MODULAR STABILITY OF THE GENERALIZED QUADRATIC FUNCTIONAL EQUATION

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***Abstract :** In this paper, authors obtain general solution and investigate the Hyers-Ulam, generalized Hyers-Ulam, generalized Hyers-Ulam-Rassias stabilities of generalized quadratic functional equation in modular space using fixed point theory.*

***IndexTerms** - Modular space, quadratic functional equation, generalized Hyers- Ulam- Rassias stability.*
***2010 Mathematics Subject Classification:** 39B52, 39B72, 39B82, 46B03.*

I. INTRODUCTION

In 2008, Wanchitra Towanlong and Paisan Nakmahachalasint [9], investigated the generalized Hyers-Ulam Rassias stability of a quadratic functional equation

$$g(3u + v) + g(3u - v) = g(u + v) + g(u - v) + 16g(u).$$

In 2017, Hark-Mahn Kim and Young Soon Hong [5], investigated an alternative generalized Hyers-Ulam stability theorem of a modified quadratic functional equation in a modular space X_ρ using Δ_3-condition without the Fatou property on the modular function ρ.

To know about Ulam problem and Hyers-Ulam, generalized Hyers-Ulam, gener- alized Hyers-Ulam-Rassias stabilities, one can refer [1], [2], [3], [6], [7], [8] and [10]. The definitions related to our main theorem can be referred in [4].

In this paper, we obtain the general solution and investigate the Hyers-Ulam, generalized Hyers-Ulam and generalized Hyers-Ulam-Rassias stabilities of the new generalized quadratic functional equation

$$z(px + y) + z(px - y) + z(x + py) + z(x - py) = z(x + y) + z(x - y) + 2p^2\{z(x) + z(y)\} \tag{1.1}$$

for $p \neq 0, \pm 1$ in modular space by using fixed point theory. The paper organized as follows:

We find general solution of (1.1) in Section-2. In Section-3, we investigate Hyers- Ulam, generalized Hyers-Ulam and generalized Hyers-Ulam-Rassias stabilities of functional equation (1.1) in modular space by using fixed point theory and given the conclusion in section-4.

II. GENERAL SOLUTION OF (1.1)

Theorem 2.1. If a function $z : X \to Y$ is a solution of the functional equation (1.1), then z is quadratic and even.

Proof. Assume z satisfies the functional equation (1.1). Letting (x, y) by (0, 0) in (1.1), we get z(0) = 0. Setting y = 0 in (1.1), we obtain

$$z(px) = p^2 z(x), \tag{2.1}$$

for all $x \in X$. Thus z is quadratic. Let $x = 0$ in (1.1) and by (2.1), we get $z(-y) = z(y)$ for all $y \in X$. Thus z is an even function.

III. STABILITY OF GENERALIZED QUADRATIC FUNCTIONAL EQUATION

Assume that π is a convex modular on $\pi-$ complete modular space X_π with the Fatou property such that satisfies the Δ_p-condition with $0 < v \leq p$. Also, let U be a linear space. We use the following abbreviation for a given function $z: U \to X_\pi$:

$$G_p z(x, y) := z(px + y) + z(px - y) + z(x + py) + z(x - py) - \{z(x + y) + z(x - y)\} - 2p^2\{z(x) + z(y)\}$$ for all $x, y \in U$ with $p \neq 0, \pm 1$.

Theorem 3.1. Let $a: U^2 \to [0, +\infty)$ be a function such that

$$\lim_{n\to\infty} \frac{1}{p^{2n}} a\{p^n x, p^n y\} = 0, \tag{3.1}$$

and

$$a\{px, py\} \leq p^2 s a\{x, y\} \tag{3.2}$$

for all $x, y \in U$ with s<1. Suppose that $z: U \to X_\pi$ satisfies the condition

$$\pi (G_p z(x, y)) \leq a(x, y), \tag{3.3}$$

for all x, y ∈ U and z(0)=0. Then there exists a unique quadratic mapping Q_p: U → X_π such that

$$\pi(Q_p(x) - z(x)) \leq \frac{1}{p^2(1-s)} a(x, 0), \tag{3.4}$$

for all x, y ∈ U.

Corollary 3.2. Let X be a Banach space, a: U^2 → [0, +∞) be a function such that

$$\lim_{n\to\infty} \frac{1}{p^{2n}} a\{p^n x, p^n y\} = 0, \tag{3.5}$$

and

$$a\{px, py\} \leq p^2 s a\{x, y\} \tag{3.6}$$

for all x, y ∈ U with s<1. Suppose that z: U → X satisfies the condition

$$\|G_p z(x, y)\| \leq a(x, y), \tag{3.7}$$

for all x, y ∈ U and z(0)=0. Then there exists a unique quadratic mapping Q_p: U → X such that

$$\|Q_p(x) - z(x)\| \leq \frac{1}{p^2(1-s)} a(x, 0), \tag{3.8}$$

for all x ∈ U.

Theorem 3.3. Let a: U^2 → [0, +∞) be a function such that

$$\lim_{n\to\infty} v^{2n} a\left(\frac{x}{p^n}, \frac{y}{p^n}\right) = 0, \tag{3.9}$$

and

$$a\left(\frac{x}{p}, \frac{y}{p}\right) \leq \frac{s}{p^2} a\{x, y\} \tag{3.10}$$

for all x, y ∈ U with s<1. Suppose that z: U → X satisfies the condition

$$\pi (G_p z(x, y)) \leq a(x, y), \tag{3.11}$$

for all x, y ∈ U and z(0)=0. Then there exists a unique quadratic mapping Q_p: U → X_π such that

$$\pi(Q_p(x) - z(x)) \leq \frac{s}{p^2(1-s)} a(x, 0), \tag{3.12}$$

for all x ∈ U.

Corollary 3.4. Let a: U^2 → [0, +∞) be a function such that

$$\lim_{n\to\infty} p^{2n} a\left(\frac{x}{p^n}, \frac{y}{p^n}\right) = 0, \tag{3.13}$$

and

$$a\left(\frac{x}{p}, \frac{y}{p}\right) \leq \frac{s}{p^2} a\{x, y\} \tag{3.14}$$

for all x, y ∈ U with s<1. Suppose that z: U → X satisfies the condition

$$\|G_p z(x, y)\| \leq a(x, y), \tag{3.15}$$

for all x, y ∈ U and z(0)=0. Then there exists a unique quadratic mapping Q_p: U → X such that

$$\|Q_p(x) - z(x)\| \leq \frac{s}{p^2(1-s)} a(x, 0), \tag{3.16}$$

for all x ∈ U.

The following corollaries are the immediate consequence of Corollary 3.2 and Corollary 3.4 which gives the Hyers-Ulam and generalized Hyers-Ulam stabilities of the functional equation (1.1).

Corollary 3.5. Let X be a Banach space, a: U^2 → [0, +∞) be a function such that

$$\lim_{n\to\infty} \frac{1}{p^{2n}} a\{p^n x, p^n y\} = 0, \tag{3.17}$$

and

$$a\{px, py\} \leq p^2 s a\{x, y\} \tag{3.18}$$

for all x, y ∈ U with s<1. Suppose that z: U → X satisfies the condition

$$\|D_p z(x,y)\| \leq \Theta, \tag{3.19}$$

for all x, y ∈ U and z(0)=0. Then there exists a unique quadratic mapping Q_p: U → X defined by

$Q_p(x) = \lim_{n\to\infty} \frac{z(p^n x)}{p^{2n}}$

such that

$$\|Q_p(x) - z(x)\| \leq \frac{\Theta}{p^2 - 1}, \tag{3.20}$$

for all x∈ U with p ≠ 0, ±1.

Corollary 3.6. Let U and X be a linear space and a Banach space, respectively. Suppose that z: U → X satisfies the inequality

$$\|G_p z(x,y)\| \leq \Theta\left(\|x\|^t + \|y\|^t\right), \tag{3.21}$$

for all x, y ∈ U and z(0)=0 with 0 ≤ t < 2 or t > 2. Then there exists a unique quadratic mapping Q_p: U → X defined by

$Q_p(x) = \lim_{n\to\infty} \frac{z(p^n x)}{p^{2n}}$

such that

$$\|Q_p(x) - z(x)\| \leq \frac{\varepsilon}{|p^2 - p^t|}\|x\|^t, \ \forall x \in U, \ p \neq 0, \pm 1. \tag{3.22}$$

for all x∈ U with p ≠ 0, ±1.

IV. Conclusion

In this paper, we introduced a new generalized quadratic functional equation and obtained the general solution and stabilities in modular space by using fixed point theory.

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