INTERVAL VALUED FUZZY SET OF FUZZY HEMIRINGS

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***Abstract** In this paper, we first present the concepts of interval valued fuzzy set of fuzzy hemirings and we introduce the notion of $A = [\mu^-, \mu^+]$. Interval valued fuzzy set of fuzzy hemirings and explore some of their attribute.*

***Index Terms** Fuzzy set, Interval valued fuzzy set, hemirings, fuzzy hemirings.*

I.INTRODUCTION

In dealing with the intricate problems in economics, engineering and environmental sciences, we are ordinarily in effective to apply the classical methods because there are various suspense in their problem. The construction of fuzzy sets and fuzzy set operations, introduced by L.A. Zadeh[12] (1965) have been all inclusive to a great deal scientific field of operation. Zadeh bring in and used interval valued fuzzy sets. In algebra, Rosenfeld[6] first used fuzzy set, he introduced fuzzy group and explore some of its attribute. So fuzzy of hemirings became significant tools in lots of branches of applied mathematics and engineering.

But greatly of the researches of fuzzy hemirings[8]s are based on their ideals. In this paper, we have the concepts of interval valued fuzzy set of fuzzy hemirings and studied some some of its attribute. The complement, union, intersection operations have been defined on the interval valued fuzzy set of fuzzy hemirings.

II.PRELIMINARIES

2.1 Definition

Let X be the collection of objects denoted generally by x. Then a fuzzy set A in X is defined as,

$$A = \{< x,\ \mu_A(x) > \ /\ x \in X\}$$

Where $\mu_A(x)$ is called the membership value of x in A and

$$0 \le \mu_A(x) \le 1.$$

2.2 Definition

An interval valued fuzzy set $\tilde{X}$ on a universe U is a mapping such that $\tilde{X} : U \to Int\ ([0, 1])$. Where Int ([0, 1]) stands for the set of all closed subinterval of [0, 1], the set of all interval valued fuzzy sets on U is denoted by $\tilde{P}(U)$.

Suppose that $\tilde{X} \in \tilde{P}(U)\ \forall\, x \in U$. $\mu_x(x) = [\mu_x^-(x), \mu_x^+(x)]$ is called the degree of membership of an element x to X. Where $\mu_x^-(x)$ and $\mu_x^+(x)$ are referred to as the lower and upper degree of membership of x to $\tilde{X}$ respectively such that

$$0 \le \mu_x^-(x) \le \mu_x^+(x) \le 1 .$$

2.3 Definition

A semiring is an algebraic system $(R, +, \cdot)$ consisting of a non- empty set R together with two binary operations on R called addition and multiplication such that $(R, +)$ and $(R, \cdot)$ are semigroups and the following distributive laws

$$a.(b + c) = a.b + a.c$$
$$(a + b).c = a.c + b.c$$

are satisfied for all $a, b, c \in R$. By zero of a semiring $(R, +, \cdot)$ we mean an element $0 \in R$ such that $0.x = x.0 = 0$ and $0 + x = x + 0 = x$ for all $x \in R$. A semiring with zero and a commutative semigroup $(R, +)$ is called a hemiring.

2.4 Definition

Let R be a hemiring. A fuzzy set μ in R is defined as a mapping from R to $[0, 1]$, the usual interval of real number. We denote by I^R the set of all fuzzy sets in R.

2.5 Definition

Suppose $\mu \in I^R$. For any $s \in]0,1]$, the set $\mu_s = \{x \in R: \mu(x) \ge s\}$ and the set $supp(\mu) = \{x \in R: \mu(x) > 0\}$ are called s- level set and supporting set of μ.

2.6 Definition

Suppose R is a hemiring and μ is a fuzzy set of R with support $\{x\}$. Then μ is called a fuzzy point if

$$\mu(y) = \begin{cases} s \in]0, 1] & if\ y = x \\ 0 & otherwise \end{cases}$$

We denote the fuzzy point with support $\{x\}$ and value s by x_s.

III.INTERVAL VALUED FUZZY SET OF FUZZY HEMIRING

3.1 Definition

Let R be a hemiring. A fuzzy set μ of R is said to be fuzzy hemiring of R if it satisfies the following conditions:

$(FH_1) : \mu(x+y) \geq \min\{\mu(x), \mu(y)\};$

$(FH_2) : \mu(x+y) \geq \min\{\mu(x), \mu(y)\} \quad \forall x, y \in R.$

3.2 Definition

Let R be a hemiring. A fuzzy set μ of R is said to be interval valued fuzzy set of fuzzy hemiring of R if it satisfies the following conditions:

$(IVFH_1) : \mu(x+y) \geq \{\min[\mu^-(x), \mu^-(y)], min[\mu^+(x), \mu^+(y)]\}$

$(IVFH_2) : \mu(xy) \geq \{\min[\mu^-(x), \mu^-(y)], min[\mu^+(x), \mu^+(y)]\}. \forall x, y \in R.$

3.3 Definition

Let R be a hemiring. A fuzzy set μ of R is said to be interval valued fuzzy set of anti-fuzzy hemiring of R if it satisfies the following conditions:

$(IVAFH_1) : \mu(x+y) \leq \{\max[\mu^-(x), \mu^-(y)], max[\mu^+(x), \mu^+(y)]\}$

$(IVAFH_2) : \mu(xy) \leq \{\max[\mu^-(x), \mu^-(y)], max[\mu^+(x), \mu^+(y)]\}. \forall x, y \in R.$

3.4 Theorem

The condition $(IVAFH_1)$ and $(IVAFH_2)$ in definition are equivalent to the conditions $(IVAFH_3)$ and $(IVAFH_4)$ respectively.

$(IVAFH_3) : x_s \in \mu$ and $y_k \in \mu \implies (x+y)_{\min(s,k)} \in \mu.$

$(IVAFH_4) : x_s \in \mu$ and $y_k \in \mu \implies (xy)_{\min(s,k)} \in \mu.$ $\forall\, s, k \in]0,1]$ and $x, y \in R.$

Proof

$(IVAFH_1) \Leftrightarrow (IVAFH_3)$

We first prove that

$(IVAFH_1) \implies (IVAFH_3)$

Assume that the condition $(IVAFH_1)$ is valid. Let $x, y \in R$ and $s, k\ ]0,1]$ be such that $x_s \in \mu$ and $y_k \in \mu$. Then $\mu(x) \geq s$ and $\mu(y) \geq k$ which imply from $(IVAFH_1)$

$\mu(x+y) \geq \{\min[\mu^-(x), \mu^-(y)], min[\mu^+(x), \mu^+(y)]\}$

$\geq \{\min[\mu^-(x), \mu^+(x)], min[\mu^-(y), \mu^+(y)]\}$

$\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$

$\mu(x+y) \geq \min\{s, k\}$

Hence $(x+y)_{\min(s,k)} \in \mu$. Hence $(IVAFH_3)$ holds.

$(IVAFH_3) \implies (IVAFH_1)$

Suppose that the condition $(IVAFH_3)$ is valid. Note that $x_{\mu(x)} \in \mu$ and $y_{\mu(y)} \in \mu;\ \forall\, x, y \in R$. Thus $(x+y)_{\min\{\mu(x),\ \mu(y)\}} \in \mu$. by $(IVAFH_3)$, and so

$\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$

$\geq \{\min[\mu^-(x), \mu^+(x)], min[\mu^-(y), \mu^+(y)]\}$

$\mu(x+y) \geq \{\min[\mu^-(x), \mu^-(y)], min[\mu^+(x), \mu^+(y)]\}$

Hence $(IVAFH_1)$ holds.

Similarly, we prove that

$(IVAFH_2) \Leftrightarrow (IVAF4).$

3.5 Proposition

Let R be a hemiring. A fuzzy set μ of R is a interval valued fuzzy set of fuzzy hemiring of R iff for any $s\ ]0,1]$, $\mu_s (\neq \emptyset)$ is a interval valued fuzzy set of sub- hemiring of R

Proof

Let $x, y \in \mu_s$ then $\mu(x) \geq s$ and $\mu(y) \geq s$.

We consider μ is a interval valued fuzzy set of fuzzy hemiring of R. then we have

$\mu(x+y) \geq \{\min[\mu^-(x), \mu^-(y)], min[\mu^+(x), \mu^+(y)]\}$

$\mu(x+y) \geq \{\min[\mu^-(x), \mu^+(x)], min[\mu^-(y), \mu^+(y)]\}$

$\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$

$\mu(x+y) \geq \min\{s, s\}$

$\mu(x+y) \geq s \qquad \therefore x + y \in \mu_s$

And

$\mu(xy) \geq \{\min[\mu^-(x), \mu^-(y)], min[\mu^+(x), \mu^+(y)]\}$

$\geq \{\min[\mu^-(x), \mu^+(x)], min[\mu^-(y), \mu^+(y)]\}$

$\mu(xy) \geq \min\{\mu(x), \mu(y)\}$

$\mu(xy) \geq \min\{s, s\}$

$\mu(xy) \geq s \qquad \therefore\ x\,y \in \mu_s$.

Hence μ_s is a interval valued sub-hemiring of R.

Conversely,

Suppose μ_s is a interval valued sub-hemiring of R. then $\forall\, x, y \in \mu_s$.

We have $x + y \in \mu_s$ and $x\,y \in \mu_s$.

Now $x + y \in \mu_s \Rightarrow \mu(x+y) \geq s$

$\mu(x+y) \geq \min\{s, s\}$

$\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$

$\geq \{\min[\mu^-(x), \mu^+(x)], min[\mu^-(y), \mu^+(y)]\}$

$\mu(x+y) \geq \{\min[\mu^-(x), \mu^-(y)], min[\mu^+(x), \mu^+(y)]\}$

And $\mu(xy) \geq s$

$\mu(xy) \geq \min\{s, s\}$

$\mu(xy) \geq \min\{\mu(x), \mu(y)\}$

$\geq \{\min[\mu^-(x), \mu^+(x)], min[\mu^-(y), \mu^+(y)]\}$

$\mu(xy) \geq \{\min[\mu^-(x), \mu^-(y)], min[\mu^+(x), \mu^+(y)]\}$

Hence μ is a interval valued fuzzy set of fuzzy hemiring of R.

3.6 Theorem

A fuzzy set μ of a hemiring R is a interval valued fuzzy set of fuzzy hemiring of R iff its complement μ^C is an interval valued fuzzy set of anti- fuzzy hemiring of R.

Proof

Let μ be a interval valued fuzzy set of fuzzy hemiring of R. we have $\forall\, x, y \in R$;

$\mu^C(x+y) = 1 - \mu(x+y)$

$\leq 1 - \min[\mu^+(x), \mu^+(y)], min[\mu^-(x), \mu^-(y)]$

$\leq 1 - \min[\mu^+(x), \mu^-(x)], min[\mu^+(y), \mu^-(y)]$

$= \max\{[1 - \mu^+(x), \mu^-(x)], [1 - \mu^+(y), \mu^-(y)]\}$

$= \max\{1 - \mu(x), 1 - \mu(y)\}$

$\mu^C(x+y) \leq \max\{\mu^C(x), \mu^C(y)\}$.

Similarly, we can show that

$\mu^C(xy) = 1 - \mu(xy)$

$\leq 1 - \min[\mu^+(x), \mu^+(y)], min[\mu^-(x), \mu^-(y)]$

$\mu^C(xy) \quad \leq 1 - \min[\mu^+(x), \mu^-(x)], min[\mu^+(y), \mu^-(y)]$

$\mu^C(xy) = \max\{[1 - \mu^+(x), \mu^-(x)], [1 - \mu^+(y), \mu^-(y)]\}$

$= \max\{1 - \mu(x), 1 - \mu(y)\}$

$\mu^C(xy) \leq \max\{\mu^C(x), \mu^C(y)\}$.

Hence μ^C is an interval valued fuzzy set of anti- fuzzy hemiring of R.

Conversely,

$\mu^C(x+y) \leq \max\{\mu^C(x), \mu^C(y)\}$.

$= \max\{1 - \mu(x), 1 - \mu(y)\}$

$= \max\{[1 - \mu^+(x), \mu^-(x)], [1 - \mu^+(y), \mu^-(y)]\}$

$\leq 1 - \min[\mu^+(x), \mu^-(x)], min[\mu^+(y), \mu^-(y)]$

$\leq 1 - \min[\mu^+(x), \mu^+(y)], min[\mu^-(x), \mu^-(y)]$

$\mu^C(x+y) = 1 - \mu(x+y)$.

$\mu^C(xy) \leq \max\{\mu^C(x), \mu^C(y)\}$.

$= \max\{1 - \mu(x), 1 - \mu(y)\}$

$\mu^C(xy) = \max\{[1 - \mu^+(x), \mu^-(x)], [1 - \mu^+(y), \mu^-(y)]\}$

$\leq 1 - \min[\mu^+(x), \mu^+(y)], min[\mu^-(x), \mu^-(y)]$

$\mu^C(xy) = 1 - \mu(xy)$

Hence A fuzzy set μ of a hemiring R is a interval valued fuzzy set of fuzzy hemiring of R

3.7 Definition

Let μ_1 and μ_2 be two interval valued fuzzy set of fuzzy hemirings of a hemiring R. we define $\mu_1 \cap \mu_2$ by

$(\mu_1 \cap \mu_2)(x) = \{\min[\mu_1^-(x), \mu_2^-(x)], \min[\mu_1^+(x), \mu_2^+(x)]\}$

3.8 Theorem

Let μ_1 and μ_2 be two interval valued fuzzy set of fuzzy hemirings of a hemiring R. we define $\mu_1 \cap \mu_2$ is also interval valued fuzzy set of fuzzy hemirings of R.

Proof

Let $x, y \in \mu_1 \cap \mu_2$. Then $x, y \in \mu_1$ and μ_2.

Now,

$(\mu_1 \cap \mu_2)\ (x+y) = \min\{\mu_1(x+y), \mu_2(x+y)\}$

$\geq \{\min[\mu_1^-(x+y), \mu_2^-(x+y)], \min[\mu_1^+(x+y), \mu_2^+(x+y)]\}$

$(\mu_1 \cap \mu_2)\ (x+y) = \min\{\{\min[\mu_1^-(x), \mu_1^-(y)], [\mu_2^-(x), \mu_2^-(y)]\}, \{\min[\mu_1^+(x), \mu_1^+(y)], [\mu_2^+(x), \mu_2^+(y)]\}$

$(\mu_1 \cap \mu_2)\ (x+y) \geq \min\{\{\min[\mu_1^-(x), \mu_1^+(x)], [\mu_1^-(y), \mu_1^+(y)]\},$

$$\{\min[\mu_2^-(x), \mu_2^+(x)], [\mu_2^-(y), \mu_2^+(y)]\}$$
$$\geq \min\{\{\min[\mu_1^-(x), \mu_1^+(x)], [\mu_2^-(x), \mu_2^+(x)]\},$$
$$\{\min\ [\mu_1^-(y), \mu_1^+(y)], [\mu_2^-(y), \mu_2^+(y)]\}$$
$$= min\ \{\{min\ [\mu_1(x), \mu_2(x)]\}, \{\min[\mu_1(y), \mu_2(y)]\}\}$$
$$(\mu_1 \cap \mu_2)\ (x+y)\ = \min\{(\mu_1 \cap \mu_2)\ (x), (\mu_1 \cap \mu_2)\ (y)\}$$

Similarly, we can prove that

$$(\mu_1 \cap \mu_2)\ (xy)\ = \min\{\{\min[\mu_1^-(x), \mu_1^-(y)], [\mu_2^-(x), \mu_2^-(y)]\},$$
$$\{\min[\mu_1^+(x), \mu_1^+(y)], [\mu_2^+(x), \mu_2^+(y)]\}$$
$$(\mu_1 \cap \mu_2)\ (xy)\ \geq \min\{\{\min[\mu_1^-(x), \mu_1^+(x)], [\mu_1^-(y), \mu_1^+(y)]\},$$
$$\{\min[\mu_2^-(x), \mu_2^+(x)], [\mu_2^-(y), \mu_2^+(y)]\}$$
$$\geq \min\{\{\min[\mu_1^-(x), \mu_1^+(x)], [\mu_2^-(x), \mu_2^+(x)]\},$$
$$\{\min\ [\mu_1^-(y), \mu_1^+(y)], [\mu_2^-(y), \mu_2^+(y)]\}$$
$$= min\ \{\{min\ [\mu_1(x), \mu_2(x)]\}, \{\min[\mu_1(y), \mu_2(y)]\}\}$$
$$(\mu_1 \cap \mu_2)\ (xy)\ = \min\{(\mu_1 \cap \mu_2)\ (x), (\mu_1 \cap \mu_2)\ (y)\}$$

Thus $\mu_1 \cap \mu_2$ is also interval valued fuzzy set of fuzzy hemirings of R.

3.9 Definition

Let μ_1 and μ_2 be two interval valued fuzzy set of fuzzy hemirings of a hemiring R. then their product is defined as

$$(\mu_1\, \mu_2)(x) = \max_{x=yz} \{\min[\mu_1^-(y), \mu_2^-(z)], \min[\mu_1^+(y), \mu_2^+(z)]\}$$

If x can be expressed as $x = yz$ otherwise $(\mu_1\, \mu_2)(x) = 0$.

3.10 Theorem

Let μ_1 and μ_2 be two interval valued fuzzy set of fuzzy hemirings of a hemiring R. then their product $\mu_1\, \mu_2$ is also interval valued fuzzy set of fuzzy hemirings of R.

Proof

Trivial.

3.11 Theorem

Let λ, μ, and ν be interval valued fuzzy set of fuzzy hemirings of a hemiring R. if $\lambda \leq \mu$, then $\lambda\nu \leq \mu\nu$ and $\nu\lambda \leq \nu\mu$.

Proof

Let $x \in R$. If x is not expressible as $x = yz$.then

$$(\lambda\nu)(x) = 0 = (\mu\nu)(x)$$

Otherwise,

$$(\lambda\nu)(x) = \max_{x=yz} \{\min[\lambda^-(y), \nu^-(z)], \min[\lambda^+(y), \nu^+(z)]\}$$
$$\leq \max_{x=yz} \{\min[\lambda^-(y), \lambda^+(y)], \min\ [\nu^-(z), \nu^+(z)]\}$$
$$\leq \max_{x=yz} \{\min\ [\lambda\ (y), \nu(z)]\}$$
$$(\lambda\nu)(x) = (\mu\nu)(x)$$

Hence $\lambda\nu \leq \mu\nu$

Similarly, we can prove that $\nu\lambda \leq \nu\mu$.

3.12 Definition

Let μ_1 and μ_2 be two interval valued fuzzy set of fuzzy hemirings of a hemiring R. then their sum is defined as

$$(\mu_1 + \mu_2)(x) = \max_{x=yz} \{\min[\mu_1^-(y), \mu_2^-(z)], \min[\mu_1^+(y), \mu_2^+(z)]\}.$$

3.13 Theorem

Let μ_1 and μ_2 be two interval valued fuzzy set of fuzzy hemirings of a hemiring R. then their sum $\mu_1 + \mu_2$ is also interval valued fuzzy set of fuzzy hemirings of R.

Proof

Trivial.

IV.CONCLUSION

In this paper, we have introduced the concepts of interval valued fuzzy set of fuzzy hemirings and studied some of its attribute. So fuzzy became significant tools in lots of branches of applied mathematics and engineering. But greatly of the researches of fuzzy hemirings are based on their ideals.

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