FUZZY PRE GENERALIZED SEMI-CLOSED SETS

M. M. SALASDEENA¹ AND M. SENTAMILSELVI²

¹Research Scholar in Mathematics, Vivekanandha college of arts and sciences for women (Autonomous) Tiruchengode Namakkal (Dt), Tamilnadu, India.
²Assistant professor Department of Mathematics, Vivekanandha college of arts and sciences for women (Autonomous) Tiruchengode Namakkal (Dt), Tamilnadu, India.

Abstract : In this paper, we introduced a new classes of sets called fuzzy pre-generalized semi-closed sets in fuzzy topological spaces is introduced and investigate their properties. As an application of this set we also introduces the new kinds of separation axioms namely, $F_{pgs}$- $T_{1/2}$-space $F_{pgs}$-continuity and $F_{pgs}$- irresolute mappings. Fuzzy pre generalized semi $T_{1/2}$ fuzzy pre generalized semi $T_{3/4}$ spaces and fuzzy semi $T_{1/3}$ and characterized them.

Key words and phrases : Fuzzy pre generalized semi-closed sets, Fuzzy pre generalized semi-continuity, Fuzzy pre generalized semi- irresoluteness, Fuzzy semi $T_{1/2}$-space.

INTRODUCTION :

We define a new class of fuzzy pre generalized sets namely, fuzzy semi-closed sets and investigate their properties. The fts $X$ denote a fuzzy topological spaces $(X, \tau)$. Fuzzy sets in $X$ will be denoted by $(\nu, \eta, \lambda, \mu)$. The operators can be denoted by fuzzy closure and fuzzy interior. The concept of fuzzy semi-preopen sets and introduced fuzzy pre generalized $T_{1/2}$ spaces, $F_{pgs}$ continuity and $F_{pgs}$- irresoluteness. The aim of this paper is to introduce the notion of fuzzy pre generalized semi-closed sets, an alternative generalization of fuzzy semi open set in fuzzy topological spaces. We introduce a class of fuzzy topological spaces, called $F_{pgs} T_{1/2}$- spaces and obtain some of its characterizations. Further, we also introduce $F_{pgs}$-continuity and $F_{pgs}$- irresoluteness.

1.1 Definition : A fuzzy set $A$ of $(X, \tau)$ is called
1) Fuzzy semi open (shortly, $F_{s}$- open) if $A \leq \text{Cl}(\text{Int}(A))$ and a fuzzy semi closed (shortly, $F_{s}$-closed) if $\text{Int}(\text{Cl}(A)) \leq A$.
2) Fuzzy pre open (shortly, $F_{p}$-open) if $A \leq \text{Int}(\text{Cl}(A))$ and a fuzzy pre closed (shortly, $F_{p}$-closed) if $\text{Cl}(\text{Int}(A)) \leq A$.
3) Fuzzy pre semi open (shortly, $F_{ps}$-open) if $A \leq \text{Int}(\text{Cl}(A))$ and a fuzzy pre semi closed (shortly $F_{ps}$-closed) if $\text{Cl}(\text{Int}(A)) \leq A$.

1.2 Definition : A fuzzy set $\nu$ in fuzzt topological space $(X, \tau)$ is called
1) Fuzzy generalized closed set if $\text{Cl}(\nu) \leq \eta$ whenever $\nu \leq \eta$ and $\eta$ is fuzzy open. We shortly denoted it as $F_{g}$-closed.
2) Fuzzy pre- generalized closed set if $\text{pCl}(\nu) \leq \eta$ whenever $\nu \leq \eta$ and $\nu$ is fuzzy semi open. We shortly denoted it as $F_{pg}$-closed set.
3) Fuzzy generalized semi- closed set if $s\text{Cl}(\nu) \leq \mu$ whenever $\nu \leq \eta$ and $\eta$ is fuzzy open. We shortly denoted it as $F_{pgs}$-closed set.
4) Fuzzy pre generalized semi- closed set if $s\text{pCl}(\nu) \leq \eta$ whenever $\nu \leq \eta$ and $\eta$ is fuzzy open. We shortly denoted it as $F_{pgs}$-closed set.

1.3 Definition : A fuzzy topological space $(X, \tau)$ is said to be a
1) Fuzzy - $T_{1/2}$-space if every $F_{g}$-closed set is fuzzy closed.
2) Fuzzy semi- $T_{1/2}$-space if every $F_{pg}$-closed set is fuzzy semi-closed.
3) Fuzzy pre- $T_{1/2}$-space if every $F_{pgs}$-closed set is fuzzy pre closed.

FUZZY SEMI – CLOSED SETS

2.1 Definition : Let $\eta$ be a fuzzy set in a fuzzy topological spaces $(X, \tau)$. Then $\nu$ is called a fuzzy semi-closed set $X$ if $\text{spCl}(\eta) \leq \nu$, whenever $\eta \leq \nu$ and $\nu$ is a $F_{g}$-open set in $X$.

2.2 Proposition : Every fuzzy semi pre-closed set in a fuzzy topological space $(X, \tau)$ is fuzzy open set.

Proof : Let $\eta$ be a fuzzy semi pre-closed set in a fuzzy topological space $(X, \tau)$. Suppose that $\eta \leq \nu$ and $\nu$ is a fuzzy generalized-open set in $X$. Since $\text{spCl}(\eta) = \eta$, it follows that $\text{spCl}(\eta) = \eta \leq \nu$. Hence, $\eta$ is fuzzy semi-closed in $X$. The reverse implication in the above proposition is not true as seen in the following example.

Example : Consider the fuzzy topological space $(X, \tau)$, where $X = \{a,b,c\}$ and
$$\tau = \{0,1,\eta = \frac{0.9}{a} + \frac{0.2}{b} + \frac{1}{c}, \nu = \frac{0.9}{a} + \frac{0.2}{b} + \frac{0}{c}\}.$$ 
Fuzzy closed sets in $X$ are...
So the family of fuzzy generalized-closed sets is
\[ \{0,1, \eta, v, \alpha_1 + \alpha_2 + \alpha_3 \text{ either } \alpha_1 > 0.8 \text{ or } \alpha_2 > 0.2 \} \]
Hence the family of generalized open sets is
\[ \{0,1, \eta, v, \alpha_1 + \alpha_2 + \alpha_3 \text{ either } \alpha_1 < 0.2 \text{ or } \alpha_2 < 0.8 \} \]
Now
\[ v = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \]
is not a fuzzy semi-pre closed set in X, for \( \text{Int } \lambda = v \) and so, \( \text{Int Cl Int } \lambda = \text{Int Cl } v = 1 > \lambda \).
Moreover, \( \lambda \) is fuzzy pre semi-closed. Indeed, let \( \lambda \leq \mu \) and \( \mu \) be fuzzy generalized open in X. Then \( \mu = 1 \) and \( \text{spCl } \lambda \leq \mu \).
From the succeeding two examples, it can be seen that fuzzy pre-semi-closedness is independent from fuzzy generalized semi-closedness.

**Example:**
Consider the fuzzy topological space \((X, \tau)\), where \(X = \{a, b, c\}\) and \(\tau = \{0,1,v = \frac{0.2}{a} + \frac{0.3}{b} + \frac{0}{c}\}\).

Fuzzy closed sets are:
\[ 0,1,v = \frac{0.2}{a} + \frac{0.9}{b} + \frac{1}{c} \]
So the family of fuzzy generalized closed sets in X is
\[ \{0,1, \eta, v, \alpha_1 + \alpha_2 + \alpha_3 \text{ either } 0.7 < \alpha_1 \text{ or } 0.3 < \alpha_2 \} \]
Hence the family of fuzzy generalized-open sets is
\[ \{0,1, \eta, v, \alpha_1 + \alpha_2 + \alpha_3 \text{ either } \alpha_1 < 0.2 \text{ or } \alpha_2 < 0.9 \} \]
Now,
\[ \lambda = \frac{0.7}{a} + \frac{0.2}{b} + \frac{0}{c} \]
is a fuzzy semi-pre-closed set in X, for if \( \lambda \leq \mu \) and \( \mu \) is fuzzy generalized open set in X. Then \( \mu = 1 \). Hence, \( \text{spCl } \lambda \leq \mu \). But, \( \text{Int Cl } \lambda = 1, \lambda \leq v \) and \( \text{Int Cl } \lambda = 1 > v \), so \( \lambda \) is not a fuzzy generalized closed set in X.

**Example:**
Consider the fuzzy topological space \((X, \tau)\) : \(X = \{a, b, c\}\) and \(\tau = \{0,1,v = \frac{0}{a} + \frac{1}{b} + \frac{1.2}{c}\}\).

Fuzzy closed sets are:
\[ 0,1,v = \frac{1}{a} + \frac{0}{b} + \frac{0}{c} \]
So the family of fuzzy generalized closed sets in X is
\[ \{0,1, \alpha_1 + \alpha_2 + \alpha_3 \text{ either } 1 > \alpha_1 \text{ or } 0 > \alpha_2 \} \]
Hence the family of fuzzy generalized-open sets in X is
\[ \{0,1, \eta, v, \alpha_1 + \alpha_2 + \alpha_3 \text{ either } \alpha_1 < 1 \text{ or } \alpha_2 < 1 \} \]
Now
\[ \lambda = \frac{1}{a} + \frac{1}{b} + \frac{0}{c} \]
is not a fuzzy semi-pre-closed set in X; if \( \lambda \leq v \) and \( \lambda \) is fuzzy generalized open set, but, \( \text{Int Cl } \lambda = 1, \text{ and hence, } \text{spCl } \lambda = 1 > \lambda \). But, \( \lambda \) is fuzzy generalized closed set in X and hence it is fuzzy generalized semi-closed.

### 2.3 Proposition:
Let \( \eta \) be a fuzzy set in a fuzzy topological spaces \((X, \tau)\). Then the following are equivalent:

i. \( \eta \) is fuzzy open and fuzzy pre semi-closed.

ii. \( \eta \) is fuzzy open and \( F_{\text{pgs}} \)-closed.

**Proof:**
Let \( \eta \) be fuzzy open and fuzzy pre semi-closed. Then, by known proposition,

"Every fuzzy pre semi-closed set in a fuzzy topological space \((X, \tau)\) is \( F_{\text{pgs}} \)-closed."

**Proof:**
Let \( \lambda \) be a fuzzy pre semi-closed set in a fuzzy topological space \((X, \tau)\). Suppose that \( \lambda \leq v \) and \( \nu \) is a fuzzy open set in X. Then, \( \text{psCl } \lambda \leq v \) and \( \nu \) is \( F_\nu \)-open in X and hence \( \lambda \) is fuzzy pre generalized semi-closed in X.

Hence, it is fuzzy pre generalized semi-closed.

### 2.3 Proposition:
Let \( \eta \) be a fuzzy pre-semi-closed set in a fuzzy topological space \((X, \tau)\). If \( \eta \) is a fuzzy set in X such that \( \eta \leq \nu \leq \text{spCl } \nu \), then \( \nu \) is also fuzzy pre-semi-closed.

**Proof:**
Let \( \nu \leq \mu \) and \( \mu \) be fuzzy generalized-open in X. Then \( \eta \leq \lambda \) and since \( \mu \) is fuzzy pre semi-closed. It is clear that, \( \text{psCl } \nu \leq \text{sp Cl } (\text{sp Cl } \eta) = \text{sp Cl } \nu \leq \nu \). Hence, \( \nu \) is fuzzy pre-semi-closed.
FUZZY PRE GENERALIZED SEMI- CLOSED SETS

3.1 Definition :
A fuzzy set $A$ of $(X, \tau)$ is called fuzzy pre generalized semi- closed (shortly, $F_{pgs}$-closed) if $psCl(v) \leq \eta$, whenever $v \leq \eta$ and $\eta$ is $F_{\tau}$- open in $X$.
By fuzzy pre generalized semi- closed $(X, \tau)$, we denote the family of all fuzzy pre generalized semi- closed sets of fuzzy topological space $X$.

Example :
Let $X = \{a, b\}$ and $Y = \{x, y, z\}$ and fuzzy sets $A, B, E, H, K$ and $M$ be defined by :

$A(a) = 0.3$, $A(b) = 0.4$, $B(a) = 0.4$, $B(b) = 0.5$ :

$E(a) = 0.3$, $E(b) = 0.7$, $H(a) = 0.7$, $H(b) = 0.6$ :

$K(x) = 0.1$, $K(y) = 0.2$, $K(z) = 0.7$ :

$M(x) = 0.9$, $M(y) = 0.2$, $M(z) = 0.5$.

Let $\tau = \{0, A, 1\}$, $\sigma = \{0, E, 1\}$ and $\gamma = \{0, K, 1\}$. Then $B$ is $F_{pgs}$- closed in $(X, \tau)$ but not $F_{gs}$- closed ; $M$ is $F_{pgs}$- closed in $(Y, \gamma)$ but not $F_{gs}$- closed because, If we consider a fuzzy set $T(x) = 0.9$, $T(y) = 0.2$, $T(z) = 0.7$, then clearly $sCl(M) \not\subseteq T$ where as $M \leq T$ and $T$ is fuzzy semi open in $(Y, \gamma)$ and $H$ is $F_{pgs}$- closed in $(X, \sigma)$ but neither $F_{pgs}$-closed because, If we consider a fuzzy set $L(a) = 0.8$, $L(b) = 0.7$, then clearly $sCl(H) \not\subseteq L$ where as $H \leq L$ and $L$ is fuzzy semiopen in $(X, \sigma)$ nor $F_{sp}$- closed because $Int(\{Cl(Int(H))\}) \not\subseteq H$.

3.2 Theorem :
If $A$ is fuzzy semi open and $F_{pgs}$- closed in $(X, \tau)$, then $A$ is a $F_{ps}$- closed in $(X, \tau)$.

Proof :
Since $A \leq A$ and $A$ is fuzzy semi open and $F_{pgs}$- closed, then $psCl(A) \leq A$. Since $A \leq psCl(A)$, we have $A = psCl(A)$ and thus $A$ is a $F_{ps}$- closed set in $X$.

3.3 Theorem :
If $A$ is a $F_{pgs}$- closed set of $(X, \tau)$ and $A \leq B \leq psCl(A)$, then $B$ is a $F_{pgs}$- closed set of $(X, \tau)$.

Proof :
Let $B$ be a $F_{\tau}$- open set of $(X, \tau)$ such that $B \leq H$. Then $A \leq H$. Since $A$ is $F_{pgs}$- closed, it follows that $spCl(A) \leq H$. Now, $B \leq psCl(A)$ implies $psCl(B) \leq psCl(psCl(A)) = psCl(A)$. Thus, $psCl(B) \leq H$. This proves that $B$ is also a $F_{pgs}$- closed set of $(X, \tau)$.

3.4 Definition :
A fuzzy set $A$ of $(X, \tau)$ is called fuzzy pre generalized semi-open (shortly, $F_{pgs}$- open) iff $(1 - A)$ is $F_{pgs}$- closed set in $X$. That is , $A$ is $F_{pgs}$- open iff $E \leq sp Int(A)$ whenever $E \leq A$ and $E$ is a $F_{\tau}$- closed set in $X$.

3.5 Theorem :
Fuzzy pre semi- open $(X, \tau)$ $\leq$ fuzzy pre generalized semi- open $(X, \tau)$.

Proof :
Let $A$ any fuzzy pre semi- open set in $X$. Then, $1 - A$ is fuzzy pre semi- closed and hence, fuzzy pre generalized semi- closed. This implies that $A$ is fuzzy pre generalized semi- open. Hence, $B$ is fuzzy pre semi- open $(X, \tau)$ $\leq$ fuzzy pre generalized semi open $(X, \tau)$.

3.6 Theorem :
Let $A$ be fuzzy pre generalized semi- open in $X$ and $ps Int(A) \leq B \leq A$, then $B$ is fuzzy pre generalized semi- open.

Proof :
Suppose $A$ is $F_{pgs}$- open in $X$ and $ps Int(A) \leq B \leq A$. Then $1 - A$ is $F_{pgs}$- closed and $1 - A \leq 1 - B \leq psCl(1 - A)$. Then $1 - A$ is $F_{pgs}$- closed and $1 - A \leq 1 - B \leq psCl(1 - A)$. Then $1 - B$ is $F_{pgs}$- closed set. Hence, $B$ is $F_{pgs}$- open set in $X$.

FUZZY PRE GENERALIZED SEMI- IRRESOLUTE MAPPINGS :

4.1 Theorem :
Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be $F_{pgs}$- irresolute, then $f$ is $F_{pgs}$- continuous.

Proof :
Proof is immediate as every fuzzy closed set is $F_{pgs}$- closed and $f$ is $F_{pgs}$- irresolute map.

4.2 Theorem :
Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be fuzzy irresolute and fuzzy semi- closed. Then for every $F_{pgs}$- closed set $A$ of $X$, $f(A)$ is a $F_{pgs}$- closed in $Y$.

Proof :
Let $A$ be a $F_{pgs}$- closed set of $X$. Let $B$ be a fuzzy semi open set of $Y$ containing $f(A)$. Since $f$ is fuzzy irresolute, $f^{-1}(V)$ is a fuzzy semi open set of $X$. As $A \leq f^{-1}(V)$ and $A$ is a $F_{pgs}$- closed in $X$, then $psCl(A) \leq f^{-1}(V)$ implies that $f(psCl(A)) \leq V$. Since $f$ is fuzzy semi pre-closed, then $f(psCl(A)) = psCl\left( f(psCl(A)) \right)$.

Then, $psCl(f(A)) \leq psCl\left( f(psCl(A)) \right) = f(psCl(A)) \leq V$.

Therefore, $f(A)$ is a $F_{pgs}$- closed set in $Y$.
FUZZY PRE- SEMI- $T_{1/2}$ SPACES :

5.1 Proposition :
Every fuzzy pre semi- $T_{1/2}$ space is a fuzzy pre semi- $T_{1/3}$ space.

Proof :
Let $(X, \tau)$ be a fuzzy pre semi- $T_{1/2}$ space and $\mu$ be a $F_{pgs^*}$ closed set in $(X, \tau)$. Then $\mu$ is fuzzy pre semi- closed. Then, $(X, \tau)$ is a fuzzy semi- $T_{1/3}$ space.

5.2 Proposition :
Every fuzzy pre semi- $T_{3/4}$ space is a fuzzy pre semi- $T_{1/2}$ space.

Proof :
Let $(X, \tau)$ be a fuzzy pre semi- $T_{3/4}$ space and $\nu$ be a fuzzy pre semi- closed set in $(X, \tau)$. Then, $\nu$ is fuzzy pre closed. Then, $(X, \tau)$ is a fuzzy pre semi- $T_{1/2}$ space. Converse of the above proposition is not true as seen in the following example.

Example :
Let $(X, \tau)$ be a fuzzy topological space, where $\tau = \{0, 1, \nu_1 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}, \nu_2 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}, \nu_3 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\}$. Fuzzy closed sets in $(X, \tau)$ are

$$0, 1, \nu' = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}, \nu' = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}, \nu' = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$ 

If $\eta$ is fuzzy generalized-closed then $\eta \leq \nu$ implies $Cl \eta \leq \nu$ whenever $\nu$ is fuzzy open. Thus, fuzzy generalized-closed sets in $(X, \tau)$ are :

$$0, 1, \nu' = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}, \nu' = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}, \nu' = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

Where $\alpha_3 \neq 0$. So, the family of fuzzy generalized-open sets in $(X, \tau)$ is

$$\{0, 1, \alpha_1 + \alpha_2 + \alpha_3 \text{ where } \alpha_i \neq 1\}.$$

It is enough to prove that, if $\eta$ is not fuzzy semi-pre-closed then it is not fuzzy semi-closed and there is a fuzzy pre-semi-closed set which is not pre-closed. Let $\eta \neq 0$ be a fuzzy set in $X$. Then,

$$Int \eta = \begin{cases} v_1, & v_2, \text{ Cl Int } \eta = \begin{cases} v'_2, & v'_3, \text{ and Int Cl Int } \eta = \begin{cases} v_4, & v_2, \text{ 1}, & 1, \text{ and } \frac{\alpha_1 + \alpha_2 + \alpha_3}{a} \end{cases} \end{cases}$$

So, $\eta$ is not fuzzy semi-pre-closed if $v_2 \leq \eta$. In that case, $\eta$ is also not fuzzy semi-pre-closed. For $\eta$ is fuzzy generalized-open and $\eta \leq \eta$. But $sp Cl \eta \geq \eta \forall Int Cl Int \nu = 1 > \eta$. Thus $X$ is a pre-semi-$T_{1/2}$ space.

$$\nu = 0 + \frac{0}{a} + \frac{0}{b} + \frac{0}{c}$$

Is a fuzzy semi-pre-closed. Hence it is fuzzy pre-semi-closed but it is not fuzzy pre-closed, as $Cl Int \nu \neq \nu$.

CONCLUSION :
In this paper, by the introduction of fuzzy pre semi- closed sets, we have equivalences of fuzzy spaces namely, fuzzy pre semi- $T_{1/3}$ space, pre semi- $T_{3/4}$ space and fuzzy pre semi- $T_{1/2}$ space.

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