NEW TYPE OF SEMI REGULAR WEAKLY OPEN SETS IN TOPOLOGICAL SPACES

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***Abstract :** This paper considers a new class of sets called semi regular weakly open sets (briefly srw-open) are introduced and studied in topological spaces. A subset G of topological spaces X is said to be semi regular weakly open set, if $\gamma \subseteq sin\,t(\lambda)$, whenever $\gamma \subseteq \lambda$ and γ is rw- closed set in X. The new class strictly lies between semi open sets, αrw-open set and gs-open sets in topological spaces. Also, as applications, using some properties of srw-open sets and srw-closed sets and their properties respectively.*

***Keywords :** srw-Closed sets, srw-open sets, srw-neighborhoods.*

I. INTRODUTION

Levine [7] introduced generalized open sets, regular open sets in topological spaces respectively, then regular weakly open sets, generalized semi closed sets, generalized α-closed sets and α-generalized closed sets semi open sets, α-regular w-closed sets, pgrw- closed sets and semi-regular weakly closed sets have been introduced and studied by Benchalli.S.S and Wali.R.S[2],Arya S.P. and Nour T.M.[1], Maki [7] and Levine [7] respectively

We introduce and study the semi-regular weakly open sets, semi-regular weakly neighborhood and operators in topological spaces and obtain some of their properties.

II. PRELIMINARIES

Throughout this paper X and Y represent the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset λ of topological spaces X, cl(λ) and int(λ) denote the closure of λ and interior of λ respectively. Let X/λ denotes the complement of λ in X. Now, we recall the following definitions.

2.1 Definition

A subset λ of a topological space X is called

i. Regular open, if $\lambda = int(cl(\lambda))$ and regular closed if $cl\big(int(\lambda)\big) = \lambda$.
ii. Pre-open, if $\lambda \subseteq int(cl(\lambda))$ and pre-closed if $cl\big(int(\lambda)\big) \subseteq \lambda$.
iii. Semi open, if $\lambda \subseteq cl(int(\lambda))$ and semi-closed if $int\big(cl(\lambda)\big) \subseteq \lambda$.
iv. α-open, if $\lambda \subseteq int(cl\big(int(\lambda)\big))$ and α-closed if $int\left(int\big(cl(\lambda)\big)\right) \subseteq \lambda$.
v. Semi pre open, if $\lambda \subseteq cl(int\big(cl(\lambda)\big))$ and semi pre closed if $int\left(cl\big(int(\lambda)\big)\right) \subseteq \lambda$.
vi. π-open, if λ is a finite union of regular open sets.

2.2 Definition

A subset λ of a fuzzy topological space X is called

i. Generalized closed, if $cl(\lambda) \subseteq \mu$ whenever $\lambda \subseteq \mu$ and μ is open in X.
ii. Semi-generalized closed, if $scl(\lambda) \subseteq \mu$ whenever $\lambda \subseteq \mu$ and μ is semi open in X.
iii. Generalized semi- closed, if $scl(\lambda) \subseteq \mu$ whenever $\lambda \subseteq \mu$ and μ is open in X.
iv. α-generalized closed, if $\alpha cl(\lambda) \subseteq \mu$ whenever $\lambda \subseteq \mu$ and μ is open in X.
v. Generalized semi pre-closed, if $spcl(\lambda) \subseteq \mu$ whenever $\lambda \subseteq \mu$ and μ is open in X.
vi. Regular generalized closed, if $cl(\lambda) \subseteq \mu$ whenever $\lambda \subseteq \mu$ and μ is regular open in X.
vii. Weakly closed, if $cl(\lambda) \subseteq \mu$ whenever $\lambda \subseteq \mu$ and μ is semi open in X.
viii. Regular weakly closed, if $cl(\lambda) \subseteq \mu$ whenever $\lambda \subseteq \mu$ and μ is regular semi open in X.
ix. α-regular weakly closed, if $\alpha cl(\lambda) \subseteq \mu$ whenever $\lambda \subseteq \mu$ and μ is rw-open set in X.

The complements of all closed sets are their respective open sets in the same topological spaces X.

The semi-pre-closure (resp. semi-closure, resp. pre-closure, resp. α –closure) of a subset λ of X is the intersection of all semi-pre-closed (resp. Semi-closed, resp. pre-closed, resp. α- closed) sets containing A and is denoted by (spcl(λ) (resp. scl(λ), resp. pcl(λ), resp. cl(λ)).

2.3 Definition

A subset λ of a space X is said to be semi regular weakly closed set, if $scl(\lambda) \subseteq \mu$ whenever $\lambda \subseteq \mu$ and μ is rw-open set in X.

We denote the family of all srw -closed sets, srw -open sets, αrw -open sets, and semi-open sets of X by $SRW(X), SRWO(X), \alpha RWO(X)$ and $SO(X)$ respectively.

2.4 Lemma

i. For a subset λ of X, αrw-closure of λ is denoted by $\alpha rw-cl(\lambda)$ and defined as $\alpha rw-cl(\lambda)=\cap\,\gamma\subset X:\lambda\subset\gamma\in\alpha RWC(X)$.

ii. For a subset λ of X, semi-closure of λ is denoted by $scl(\lambda)$ and defined as $scl(\lambda)=\cap\,\gamma\subset X:\lambda\subset\gamma\in SC(X)$.

iii. For a subset λ of X, gs-closure of λ is denoted by $gs-cl(\lambda)$ and defined as $gs-cl(\lambda)=\cap\,\gamma\subset X:\lambda\subset\gamma\in GSC(X)$.

III. SEMI REGULAR WEAKLY OPEN SETS

In this section, we introduce and study srw-open sets in topological spaces and obtain some of their basic properties.

3.1 Definition

A subset λ of X is called semi regular weakly open set, if $X\setminus\lambda$ is srw-closed set in X. The family of all semi regular weakly open sets in X is denoted by $SRWO(X)$.

3.2 Theorem

If a subset λ of space X is αrw-open, then it is srw-open in X but not conversely.

Proof:

Let λ be a αrw-open set in a space X. Then $X\setminus\lambda$ is a αrw-closed set. By theorem 3.2 $X\setminus\lambda$ is srw-closed. There λ is a srw-open set in X.

The converse of the above theorem need not be true as shown in example 3.3

3.3 Example

Let $X=\{a,b,c,d\}$ with $\tau=\{X,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$. Then {a, d} and {b, c, d} are srw- open sets in X but it is not αrw-open sets in X.

3.4 Theorem

If a subset λ of space X is semi-open, then it is semi-open in X but converse is not true.

Proof:

Let λ be a semi-open set in a space X. Then $X\setminus\lambda$ is a semi-closed set. By Theorem 3.6 of, $X\setminus\lambda$ is srw-closed. Therefore λ is a srw-open set in X.

The converse of the above theorem need not be true as shown in example 3.5

3.5 Example

Let $X=\{a,b,c,d\}$ with $\tau=\{X,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$. then {b} and {c} are srw-open sets in X but not semi-open sets in X.

3.6 Corollary

It is evident that every open set is semi-open set but not conversely. By Theorem 3.4 every semi-open set is srw-open set in X but not conversely and hence every open set is srw-open set in X.

3.7 Corollary

It is evident that every α-open set is αrw-open set is srw-open set but not conversely and hence every α-open set is srw-open set but not conversely.

3.8 Corollary

It is evident that every regular open set is open, but not conversely. By corollary 3.7, every open set is srw-open set but conversely and hence every regular open set is srw-open set in X.

3.9 Corollary

It is evident that every θ-open set is open but not conversely. By corollary 3.7, every open set is srw-open set but not conversely and hence every θ-open set is srw-open set in X.

3.10 Theorem

If a subset λ of a space X is srw-open, then it is a gs-open set in X.

Proof:

Let λ be a srw-open set in X, then $X\setminus\lambda$ is a srw-closed set in X. by Theorem 3.4 of, every srw-closed set is gs-closed set in X. (i.e) $X\setminus\lambda$ is a gs-closed set in X. Therefore λ is a gs-open set in X.

The converse of the above theorem need not be true as shown in example 3.11.

3.11 Example

Let $X=\{a,b,c,d\}$ with $\tau=\{X,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$. Then {a, c} and {a, d} are gs-open set in X but not srw-open sets in X.

3.12 Theorem

If a subset λ of a space X is srw-open, then it is a gs-open set in X, but not conversely.

Proof:

Let λ be a srw-open set in X, then $X\setminus\lambda$ is a srw-closed set in X. By theorem 3.10 of, every srw-closed set is sp-closed set in X. (i.e) $X\setminus\lambda$ sp-closed set in X. Therefore λ is a sp-open set in X.

The converse of the above theorem need not be true as shown in example 3.13.

3.13 Example

Let $X=\{a,b,c,d\}$ with $\tau=\{X,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$. Then {a, b} and {c, d} are gsp-open sets in X but not srw-open sets in X.

The concepts of g-open, w-open, αg-open and $w\alpha$-open sets are independent with the concept of srw-open set as shown in the following example 3.14.

3.14 Example

Let $X=\{a,b,c,d\}$ with $\tau=\{X,\emptyset,\{a\},\{b,c\},\{a,b,c\}\}$. then {a, d} is a srw-open, however it can be verified that it is not g-open, w-open, g-open and w-open set. Also, the set {a, b} and {a, c} are g-open, w-open, αg-open and $w\alpha$-open set but not srw-open set in X.

3.15 Remark

Union and intersection of two srw-open sets need not be srw-open sets as shown in the following example 3.16

3.16 Example

Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then $SRWO(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$let A= {b}, B= {a, d} and C= {b, c, d}. Here A and B are srw-open sets but $A \cup B = \{a, b, d\}$ is not srw-open. Also B and C are srw-open sets but $B \cap C = \{d\}$ is not srw-open set in X.

3.17 Theorem

If $\lambda \subseteq X$ is srw-closed, then $scl(\lambda) \setminus \lambda$ is srw-open set in X.

Proof:

If $\lambda \subseteq X$ is srw-closed and let γ be a rw-closed set such that$\gamma \subseteq scl(\lambda) \setminus \lambda$. Then by Theorem 3.19 of, $\gamma = \emptyset$ that implies $\gamma \subseteq sint(scl(\lambda) \setminus \lambda)$ and Theorem 3.17 $scl(\lambda) \setminus \lambda$ is srw-open set in X.

3.18 Theorem

A subset λ of a topological space X is srw-open if and only if $\gamma \subseteq sint(\lambda)$ whenever γ is rw-closed and $\gamma \subseteq \lambda$.

Proof:

Let $\gamma \subseteq \lambda$ is srw-closed and let γ be a rw-closed set and $\gamma \subseteq \lambda$. Then $X \setminus \lambda \subseteq X \setminus \gamma$ where $X \setminus \gamma$ is rw-open. Since $X \setminus \lambda$ is srw-closed, $scl(X \setminus \lambda) \subseteq X \setminus \gamma$ and hence $X \setminus sint(\lambda) \subseteq X \setminus \gamma$ that implies $\gamma \subseteq sint(\lambda)$.

Conversely, suppose $\gamma \subseteq sint(\lambda)$ whenever $\gamma \subseteq \lambda, \gamma$ is rw-closed. To prove: λ is srw- open. Suppose $X \setminus U \subseteq \lambda$ where U is rw-open. Then $X \setminus U \subseteq \lambda$ where $X \setminus U$ is rw-closed. By assumption $X \setminus U \subseteq sint(\lambda)$ that implies $scl(X \setminus \lambda) \subseteq U$. This proves that $X \setminus \lambda$ is srw-closed and hence λ is srw-open set in X

3.19 Theorem

Every singleton point set in a space X is either srw-open or rw-open in X.

Proof:

Let $x \in X$ where X is a topological space. To prove: {x} is either srw-open or rw-open in X. (i.e) to prove that $X \setminus \{x\}$ is either srw-open or rw-open, which follows from Theorem 3.25 of, the next theorem shows that all the sets between $sint(\lambda)$ and λ are srw-open whenever λ is srw-open.

3.20 Theorem

If $sint(\lambda) \subseteq B \subseteq A$ and A is a srw-open set in X, Then B is srw-open set in X.

Proof:

Let $sint(A) \subseteq B \subseteq A$ and A is a srw-open set. Then $X \setminus A \subseteq X \setminus B \subseteq X \setminus sint(\lambda)$ that implies $X \setminus A \subseteq X \setminus B \subseteq sint(X \setminus \lambda)$, since $X \setminus \lambda$ is srw-closed set, by Theorem 3.23 of, $X \setminus B$ is srw-closed set. Therefore B is srw-open set in X.

3.21 Theorem

If $\lambda \subseteq X$ is a srw-closed, then $scl(\lambda) \setminus \lambda$ is srw-open set in X.

Proof:

Let $\lambda \subseteq X$ is a srw-closed set and γ be a rw-closed set such that $\gamma \subseteq sint(\lambda) \setminus \lambda$. By Theorem 3.19 of,$\gamma = \emptyset$, so $\gamma \subseteq sint(scl(\lambda) \setminus \lambda)$ by Theorem 3.18 $scl(\lambda) \setminus \lambda$ is srw-open set in X.

The converse of above theorem does not hold shown by example 3.22

3.22 Example

Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then λ ={c, d} then $scl(\lambda) = \{b, c, d\}$ and $scl(\lambda) \setminus \lambda = \{b\}$ is an srw-open set, but λ is not an srw-closed set in X.

3.23 Theorem

If a subset λ is srw-open in X and if G is rw-open in X with $sint(\lambda) \cup (X \setminus \lambda) \subseteq G$ then G=X.

Proof:

Suppose that G is an rw-open set and $sint(\lambda) \cup (X \backslash \lambda) \leq G$. Now $(X \backslash \lambda) \subseteq X \backslash scl(\lambda) \cap X \backslash (X \backslash \lambda)$ implies that $(X \backslash G) \subseteq scl(X \backslash \lambda) \cap \lambda$. suppose λ is srw-open. Since $X \backslash G$ is rw-closed and $X \backslash \lambda$ is rw-closed, then by Theorem 3.19 of, $X \backslash G = \emptyset$ and hence G=X.

The converse of the above theorem need not be true in general as shown in example 3.24.

3.24 Example

Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then$SRWO(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$, and $RWO(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{b, d\}, \{a, c\}, \{a, b, c\}\}$. let A={a, b, d}{a, b, d} is not an srw-open set in X. However $sint(\lambda) \vee (X \setminus \lambda) = \{a, d\} \vee \{c\} = \{a, c, d\}$. So for some rw-open set G, such that $sint(\lambda) \vee (X \backslash \lambda) = \{a, c, d\} < G$ gives G=X but λ is not srw-open set in X.

3.25 Theorem

Let X be a topological space and A, $B \subseteq X$. If B is srw-open and $sint(B) \subseteq A$. then $A \cap B$ is srw-open in X.

Proof:

Since B is srw-open and $sint(B) \subseteq A$. then $sint(B) \subseteq A \wedge B \subseteq B$, then by theorem 3.20 of, $A \cap B$ is srw-open set in X.

IV. SEMI REGULAR WEAKLY NEIGHBORHOODS

4.1 Definition

Let (X, τ) be a topological space and let$x \in X$. A subset N is said to be srw-nbd of x, if and only if there exists a srw-open set G such that $x \in G \subseteq N$.

4.2 Definition

i. A subset N of X is a srw-nbd of $\lambda \subseteq X$ in topological space (X, τ), if there exists an srw-open set G such that $\lambda \subseteq G \subseteq N$.

ii. The collection of all srw-nbd of $x \in X$ is called srw-nbd system at $x \in X$ and shall be denoted by srw-N(x).

4.3 Theorem

Every neighborhood N of $x \in X$ is a srw-nbd of x.

Proof:

Let N be neighborhood of point$x \in X$. To prove that N is a srw-nbd of x. by definition of neighborhood, there exists an open set G such that $x \in G \subseteq N$. Hence N is srw-nbd of x.

4.4 Remark

In general, a srw-nbd N of x in X, as shown from example 4.5

4.5 Example

Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. then $SRWO(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. the set {a, b, d} is srw-nbd of the point b, since the srw-open set {b} is such that $b \in \{b\} \subset \{a, b, d\}$. However, the set {a, b, d} is not a neighborhood of the point b, since no open set G exists such that $b \in \{b\} \subset \{a, b, d\}$.

4.6 Theorem

If a subset N of a space X is srw-open and then N is srw-nbd of each of its points.

Proof:

Suppose N is srw-fuzzy open. Let $x \in N$ we claim that N is a srw-nbd of x. For N is a srw-open set such that $b \in N \subset N$. since x is an arbitrary point of N, it follows that N is a srw-nbd of each of its points.

The converse of the above theorem is not true in general as seen from the following example 4.7

4.7 Example

Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. then $SRWO(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$.

The set {a, c} is srw-nbd of the point a, since the srw-open set {a} is such that$a \in \{a\} < \{a, c\}$. Also the set {a, c} is a srw-nbd of the point c, since the srw-open set {c} is such that $c \in \{c\} < \{a, c\}$ (i.e). {a, c} is a srw-nbd of each of its points. However the set {a, c} is not a srw-open set in X.

4.8 Theorem

Let X be a topological spaces. If F is a srw-closed set subset of X and $x \in (X \backslash \lambda)$, then there exists a srw-nbd N of x such that $N \cap \gamma = \emptyset$.

Proof:

Let γ be a srw-closed subset of X and $x \in (X \backslash \gamma)$. then $(X \backslash \gamma)$ is a srw-open set of X. By theorem 4.6, $(X \setminus \gamma)$ contains a srw-nbd of each of its points. Hence there exists a srw-nbd N of x such that $N \cap \gamma = \emptyset$.

4.9 Theorem

Let X is a topological space and for each$x \in X$, let srw-N(x) is the collection of all srw-nbds of x, then we have the following results.

i. $\forall x \in X, srw - N(x) \neq \emptyset$.

ii. $X \in srw - N(x) \Rightarrow x \in N$.

iii. $N \in srw - N(x)$ And $N \subset M \Rightarrow M \in srw - N(x)$.

iv. $N \in srw - N(x) \Rightarrow \exists M \in srw - N(y)$ for every $y \in M$.

Proof:

i. Since X is an srw-open set, it is a srw-nhd of every $x \in X$. Hence there exists at least one srw-nbd(X) for each $x \in X$. Hence $srw - N(x) \neq \emptyset$ for every $x \in X$.

ii. If $N \in srw - N(x)$, then N is a srw-nhd of x. So, by definition of srw- nhd $x \in X$.

iii. let $N \in srw - N(x)$ and $N \subset M$,then there is a srw –open set in G such that $x \in G \subset N$. Since$N \subset M$, $x \in G \subset M$ and so M is a srw – nbd of x. Hence$M \in srw - N(x)$.

iv. If $N \in srw - N(x)$, then there exists an srw –open set M is an srw –open set, it is a srw – nhd of each of its points. Therefore $N \in srw - N(y)$ for$y \in M$.

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