies the properties \( f_{r_1} \) and \( f_{r_2} \). Hence \( f_F \) is a fuzzy soft filter of \( f_L \).

(d) Consider the fuzzy soft set \( f_F = \{ f_\emptyset, f_I, f_Q, f_L \} \subseteq f_L \) clearly \( f_F \neq f_\emptyset \). It also satisfies the properties \( f_{r_1} \) but \( f_F(P) \subseteq f_F \) and \( f_F(P) \subseteq f_{r_1} \) satisfies the properties \( f_{r_1} \).

2.5 Theorem

Every fuzzy soft ideal and fuzzy soft filter of a fuzzy soft lattice \( f_L \) is a convex fuzzy soft sublattice of \( f_L \). Conversely, every convex fuzzy soft sublattice of \( f_L \) is a fuzzy soft filter.

Proof

Let \( f_1 \) be an fuzzy soft ideal of \( f_L \). Let \( f_1(a), f_1(b) \in f_1 \) then by \( f_1, f_1(a) \cup f_1(b) \in f_1, f_1(a) \land f_1(b) \subseteq f_1(a), f_1(a) \in f_1 \) implies \( f_1(a) \land f_1(b) \subseteq f_1 \). Therefore \( f_1 \) is a fuzzy soft sublattice of \( f_L \). Let \( f_1(x), f_1(y) \in f_1 \) and \( f_1(x) \subseteq f_1(y) \). Then \( f_1(y) = f_1(a) \subseteq f_1(f_1(x) \subseteq f_1(y)) \subseteq f_1, \) therefore \( f_1(x) \subseteq f_1 \) for \( f_1(x) \subseteq f_1 \). Clearly \( f_1(\emptyset) \subseteq f_1 \) and hence \( f_1 \) is non-empty. Let \( f_1(a), f_1(b) \in f_1 \). Then there exist \( f_1(v_1), f_1(v_2) \in f_1 \) such that \( f_1(a) \subseteq f_1(v_1) \) and \( f_1(b) \subseteq f_1(v_2) \) since \( f_1(v_1), f_1(v_2) \subseteq f_1 \) implies \( f_1(v_1) \subseteq f_1 \). Therefore \( f_1(a) \subseteq f_1(v_1) \subseteq f_1 \). Clearly \( f_1(\emptyset) \subseteq f_1 \) and hence \( f_1 \) is a fuzzy soft ideal of \( f_L \).

Let \( f_F = \{ f_F(a) \subseteq f_L \} \subseteq f_F \) since \( f_F(w) \subseteq f_F \). Clearly \( f_F(\emptyset) \subseteq f_F \) and hence \( f_F \) is non-empty. Let \( f_F(a), f_F(b) \subseteq f_F \). Then there exist \( f_F(w_1), f_F(w_2) \subseteq f_F \) such that \( f_F(w_1) \subseteq f_F \) and \( f_F(w_2) \subseteq f_F \). Since \( f_F(w_1), f_F(w_2) \subseteq f_F \) implies \( f_F(w) \subseteq f_F \). Therefore \( f_F(a) \subseteq f_F \) and \( f_F(b) \subseteq f_F \). Clearly \( f_F(\emptyset) \subseteq f_F \) and hence \( f_F \) is a fuzzy soft sublattice of \( f_F \).

2.6 Definition

A fuzzy soft ideal \( f_I \) of the fuzzy soft lattice \( f_L \) is said to be a prime fuzzy soft ideal if and only if at least one of an arbitrary pair of elements whose meet is in \( f_I \) is contained in \( f_I \).

2.7 Definition

A fuzzy soft filter \( f_F \) of the fuzzy soft lattice \( f_L \) is said to be a prime fuzzy soft filter, if \( f_F(a) \cup f_F(b) \subseteq f_F \) implies \( f_F(a) \subseteq f_F \) or \( f_F(b) \subseteq f_F \).

2.8 Definition

Let \( f_2 \) be a fuzzy soft lattice. Let \( f_2(x) \subseteq f_2 \). Then \( f_2(x) \subseteq f_2 \) is a fuzzy soft ideal and is called the principal fuzzy soft ideal generated by \( f_2(x) \) and is denoted by \( (f_2(x)) \).

2.9 Definition

Let \( f_2 \) be a fuzzy soft lattice. Let \( f_2(x) \subseteq f_2 \). Then \( f_2(x) \subseteq f_2 \) is a fuzzy soft filter and is called the principal fuzzy soft filter generated by \( f_2(x) \) and is denoted by \( (f_2(x)) \).
2.10 Example

Let \( X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\} \), be the universe and \( E = \{e_1, e_2, e_3\} \) be the set of parameters, \( A = \{e_1\}, B = \{e_2, e_3\} \), 
\( C = \{e_1, e_3\}, D = \{e_1, e_2, e_3\} \) where \( A, B, C, D \subseteq E \) and
\( f_i = \{f_i(\emptyset), f_i(A), f_i(B), f_i(C), f_i(D)\} \subseteq F(X) \) with the operations \( \tilde{U} \) and \( \tilde{\cap} \).
Assume that,
\[
\begin{align*}
    f_i(\emptyset) &= \emptyset \\
    f_i(A) &= \{(e_1, \{x_1, x_2\})\} \\
    f_i(B) &= \{(e_1, \{x_1, x_2, x_3\}\}, (e_2, \{x_5, x_6\})\} \\
    f_i(C) &= \{(e_1, \{x_1, x_2, x_4\}\}, (e_3, \{x_7, x_8\})\} \\
    f_i(D) &= \{(e_1, \{x_1, x_2, x_3, x_4\}\}, (e_2, \{x_5, x_6, x_9\}\), (e_3, \{e_7, e_8, e_{10}\})\}
\end{align*}
\]
Then \( (f_i(\tilde{U}, \tilde{\cap}) \) is a fuzzy soft lattice. The Hasse diagram of it appears in figure 2.

![Diagram](image_url)

Figure 2

(a) Consider the fuzzy soft ideal \( f_i = \{f_i(\emptyset), f_i(A), f_i(B), f_i(C), f_i(D)\} \). Now, \( f_i(B) \land f_i(C) = f_i(A) \in f_i \) implies \( f_i(B) \in f_i \). Hence \( f_i \) is a prime fuzzy soft ideal of \( f_i \).

(b) \( (f_i(B)) = \{f_i(\emptyset), f_i(A), f_i(B)\} \) is a principal fuzzy soft ideal generated by \( f_i(B) \).

(c) Consider the fuzzy soft ideal \( f_i = \{f_i(\emptyset), f_i(A)\} \). Let \( f_i(B), f_i(C) \in f_i \). Then \( f_i(B) \land f_i(C) = f_i(A) \in f_i \) implies \( f_i(B) \in f_i \). Hence \( f_i \) is a prime fuzzy soft ideal of \( f_i \).

2.11 Example

Let \( X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\} \), be the universe and \( E = \{e_1, e_2, e_3\} \) be the set of parameters, \( A = \{e_1\}, B = \{e_2\}, C = \{e_1, e_2\}, D = \{e_1, e_2, e_3\} \) where \( A, B, C, D \subseteq E \) and
\( f_i = \{f_i(\emptyset), f_i(A), f_i(B), f_i(C), f_i(D)\} \subseteq F(X) \) with the operations \( \tilde{U} \) and \( \tilde{\cap} \).
Assume that,
\[
\begin{align*}
    f_i(\emptyset) &= \emptyset \\
    f_i(A) &= \{(e_1, \{x_1, x_2\})\} \\
    f_i(B) &= \{(e_1, \{x_3, x_4\})\} \\
    f_i(C) &= \{(e_1, \{x_1, x_2, x_5\}\), (e_3, \{x_3, x_4, x_6\})\} \\
    f_i(D) &= \{(e_1, \{x_1, x_2, x_5, x_7\}\), (e_2, \{x_3, x_4, x_8\}\), (e_3, \{e_9, e_{10}\})\}
\end{align*}
\]
Then \( (f_i(\tilde{U}, \tilde{\cap}) \) is a fuzzy soft lattice. The Hasse diagram of it appears in figure 2.
Consider the fuzzy soft filter \( f_P = \{ f_P, f_C, f_D \} \). Now, \( f_I(A) \lor f_I(B) = f_P(C) \in f_P \) implies \( f_P(A) \in f_P \). Hence \( f_P \) is a prime fuzzy soft filter of \( f_I \).

(b) Consider the fuzzy soft filter \( f_P = \{ f_P(C), f_P(D) \} \). Let \( f_P(B), f_P(C) \in f_I \). Then \( f_P(B) \lor f_P(A) = f_P(C) \in f_P \) implies \( f_P(B) \in f_P \). Hence \( f_P \) is a prime fuzzy soft filter of \( f_I \).

(c) \( (f_P(A) = \{ f_P(A), f_P(C), f_P(D) \} \) is a principal fuzzy soft filter generated by \( f_P(A) \).

2.12 Theorem

Every fuzzy soft lattice has almost one minimal and one maximal element. These elements are at the same time the least and greatest element of that fuzzy soft lattice.

Proof

If possible, let there be two minimal elements \( f_L(m), f_L(n) \in f_L \) then \( f_L(m) \land f_L(n) \leq f_L(m) \). Since \( f_L(m) \) is a minimal element, \( f_L(m) \land f_L(n) = f_L(m) \) and hence \( f_L(m) \leq f_L(n) \).

Similarly, we take \( f_L(m) \land f_L(n) \leq f_L(n) \), then \( f_L(n) \leq f_L(m) \). Therefore \( f_L(m) = f_L(n) \). Hence the fuzzy soft lattice \( f_L \) has almost one minimal element and it is the least element of the lattice. By the principle of duality, every fuzzy soft lattice has almost one maximal element and it is the greatest element of that fuzzy soft lattice.

2.13 Definition

An element \( f_L(x) \) of a fuzzy soft lattice \( f_L \) is called a greatest element of the fuzzy soft lattice if \( f_L(a) \leq f_L(x) \) for all \( f_L(a) \in f_L \). Similarly, an element \( f_L(a) \) of a fuzzy soft lattice \( f_L \) is called a least element if \( f_L(x) \leq f_L(a) \) for all \( f_L(a) \in f_L \).

III. THE MAXIMUM AND MINIMUM CONDITIONS

In this section, we define the maximum and minimum conditions in fuzzy soft lattice. We also obtain a necessary and sufficient conditions for a fuzzy soft lattice to satisfy the maximum condition. We also define \( \land \) and \( \lor \) of two fuzzy soft ideals and we prove that the set of all fuzzy soft ideals of a fuzzy soft lattice.

3.1 Definition

Let \( f_C \) be any element of a poset \( f_P \) in the fuzzy soft lattice. Let us form the subchain of \( f_C \) in the following way: let the greatest element of the subchain be \( f_C \). Let \( f_C(K) \geq 1 \) be an element of \( f_P \) such that \( f_{C_{K'}} \leq f_{C_{K-1}} \). If each of the chains so formed, commencing at any \( f_{C_{K}} \) is finite, then \( f_P \) is said to satisfy the maximum condition.

3.2 Definition

Let the least element of the subchain be \( f_C \). Let \( f_C(K) \geq 1 \) be an element of \( f_P \) such that \( f_{C_{K-1}} \leq f_{C_{K}} \). If each of the chains so formed, commencing at any \( f_{C_{K}} \) is finite, then \( f_P \) is said to satisfy the minimum condition.

3.3 Result

If a poset \( f_P \) in a fuzzy soft lattice satisfies the minimum condition then for any \( f_P(a) \in f_P \), there exist at least one minimal element \( f_P(m) \) of \( f_P \) such that \( f_P(m) \leq f_P(a) \).

3.4 Result

If a poset \( f_P \) in a fuzzy soft lattice satisfies the maximum condition then for any \( f_P(a) \in f_P \), there exist at least one maximal element \( f_P(m) \) of \( f_P \) such that \( f_P(a) \leq f_P(m) \).

3.5 Corollary

Every fuzzy soft lattice satisfying minimum (maximum) conditions has a least (greatest) element.
3.6 Note
By a fuzzy soft ideal chain of a fuzzy soft lattice $f_1$, we shall mean a set of fuzzy soft ideals in $f_1$ in which one of every pair of fuzzy soft ideals includes the other.

3.7 Lemma
The fuzzy soft union of any fuzzy soft ideal chain of a fuzzy soft lattice $f_1$ is itself a fuzzy ideal in $f_1$.

**Proof**
Let $f_1$ be a chain of fuzzy soft ideals of $f_1$, let $f_1$ denote the fuzzy soft union of all fuzzy soft ideals of $f_1$ in $f_1$. Let $f_1(x), f_1(y) \in f_1$. Then there exists fuzzy soft ideals $f_1$ and $f_2$ in $f_1$ such that $f_1(x) \in f_1$ and $f_1(y) \in f_1$. Since either $f_1 \subseteq f_2$, or $f_2 \subseteq f_1$. Let $f_1 \subseteq f_2$, then $f_1(x) \subseteq f_2$, therefore $f_2(x) \subseteq f_2$. Hence $f_1(x) \wedge f_1(y) \in f_1$. Let $f_1(a) \in f_1$, then $f_1(x) \wedge f_1(a) \in f_1 \subseteq f_1$. Therefore $f_1(x) \wedge f_1(a) \in f_1$.

Hence $f_1$ is a fuzzy soft ideal.

3.8 Theorem
A necessary and sufficient condition for a fuzzy soft ideal $f_1$ in a fuzzy soft lattice $f_1$ to be a principal fuzzy soft ideal is that the fuzzy soft lattice $f_1$ satisfies the maximum condition.

**Proof**
Suppose the fuzzy soft lattice $f_1$ satisfies the maximum condition, then it is also satisfied in every fuzzy soft ideal $f_1$ of $f_1$. By corollary the fuzzy soft ideal $f_1$ includes a greatest element $f_1(x)$. Hence $f_1(x)$ is a fuzzy soft ideal in $f_1$. Conversely, suppose that every fuzzy soft ideal is a principal fuzzy soft ideal, we have to prove fuzzy soft lattice satisfies the maximum condition.

Suppose not, then we can find an infinite subchain of the form $f_1 = f_0 \prec f_1 \prec \ldots$. The set $f_1 = \bigcup_{n=0}^{\infty} f_{c_n}$ being the fuzzy soft union of the elements of the fuzzy soft ideal chain is itself a fuzzy soft ideal by lemma. Hence $f_1$ cannot be a principal fuzzy soft ideal since every one of its elements is less than the other of its elements.

Therefore $f_1$ has no greatest element which is a contradiction.

3.9 Theorem
Let $f_1$ and $f_2$ be fuzzy soft ideals of a fuzzy soft lattice $f_1$. Define $f_1 \wedge f_2 = \{f_1(a) \in f_1 \cap f_2(a) \in f_2 \}$. Then $f_1 \wedge f_2$ satisfies the maximum condition. Then the set of all fuzzy soft ideals $f_1 \wedge f_2$ is a fuzzy soft lattice.

**Proof**
Clearly $f_1 \wedge f_2 \neq \emptyset$ for $f_1(0) \in f_1$ and $f_2(0) \in f_2$. Let $f_1(0), f_2(0) \in f_1$. Let $f_1(0) = f_1(a), f_2(0) = f_2(b)$. Therefore $f_1(a), f_2(b) \in f_1$. Hence $f_1(a) = f_1(0)$. Moreover $f_1(a) \wedge f_2(b) = (f_1(a) \wedge f_2(b)) = (f_1(a) \wedge f_2(b)) = (f_1(a) \wedge f_2(b)) = \{f_1(x) \wedge f_2(y) \mid f_1(x) \wedge f_2(y) = f_1(a) \wedge f_2(b)\}

Therefore $f_1(a) \wedge f_2(b) \subseteq f_1(a) \wedge f_2(b)$ since $f_1(a) \wedge f_2(b) \subseteq f_1(a) \wedge f_2(b)$. Furthermore $f_1(a) \wedge f_2(b)$ is a fuzzy soft ideal.

3.10 Theorem
The set of all principal fuzzy soft ideals $f_0(f_1)$ of a fuzzy soft lattice $f_1$ is a fuzzy soft sublattice of $f_1(f_1)$ and is fuzzy soft isomorphic to $f_1$.

**Proof**
We claim that $f_0(f_1) = \{f_1(x) \wedge f_1(y) \mid (f_1(x) \wedge f_1(y)) \wedge (f_1(x) \wedge f_1(y)) = (f_1(x) \wedge f_1(y)) \}$ holds for every pair of elements $f_1(x), f_1(y)$ of $f_1$.

First to prove $f_0(f_1) \subseteq f_0(f_1)$. Let $f_0(f_1) \subseteq f_0(f_1)$. Then $f_0(f_1) \subseteq f_0(f_1)$. Therefore, $f_0(f_1) \subseteq f_0(f_1)$. Hence $f_0(f_1) \subseteq f_0(f_1)$.

Next to prove $f_0(f_1) \subseteq f_0(f_1)$. Let $f_0(f_1) \subseteq f_0(f_1)$.

Let $f_0(f_1) = f_0(f_1) \wedge (f_1(x) \wedge f_1(y))$. Then $f_0(f_1) \subseteq f_0(f_1)$. Therefore $f_0(f_1) \subseteq f_0(f_1)$.

Let us define $\eta: f_1 \to f_1$ by $\eta(f_1(a)) = f_1(a)$. Suppose $\eta(f_1(a)) = f_1(a)$. Then $f_1(a) = f_1(a)$.

Let us define $\eta: f_1 \to f_1$ by $\eta(f_1(a)) = f_1(a)$. Suppose $\eta(f_1(a)) = f_1(a)$.

Therefore $f_0(f_1)$ is a fuzzy soft lattice.
Since $f_{(f_{(L)})}(b) \in f_{(f_{(L)})} = f_{(f_{(a)})} \Rightarrow f_{(f_{(L)})}(b) \in f_{(f_{(a)})} \Rightarrow f_{(L)}(b) \leq f_{(a)}(b)$ Therefore $f_{(L)}(a) = f_{(L)}(b)$ and hence $\eta$ is one-one. For every $f_{(L)}(a) \in f_{(L)}(a)$, there exist an element $f_{(L)}(a) \in f_{L}$ such that $\eta(f_{(L)}(a)) = (f_{(L)}(a))$. Therefore $\eta$ is onto. To prove $\eta$ is a fuzzy soft lattice homomorphism.

\[
\begin{align*}
\eta(\bigvee f_{(L)}(a)) &= (\bigvee f_{(L)}(a)) = \bigvee f_{(L)}(a) = \bigvee (f_{(L)}(a)) = f_{(L)}(a), \\
\eta(\bigwedge f_{(L)}(a)) &= \bigwedge f_{(L)}(a) = \bigwedge (f_{(L)}(a)) = f_{(L)}(a), \\
\eta(\bigwedge f_{(L)}(a)) &= \bigwedge f_{(L)}(a) = \bigwedge (f_{(L)}(a)) = f_{(L)}(a), \\
\eta(\bigvee f_{(L)}(a)) &= \bigvee f_{(L)}(a) = \bigvee (f_{(L)}(a)) = f_{(L)}(a).
\end{align*}
\]

Therefore $\eta$ is a fuzzy soft homomorphism.

Hence the map $\eta : f_{L} \rightarrow f_{(L)}(a)$ is a fuzzy soft isomorphism and $f_{L} \cong f_{(L)}(a) \notin f_{(L)}(a)$.

3.11 Theorem

The fuzzy soft lattice $f_{L}$ is modular if and only if the fuzzy soft ideal lattice $f_{(L)}(a)$ is modular.

**Proof**

Suppose $f_{(L)}(a)$ is a modular fuzzy soft lattice, then the set of all principal fuzzy soft ideals $f_{(L)}(a)$ of a fuzzy soft lattice $f_{L}$ is a fuzzy soft sublattice of $f_{(L)}(a)$ and is fuzzy soft isomorphic to $f_{L}$.

That is $f_{L} \cong f_{(L)}(a) \notin f_{(L)}(a)$.

$f_{(L)}(a)$ is a modular fuzzy soft lattice implies its fuzzy soft sublattice $f_{(L)}(a)$ is a modular fuzzy soft lattice. $f_{(L)}(a)$ is a modular fuzzy soft lattice implies its fuzzy soft isomorphic copy $f_{L}$ is a modular fuzzy soft lattice. Hence $f_{(L)}(a)$ is a modular fuzzy soft lattice implies that $f_{L}$ is a modular fuzzy soft lattice. Conversely, let $f_{L}$ be a modular fuzzy soft lattice. To prove that $f_{(L)}(a)$ is a modular fuzzy soft lattice. Let $f_{L}, f_{j}, f_{K}$ be fuzzy soft ideals of $f_{L}$ such that $f_{L} \leq f_{K}$ and $f_{j} \leq f_{K}$ clearly $f_{j} \vee f_{j} \leq f_{j} \vee f_{K}$. Hence it is enough to prove that $f_{j} \leq f_{j} \leq f_{j} \vee f_{K}$. Let $f_{j} \leq f_{j} \leq f_{j} \vee f_{K}$. Then $f_{j} = f_{j} \leq f_{j} \vee f_{K}$ and $f_{j} \leq f_{j} \vee f_{K}$ Where $f_{j}(y) \in f_{j} \cup f_{j} \vee f_{K}$. Similarly $f_{j} \cup f_{j} \vee f_{K}$ is distributive fuzzy soft lattice. Hence $f_{(L)}(a)$ is a modular fuzzy soft lattice.

3.12 Theorem

The fuzzy soft lattice $f_{L}$ is distributive if and only if the fuzzy soft ideal lattice $f_{(L)}(a)$ is distributive.

**Proof**

Suppose $f_{(L)}(a)$ is a distributive fuzzy soft lattice, then the set of all principal fuzzy soft ideals $f_{(L)}(a)$ of a fuzzy soft lattice $f_{L}$ is a fuzzy soft sublattice of $f_{(L)}(a)$ and is fuzzy soft isomorphic to $f_{L}$.

That is $f_{L} \cong f_{(L)}(a) \notin f_{(L)}(a)$.

$f_{(L)}(a)$ is a distributive fuzzy soft lattice implies its fuzzy soft sublattice $f_{(L)}(a)$ is a distributive fuzzy soft lattice. $f_{(L)}(a)$ is a distributive fuzzy soft lattice implies its fuzzy soft isomorphic copy $f_{L}$ is a distributive fuzzy soft lattice. Hence $f_{(L)}(a)$ is a distributive fuzzy soft lattice implies that $f_{L}$ is a distributive fuzzy soft lattice. Conversely, let $f_{L}$ be a distributive fuzzy soft lattice. To prove that $f_{(L)}(a)$ is a distributive fuzzy soft lattice. Let $f_{L}, f_{j}, f_{K}$ be fuzzy soft ideals of $f_{L}$ such that $f_{L} \leq f_{K}$, clearly $f_{j} \wedge f_{j} \leq f_{j} \wedge f_{K}$ and $f_{j} \wedge f_{j} \leq f_{j} \wedge f_{K}$. It is enough to prove that $f_{j} \wedge (f_{j} \wedge f_{K}) \leq f_{j} \wedge f_{K}$. Let $f_{j} \wedge (f_{j} \wedge f_{K}) \leq f_{j} \wedge f_{K}$. Then $f_{j} = f_{j} \wedge (f_{j} \wedge f_{K}) \leq f_{j} \wedge f_{K}$ Where $f_{j}(y) \in f_{j} \cup f_{j} \wedge f_{K}$. Similarly $f_{j} \wedge (f_{j} \wedge f_{K})$ is a distributive fuzzy soft lattice. Hence $f_{(L)}(a)$ is a distributive fuzzy soft lattice.

IV CONCLUSION

In this study, we have defined fuzzy soft ideals and filters, prime fuzzy soft ideals and filters, principal fuzzy soft ideals and filters, discussed their properties and illustrated them with some examples. We have shown that the set of fuzzy soft ideals of a fuzzy soft lattice is modular, and we have also proved modular fuzzy soft lattice and distributive fuzzy soft lattice. An interesting topic for further study is to discuss methods of fuzzy soft filters of a fuzzy soft lattice.

V REFERENCES


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