# LIGHT-FRONT HIGHER-SPIN THEORIES IN FLAT SPACE

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***ABSTRACT: This concept revisit the problem of interactions of higher-spin fields in flat space. This presentation argue that all no-go theorems can be avoided by the light-cone approach, which results in more interaction vertices as compared to the usual covariant approaches. It is stressed that there exist two-derivative gravitational couplings of higher-spin fields. It show that some reincarnation of the equivalence principle still holds for higher-spin fields—the strength of gravitational interaction does not depend on spin. Moreover, it follows from the results by Metsaev that there exists a complete chiral higher-spin theory in four dimensions. Finally it gives a simple derivation of this theory and show that the four-point scattering amplitude vanishes. Also, reconstruct the quartic vertex of the scalar field in the unitary higher-spin theory, which turns out to be perturbatively local.***

## INTRODUCTION

Since the early days of quantum field theory there have been many no-go results that prevent non-trivial interacting theories with massless higher-spin fields to exist. Notable examples are the Weinberg low energy theorem [1] and the Coleman–Mandula theorem [2]. One possible way out is to switch on the cosmological constant [3–5], which simultaneously avoids the no-go theorems that are formulated for QFT in flat space. Higher-spin theories in anti-de Sitter space later received a solid ground on the base of AdS/CFT correspondence [6–8] where higher-spin theories are supposed to be generic duals of free CFT's [9–12] with certain interacting ones accessible via an alternate choice [13] of boundary conditions [9, 11, 12, 14, 15]. The fate of higher-spin theories in flat space is still unclear and is a source of controversy. The no-go theorems are still true. Also, within the local field theory approach one immediately faces certain obstructions: Aragone–Deser argument forbids minimal gravitational interactions of massless higher-spin fields [16, 17] and, even if relaxing this assumption, it is still impossible to deform the gauge algebra [18, 19]. These results are based on the gauge invariant and manifestly Lorentz covariant field description in terms of Fronsdal fields [20], which suggests another possible way out. Indeed, gauge symmetry can be thought of as just a redundancy of description, though it turns out to be exceptionally useful in many cases. Therefore, in order to look for higher-spin theories in flat space it can be useful to turn to methods that deal with physical degrees of freedom only and thereby avoid any problems that originate from specific field descriptions. One such method is the light-cone approach, which still allows one to have a local field theory.

It is in the light-cone approach that the first examples of non-trivial cubic interactions between higher-spin fields were found in [21–23]. The covariant results followed soon after [24, 25]. A detailed classification of cubic vertices within the light cone approach is now available in all dimensions for massive and massless fields of arbitrary spin and symmetry type [26–29].

In this paper we revisit the problem of constructing higher-spin theories in flat space, specifically in four-dimensions. First of all, we argue that at least formally the most powerful no-go theorems are avoided by the light-cone approach. Also, we recall that there is a mismatch between the covariant cubic vertices and those found in [21–23] by the light-cone methods: there exist exceptional vertices not seen by some of the covariant methods. In particular, there does exist a two-derivative gravitational vertex for a field of any spin [3–2], which is also evident in the language of amplitudes .Having the gravitational higher-spin vertex at our disposal we prove that fields of any spin couple to gravity universally, i.e. some form of the equivalence principle is still true for higher-spin fields. In fact, the strength of the gravitational coupling does not depend on spin at all. A remarkable result obtained by Metsaev in [3] is that one can fix the cubic vertex without having to perform the full quartic analysis. We present a simple derivation of this result, which clarifies the assumptions. Based on this solution, we note that there exists a consistent non-trivial higher-spin theory in flat space. This theory contains graviton, massless higher-spin fields, the two-derivative gravitational vertices as well as other vertices. The action terminates at cubic vertices. Like in the self-dual Yang–Mills theory the four-point scattering amplitude vanishes. The only feature is that it breaks parity and is non-unitary. Nevertheless, it provides a counterexample to a widespread belief that higher-spin theories in flat space do not exist at all. Aiming at the unitary and parity preserving higher-spin theory in flat space we reconstruct the part of the quartic Hamiltonian that contains self-interactions of the scalar field, which can be regarded as the flat space counterpart of the $AdS_4$ result.

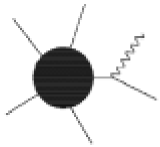
## 2. AVOIDING NO–GO THEOREMS

In the distant past it was a common belief that higher-spin theories, i.e. the theories with massless fields with spin greater than two, are not consistent. The most notable examples of such no-go theorems are Weinberg low energy theorem [1], Coleman–Mandula theorem [2] and the Aragone–Deser argument [16]. We briefly discuss them below, see also a very nice review [9], as to point out how all of them can be avoided.

Our conclusion is that there are still good chances to have nontrivial higher-spin theories in flat space. Moreover, we will present an example of consistent chiral theory in section 4. However, it should be stressed that while higher-spin theories may avoid the assumptions of the no-go theorems they may not defy the spirit of these theorems: there are strong indications that S-matrix should be trivial in some sense. For example, for the case of conformal higher-spin theories the S-matrix is a combination of $\delta(s,t,u)$ [31] and the AdS/CFT duals of unbroken higher-spin theories must be free CFT's which should be thought of as examples of trivial holographic S-matrices.

**2.1. Weinberg low energy theorem**

A serious restriction comes from the Weinberg low energy theorem [1] that eventually leads to too many conservation laws, when massless higher-spin fields are present. As a result of checking linearized gauge invariance or Lorentz invariance of the n-particle amplitude with one soft spin-s particle attached one finds



$$\Longleftrightarrow \qquad \sum_i g_s^i\, p^i_{\mu_1} \cdots p^i_{\mu_{s-1}} = 0 \qquad (2.1)$$

where $g_s^i$ is the coupling constant of the ith species to a spin-s field. For s = 1 one discovers that the total (electric) charge is conserved. For s = 2 one finds a linear combination of momenta weighted by $g_2^i$ whose clash with the momentum conservation law $\sum_i p^i_\mu = 0$ can only be resolved by the equivalence principle, i.e. all fields must couple to gravity universally, $g_2^i = \mathrm{const}$.

For the higher-spin case s > 2 one finds too many conservations laws, which is a rank (s − 1) tensorial expression, with the only solution given by permutations of momenta at the condition that all coupling constants are the same.

In the course of the proof of the theorem one makes an explicit use of Lorentz covariant vertices. In particular, the expressions are manifestly Lorentz covariant. This is not the case in the light-cone approach where the vertices do not have a manifestly Lorentz covariant form. It would be interesting to reconsider the Weinberg theorem as to see whether these assumptions can be weakened4.

**2.2. COLEMAN–MANDULA THEOREM**

The famous Coleman–Mandula theorem [2] prevents S-matrix from having symmetry generators, beyond those of the Poincare group, that transform under the Lorentz group. Under assumptions of non-triviality of the symmetry action, discrete mass spectrum and the analyticity of the S-matrix in Mandelstam invariants, it can be shown that the symmetry algebra can only be a product of the Poincare group and a group of internal symmetries whose generators are Lorentz scalars. It does not apply to the case of d = 1 + 1 QFT, where only forward/backward scattering is possible, so S-matrix must have scattering angles $\theta = 0, \pi$ and thereby it is not analytic. The essence of the proof is that the scattering process is a map from one set of momenta to another one and the momenta are restricted by energy-momentum conservation, which is a Lorentz vector equation. Existence of some other charges that transform non-trivially under the spacetime symmetry would impose tensorial equations on momenta, e.g. like in Weinberg theorem, which would restrict possible processes to exchanges of momenta like in 1 + 1 or trivialize the scattering completely. One way the original Coleman–Mandula theorem can be avoided is by assuming that symmetry generators transform as spinors, which leads to supersymmetry.

One of the assumptions of the theorem is to have a finite number of particles below any mass-shell. This is certainly not true in higher-spin theories where the spectrum should contain infinitely many massless particles . It would be interesting to weaken the assumptions of the theorem5.

**2.3. Aragone–Deser argument/No canonical gravity coupling**

Contrary to the Weinberg and Coleman–Mandula theorems, this argument is local and is attached to specific field variables [16, 17]. It says that the canonical way of putting fields on a curved background by replacing partial derivatives with covariant ones does not work for massless higher-spin fields. Indeed, in checking the gauge invariance of the action we have to commute derivatives, which brings the Riemann tensor:

$$S = \int \nabla\phi\nabla\phi + ..., \qquad \delta\phi = \nabla\xi, \qquad \delta S = \int (\phi_{..})(\nabla_. \xi R_{..,..} + \xi \nabla_. R_{..,..}). \qquad (2.2)$$

Unlike low-spin examples, we find the full four-index Riemann tensor—the structure that cannot be compensated by any modifications of the action/gauge transformations. For s = 1 the action is manifestly gauge invariant, while for s = 3/2 we find not the full Riemann tensor but its trace, the Ricci tensor, which allows to overcome the problem by going to supergravities.

The argument above makes use of the specific field variables and of the manifestly Lorentz covariant methods. Obviously, this is avoided by the light-cone approach. We will emphasize in section 3.7 that there exists in fact a two-derivative gravitational coupling of massless higher-spin fields to gravity [21–23], which is not captured by covariant studies.

**2.4. BCFW**

A relatively new no-go type result came from the BCFW approach [33]. However, higher-spin theories are clearly different from Yang–Mills theory and even gravity and are not expected to have an S-matrix that is analytic. Moreover, BCFW approach is essentially based on the assumption of certain behavior of amplitudes for infinite BCFW shifts. It is not a priori clear whether these assumptions can be justified in the higher-spin case. Some works towards weakening these assumptions include.

**2.5. Three dimensions**

Massless higher-spin fields do not have local degrees of freedom in three-dimensions [5] and therefore the no-go theorems discussed above do not apply.

**2.6. AdS**

Another option to avoid the no-go theorems is to simply abandon the flat space and go to anti-de Sitter background [3–5] since the no-go theorems discussed above were formulated for QFT's in flat space.

3. Living on light-front

In this section we review the light-cone approach to relativistic dynamics. Next, we discuss the classification of cubic vertices that results from the light-cone dynamics and confront it with the covariant methods. The main lesson is that there are more vertices in the light-cone approach. In particular there are two-derivative interaction vertices s − s − 2 of a spin-s field and a graviton, which can be called gravitational. It is worth stressing that the Yang–Mills theory, when rewritten in the light-cone approach, is a theory of scalar fields in the adjoint of the global symmetry group. Similarly, gravity is a theory of two scalar fields with no symmetries like diffeomorphisms whatsoever.

3.1. Free field realization.

We have just discussed which commutation relations need to be solved. Further progress can only be made for specific theories. The general comment is that the quantization on the light-front leads to second-class constraints10. Indeed, the kinetic term $\frac{1}{2}(\partial\phi)^2$, when written in the light-cone coordinates, $\partial^+\phi\partial^-\phi+\frac{1}{2}(\nabla\phi)^2$, is linear in the velocity $\partial^-\phi$ and hence the momenta, i.e. the primary constraints, cannot be solved for $\partial^-\phi$. Therefore, the bracket is the Dirac bracket.

From now on we confine ourselves to live in the four-dimensional world. The nice feature of the 4d world is that all massless spinning particles have two degrees of freedom, i.e. made of two scalar fields except for the spin-zero particle, which equals one scalar field. A spin-s particle has two states with helicities ±s and can be described as two fields $\Phi^{\pm s}(x)$ that are complex conjugate. It is convenient to work with the fields that are Fourier transformed with respect to $x^-$ and transverse coordinates xa:

$$\Phi(x,x^+)=(2\pi)^{-\frac{d-1}{2}}\int e^{+i(x^-p^++x\cdot p)}\Phi(p,x^+)\,d^{d-1}p, \tag{3.16}$$

$$\Phi(p,x^+)=(2\pi)^{-\frac{d-1}{2}}\int e^{-i(x^-p^++p\cdot x)}\Phi(x,x^+)\,d^{d-1}x. \tag{3.17}$$

In the 4d world the equal time commutation relations that follow from the Dirac bracket are:

$$[\Phi^\mu(p,x^+),\Phi^\lambda(q,x^+)]=\delta^{\mu,-\lambda}\frac{\delta^3(p+q)}{2p^+}. \tag{3.18}$$

From now on we set $x^+ = 0$ and will omit the arguments in most of the cases. It is very easy to find the kinematical generators of the Poincare algebra in the Fourier space11:

$$\hat{P}^+=\beta, \qquad \hat{P}=p, \qquad \hat{\bar{P}}=\bar{p}, \tag{3.19a}$$

$$\hat{J}^{z+}=-\beta\frac{\partial}{\partial\bar{p}}, \qquad \hat{J}^{\bar{z}+}=-\beta\frac{\partial}{\partial p}, \qquad \hat{J}^{-+}=-N_\beta-1=-\frac{\partial}{\partial\beta}\beta, \tag{3.19b}$$

$$\hat{J}^{z\bar{z}}=N_p-N_{\bar{p}}-\lambda, \tag{3.19c}$$

where $N_p=p\partial_p$ is the Euler operator, idem. for $N_{\bar{p}}$, $N_\beta$ and we sometimes use $\partial_\beta=\partial/\partial\beta$, etc. The generators are supposed to act on $\Phi^\lambda\equiv\Phi^\lambda_p\equiv\Phi^\lambda(\beta,p,\bar{p},x^+=0)$. The dynamical generators at the free level are:

$$H_2=-\frac{p\bar{p}}{\beta}, \qquad \begin{aligned}\hat{J}^{z-}_2 &= \frac{\partial}{\partial\bar{p}}\frac{p\bar{p}}{\beta}+p\frac{\partial}{\partial\beta}+\lambda\frac{p}{\beta},\\ \hat{J}^{\bar{z}-}_2 &= \frac{\partial}{\partial p}\frac{p\bar{p}}{\beta}+\bar{p}\frac{\partial}{\partial\beta}-\lambda\frac{\bar{p}}{\beta}.\end{aligned} \tag{3.20}$$

The Poincare charges can be built in a standard way:

$$Q_\xi=\int p^+\,d^3p\,\Phi^{-\mu}_{-p}O_\xi(p,\partial_p)\Phi^\mu_p, \tag{3.21}$$

where $O_\xi$ is the generator of the Poincare algebra associated with a Killing vector ξ. We draw reader's attention to the fact that the integration measure is $p^+$. The Poincare algebra is then realized via commutators

$$\delta_\xi\Phi^\mu(p,x^+)=[\Phi^\mu(p,x^+),Q_\xi]. \tag{3.22}$$

Due to the nontrivial integration measure the conjugate operators are defined as

$$O^\dagger=-\frac{1}{p^+}O^T(-p)p^+, \tag{3.23}$$

where the transposed operator is defined via integration by parts as usual, e.g. pT = p, $\partial^T_p=-\partial_p$. The generators of the Poincare algebra given above are Hermitian, $O^\dagger=O$. In particular, we find $p^\dagger=p$. With the help of (3.18) and

$$\delta_\xi\Phi^\mu(p,x^+)=\frac{1}{2}O_\xi(p,\partial_p)\Phi^\mu(p,x^+)+\frac{1}{2}O^\dagger_\xi\Phi^\mu(p,x^+)=O_\xi(p)\Phi^\mu_p \tag{3.24}$$

one can verify all the commutation relations:

$$[Q_\xi,Q_\eta]=Q_{[\xi,\eta]}, \qquad [\delta_\xi,\delta_\eta]\Phi=+\delta_{[\xi,\eta]}\Phi. \tag{3.25}$$

(3.24) follows from a more general formula for the action of Q on an arbitrary functional $F[\Phi]$:

$$[F(\Phi), Q_\xi] = \int \mathrm{d}v\, O_\xi(v)\Phi^\nu_v \frac{\partial}{\partial \Phi^\nu_v} F[\Phi], \qquad (3.26)$$

which we will immediately apply to read off the constraints imposed by kinematical generators on the dynamical ones.

**3.2. Kinematical constraints.**

An appropriate ansatz for the Hamiltonian H and dynamical boosts Ja⁻ reads12:

$$H = H_2 + \sum_n \int \mathrm{d}^{3n}q\, \delta\big(\textstyle\sum q_i\big) h^{q_1,\dots,q_n}_{\lambda_1\dots\lambda_n} \Phi^{\lambda_1}_{q_1}\dots\Phi^{\lambda_n}_{q_n}, \qquad (3.27a)$$

$$J^{z-} = J^{z-}_2 + \sum_n \int \mathrm{d}^{3n}q\, \delta\big(\textstyle\sum q_i\big) \left[ j^{q_1,\dots,q_n}_{\lambda_1\dots\lambda_n} - \frac{1}{n} h^{q_1,\dots,q_n}_{\lambda_1\dots\lambda_n}\left(\sum_j \frac{\partial}{\partial \bar{q}_j}\right)\right] \Phi^{\lambda_1}_{q_1}\dots\Phi^{\lambda_n}_{q_n}, \qquad (3.27b)$$

$$J^{\bar{z}-} = J^{\bar{z}-}_2 + \sum_n \int \mathrm{d}^{3n}q\, \delta\big(\textstyle\sum q_i\big) \left[ \bar{j}^{q_1,\dots,q_n}_{\lambda_1\dots\lambda_n} - \frac{1}{n} h^{q_1,\dots,q_n}_{\lambda_1\dots\lambda_n}\left(\sum_j \frac{\partial}{\partial q_j}\right)\right] \Phi^{\lambda_1}_{q_1}\dots\Phi^{\lambda_n}_{q_n}, \qquad (3.27c)$$

where the delta function imposes the conservation of the total $q^+$ and transverse momenta $q, \bar{q}$, which is a consequence of the translation invariance imposed by Pa and $P^+$, (3.12) and (3.13). The rest of the kinematical generators imposes the following constraints:

$$J^{a+}: \quad \left(\sum_k \beta_k \frac{\partial}{\partial q^a_k}\right) h^{q_1,\dots,q_n}_{\lambda_1,\dots,\lambda_n} \sim 0, \qquad (3.28a)$$

$$J^{a+}: \quad \left(\sum_k \beta_k \frac{\partial}{\partial q^a_k}\right) j^{q_1,\dots,q_n}_{\lambda_1,\dots,\lambda_n} \sim 0, \qquad \text{same for } \bar{j}, \qquad (3.28b)$$

$$J^{z\bar{z}}: \quad \left[\sum_k (N_{q_k} - N_{\bar{q}_k}) + \sum \lambda_k\right] h^{q_1,\dots,q_n}_{\lambda_1,\dots,\lambda_n} \sim 0, \qquad (3.28c)$$

$$J^{-+}: \quad \sum_k \beta_k \frac{\partial}{\partial \beta_k} h^{q_1,\dots,q_n}_{\lambda_1,\dots,\lambda_n} \sim 0, \qquad (3.28d)$$

$$J^{-+}: \quad \sum_k \beta_k \frac{\partial}{\partial \beta_k} j^{q_1,\dots,q_n}_{\lambda_1,\dots,\lambda_n} \sim 0, \qquad \text{same for } \bar{j}, \qquad (3.28e)$$

$$J^{z\bar{z}}: \quad \left[\sum_k (N_{q_k} - N_{\bar{q}_k}) + \sum \lambda_k - 1\right] j^{q_1,\dots,q_n}_{\lambda_1,\dots,\lambda_n} \sim 0, \qquad (3.28f)$$

$$J^{z\bar{z}}: \quad \left[\sum_k (N_{q_k} - N_{\bar{q}_k}) + \sum \lambda_k + 1\right] \bar{j}^{q_1,\dots,q_n}_{\lambda_1,\dots,\lambda_n} \sim 0, \qquad (3.28g)$$

where $\sim 0$ means an equality up to an overall delta-function $\delta^{d-1}(\sum q_k)$.

In practice it is tedious to keep all delta-functions unresolved and it is more convenient to choose some independent momenta as basic variables. Moreover, (3.28a) and (3.28b) imply that everything depends on specific combinations of momenta $\mathbb{P}_{km}$:

$$J^{a+}: \quad \sum_k \beta_k \frac{\partial}{\partial q^a_k} \sim 0 \qquad \Longrightarrow \qquad \mathbb{P}^a_{km} = q^a_k \beta_m - q^a_m \beta_k. \qquad (3.29)$$

There are N − 2 such independent variables for N-point function. In the 4d case we have

$$\mathbb{P}_{km} = q_k \beta_m - q_m \beta_k, \qquad \bar{\mathbb{P}}_{km} = \bar{q}_k \beta_m - \bar{q}_m \beta_k. \qquad (3.30)$$

Therefore, we assume that some N − 2 variables out of all $\mathbb{P}$'s have been chosen and

$$h_{\lambda_1\dots\lambda_n}(q_1,\dots,q_n) = h_{\lambda_1\dots\lambda_n}(\mathbb{P}_{km}, \bar{\mathbb{P}}_{km}, \beta_k), \qquad (3.31)$$

$$j_{\lambda_1\dots\lambda_n}(q_1,\dots,q_n) = j_{\lambda_1\dots\lambda_n}(\mathbb{P}_{km}, \bar{\mathbb{P}}_{km}, \beta_k), \qquad \text{same for } \bar{j}. \qquad (3.32)$$

The rest of the system of kinematical constraints can be rewritten as

$$J^{z\bar{z}}: \quad \left[\mathbb{P}\frac{\partial}{\partial \mathbb{P}} - \bar{\mathbb{P}}\frac{\partial}{\partial \bar{\mathbb{P}}} + \sum \lambda_k\right] h^{q_1,\dots,q_n}_{\lambda_1,\dots,\lambda_n} \sim 0, \qquad (3.33a)$$

$$J^{-+}: \quad \left[\mathbb{P}\frac{\partial}{\partial \mathbb{P}} + \bar{\mathbb{P}}\frac{\partial}{\partial \bar{\mathbb{P}}} + \sum_k \beta_k \frac{\partial}{\partial \beta_k}\right] h^{q_1,\ldots,q_n}_{\lambda_1,\ldots,\lambda_n} \sim 0, \tag{3.33b}$$

$$J^{-+}: \quad \left[\mathbb{P}\frac{\partial}{\partial \mathbb{P}} + \bar{\mathbb{P}}\frac{\partial}{\partial \bar{\mathbb{P}}} + \sum_k \beta_k \frac{\partial}{\partial \beta_k}\right] j^{q_1,\ldots,q_n}_{\lambda_1,\ldots,\lambda_n} \sim 0, \tag{3.33c}$$

$$J^{-+}: \quad \left[\mathbb{P}\frac{\partial}{\partial \mathbb{P}} + \bar{\mathbb{P}}\frac{\partial}{\partial \bar{\mathbb{P}}} + \sum_k \beta_k \frac{\partial}{\partial \beta_k}\right] \bar{j}^{q_1,\ldots,q_n}_{\lambda_1,\ldots,\lambda_n} \sim 0, \tag{3.33d}$$

$$J^{z\bar{z}}: \quad \left[\mathbb{P}\frac{\partial}{\partial \mathbb{P}} - \bar{\mathbb{P}}\frac{\partial}{\partial \bar{\mathbb{P}}} + \sum \lambda_k - 1\right] j^{q_1,\ldots,q_n}_{\lambda_1,\ldots,\lambda_n} \sim 0, \tag{3.33e}$$

$$J^{z\bar{z}}: \quad \left[\mathbb{P}\frac{\partial}{\partial \mathbb{P}} - \bar{\mathbb{P}}\frac{\partial}{\partial \bar{\mathbb{P}}} + \sum \lambda_k + 1\right] \bar{j}^{q_1,\ldots,q_n}_{\lambda_1,\ldots,\lambda_n} \sim 0. \tag{3.33f}$$

The above conditions are very simple homogeneity constraints and need no further comments.

### 3.3. Cubic vertices.

The first nontrivial dynamical constraints arise at the cubic order. First of all, the kinematics of three (d − 1)-dimensional momenta restricted by the conservation delta-function is very simple. There is one independent $\mathbb{P}^a$ variable since $\mathbb{P}^a_{12} = \mathbb{P}^a_{23} = \mathbb{P}^a_{31}$. Therefore, in 4d we have just $\mathbb{P}$ and $\bar{\mathbb{P}}$. It is advantageous to represent it in a manifestly cyclic-invariant way:

$$\mathbb{P}^a_{12} = \ldots = \mathbb{P}^a = \frac{1}{3}\left[(\beta_1 - \beta_2)q^a_3 + (\beta_2 - \beta_3)q^a_1 + (\beta_3 - \beta_1)q^a_2\right], \tag{3.34}$$

$$\sigma_{123}\mathbb{P} = \mathbb{P}, \qquad \sigma_{12}\mathbb{P} = \sigma_{23}\mathbb{P} = \sigma_{13}\mathbb{P} = -\mathbb{P}. \tag{3.35}$$

Therefore, $\mathbb{P}$ belongs to the totally anti-symmetric representation of $S_3$. There is an identity that is of utter importance for the cubic approximation:

$$\sum_j \frac{\partial}{\partial q_j}\mathbb{P} = 0. \tag{3.36}$$

Also, at the three-point level we find

$$\sum_i H_2(q_i) = \frac{\mathbb{P}\bar{\mathbb{P}}}{\beta_1\beta_2\beta_3} = \frac{\mathbb{P}\cdot\mathbb{P}}{2\beta_1\beta_2\beta_3}. \tag{3.37}$$

Now we proceed to the dynamical constraints. The first one is $[H, J^{a-}] = 0$ restricted to the cubic order in fields $\Phi$:

$$[H, J^{a-}]\Big|_3 = [H_3, J^{a-}_2] - [J^{a-}_3, H_2] = 0, \tag{3.38}$$

which, after using the magic identity (3.36), can be shown to lead to

$$\sum_i H_2(q_i) j_3 = \sum_i (\hat{J}^{z-}_2)^T h_3, \qquad \sum_i H_2(q_i)\bar{j}_3 = \sum_i (\hat{J}^{\bar{z}-}_2)^T \bar{h}_3, \tag{3.39}$$

where the transposed generators are

$$(\hat{J}^{z-}_2)^T = -\frac{q\bar{q}}{\beta}\frac{\partial}{\partial \bar{q}} - q\frac{\partial}{\partial \beta} + \lambda\frac{q}{\beta}, \qquad (\hat{J}^{\bar{z}-}_2)^T = -\frac{q\bar{q}}{\beta}\frac{\partial}{\partial q} - \bar{q}\frac{\partial}{\partial \beta} - \lambda\frac{\bar{q}}{\beta}. \tag{3.40}$$

Now one can make an appropriate ansatz for $h_3$ that solves the kinematical constraints (3.33), act with $J^T_2$ and read off $j_3$ and $\bar{j}_3$ up to possible redefinitions. The most general case is studied in appendix A, while below we simply quote the representation given by Metsaev in [35, 36]. The first results on cubic interactions of HS fields were obtained in [21–23] in a slightly different base.

At the interaction level there is always a problem of fixing the field redefinitions. The light-cone approach is not free of this ambiguity too. At the cubic order redefinitions allow one to eliminate powers of $\mathbb{P}\bar{\mathbb{P}} \sim H_2$, but not each of the two separately. Therefore, the most natural choice of the redefinition frame is to have purely holomorphic vertices. It is worth stressing that this is not the most natural choice in the covariant approaches. The vertices are [5, 3]:

$$\boxed{\begin{aligned} h_{\lambda_1,\lambda_2,\lambda_3} &= C^{\lambda_1,\lambda_2,\lambda_3}\frac{\mathbb{P}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} + \bar{C}^{-\lambda_1,-\lambda_2,-\lambda_3}\frac{\mathbb{P}^{-\lambda_1-\lambda_2-\lambda_3}}{\beta_1^{-\lambda_1}\beta_2^{-\lambda_2}\beta_3^{-\lambda_3}}, & (3.41a)\\ j_{\lambda_1,\lambda_2,\lambda_3} &= +\frac{2}{3}C^{+\lambda_1,+\lambda_2,+\lambda_3}\frac{\bar{\mathbb{P}}^{+\lambda_1+\lambda_2+\lambda_3-1}}{\beta_1^{+\lambda_1}\beta_2^{+\lambda_2}\beta_3^{+\lambda_3}}\Lambda^{\lambda_1,\lambda_2,\lambda_3}, & (3.41b)\\ \bar{j}_{\lambda_1,\lambda_2,\lambda_3} &= -\frac{2}{3}\bar{C}^{-\lambda_1,-\lambda_2,-\lambda_3}\frac{\mathbb{P}^{-\lambda_1-\lambda_2-\lambda_3-1}}{\beta_1^{-\lambda_1}\beta_2^{-\lambda_2}\beta_3^{-\lambda_3}}\Lambda^{\lambda_1,\lambda_2,\lambda_3}, & (3.41c)\end{aligned}}$$

where

$$\Lambda = \beta_1(\lambda_2-\lambda_3)+\beta_2(\lambda_3-\lambda_1)+\beta_3(\lambda_1-\lambda_2). \qquad (3.42)$$

Here $C^{\lambda_1,\lambda_2,\lambda_3}$ and $\bar{C}^{-\lambda_1,-\lambda_2,-\lambda_3}$ are two sets of coupling constants which are a priori independent. For dimensional reasons we have to introduce a parameter $l_P$ with the dimension of length as to compensate for the higher powers of momenta, as was noted as early as [21–23]:

$$C^{\lambda_1,\lambda_2,\lambda_3} = (l_P)^{\lambda_1+\lambda_2+\lambda_3-1}c^{\lambda_1,\lambda_2,\lambda_3}, \qquad \text{same for } \bar{C}. \qquad (3.43)$$

In higher-spin theories the parameter will be naturally associated with the Planck length as the Einstein–Hilbert vertex is a part of the set above and corresponds to $C^{2,2,-2}$.

The light-cone locality implies that the powers of $\mathbb{P}$, $\bar{\mathbb{P}}$ must be non-negative or whenever $\sum \lambda_i = 0$ we should have $\lambda_i = 0$. The latter is due to the fact that $j_3^{a-}$ has one power of $\mathbb{P}$ or $\bar{\mathbb{P}}$ less. The exception is when all $\lambda_i = 0$, which is the scalar self-interaction vertex, since it leads to $j_3 = 0$, which is implied in (3.41).

Let us stress that the light-cone approach deals only with physical degrees of freedom, so the light-cone gauge is a unitary gauge, but it is not an on-shell method. Nevertheless, there is a striking relation between the on-shell amplitude methods and the light-cone approach . One can introduce

$$|i] = \frac{2^{1/4}}{\sqrt{\beta_i}}\begin{pmatrix}\bar{q}_i\\ -\beta_i\end{pmatrix} = 2^{1/4}\begin{pmatrix}\bar{q}_i\beta_i^{-1/2}\\ -\beta_i^{1/2}\end{pmatrix}, \qquad (3.44)$$

so that the basic building blocks of cubic vertices can be found in

$$[i|j] = \sqrt{\frac{2}{\beta_i\beta_j}}\,\bar{\mathbb{P}}_{ij}, \qquad (3.45)$$

and analogously one can define $|i\rangle$. As a result, the cubic vertices, i.e. Hamiltonian density $h_3$, can be rewritten in a more suggestive form:

$$C^{s_1,s_2,s_3}[12]^{s_1+s_2-s_3}[23]^{s_2+s_3-s_1}[13]^{s_1+s_3-s_2} + \text{c.c.}, \qquad (3.46)$$

which are the usual amplitudes for three helicity fields [33, 34].

**4.1. Examples of low spin fields**

What we try to see below is the conditions that arise at the quartic level when some set of cubic vertices is activated, i.e. to probe the holomorphic constraints (4.5) that decouple from $H_4$ and $J_4$, but, as we have seen, can restrict couplings.

**4.1.1. Scalar cubed theory.**

This is the simplest and somewhat trivial example:

$$h_3 = \Phi^0\Phi^0\Phi^0\left[C^{0,0,0}+\text{c.c.}\right], \qquad J_3 = 0. \qquad (6.1)$$

Thanks to $J_3 = 0$ the commutator $[J_3, H_3]$ vanishes identically revealing that the cubic vertex provides a self-consistent theory and solves (4.4), which is expected, of course.

**4.1.2. Yang–Mills theory.**

For the case of spin-one self-interaction we have to have a colored set of fields since $\mathbb{P}$ is totally anti-symmetric. Therefore, we introduce some anti-symmetric structure constants fabc and let fields carry additional indices too, $\Phi^\lambda_a$. The cubic vertex reads

$$h_3 = f^{abc}\Phi^1_a\Phi^1_b\Phi^{-1}_c\left[\frac{\bar{\mathbb{P}}_{12}C^{1,1,-1}\beta_3}{\beta_1\beta_2}+\text{c.c.}\right]. \qquad (6.2)$$

After summing over cyclic permutations we find that (4.5) is satisfied provided the Jacobi identity for the structure constants is true.

**4.1.3. Yang–Mills theory coupled to scalar matter.**

It is also interesting to see how the Yang–Mills fields can couple to matter15. To this effect we add a one-derivative $0-0-1$ vertex, where the current built of the scalar fields couples to the Yang–Mills field:

$$h_3 = \left[\frac{\beta_3}{\beta_1\beta_2}f^{abc}\Phi^1_a\Phi^1_b\Phi^{-1}_c\bar{\mathbb{P}}_{12}C^{1,1,-1} + T^{aij}\Phi^1_a\Phi^0_i\Phi^0_j C^{0,0,1}\frac{\bar{\mathbb{P}}_{23}}{\beta_1}+\text{c.c.}\right]. \qquad (6.3)$$

Interestingly, after symmetrizing over the permutations (4.5) implies $C^{1,1,-1} = C^{0,0,1}$, i.e. the coupling constants must be equal.

**4.1.4. Pure gravity.**

In the case of pure gravity we inject the Einstein–Hilbert two-derivative cubic vertex, i.e. $C^{2,2,-2} \neq 0$, while all other constants are zero:

$$h_3 = \Phi^2\Phi^2\Phi^{-2}\left[\frac{\bar{\mathbb{P}}_{12}^2\beta_3^2 C^{2,2,-2}}{\beta_1^2\beta_2^2} + \text{c.c.}\right] \tag{6.4}$$

Then the holomorphic part (4.5) of the commutator $[J_3, H_3]$ can be found to identically vanish after symmetrizing over permutations of all four legs, which, at this order, just tells us that gravity might be a consistent theory.

**4.1.5. Higher-derivative gravity.**

From the covariant approach it is known that one can add a six-derivative $R^3$-type vertex, the resulting theory being consistent. In the light-cone approach we start with

$$h_3 = \Phi^2\Phi^2\Phi^{-2}\left[\frac{\bar{\mathbb{P}}_{12}^2\beta_3^2 C^{2,2,-2}}{\beta_1^2\beta_2^2}\right] + \Phi^2\Phi^2\Phi^2\left[\frac{\bar{\mathbb{P}}_{12}^6 C^{2,2,2}}{\beta_1^2\beta_2^2\beta_3^2}\right] + \text{c.c.}. \tag{6.5}$$

In the commutator one finds two types of CC terms:

$$(4.5) \sim (...)C^{2,2,-2}C^{2,2,-2} + (...)C^{2,2,-2}C^{2,2,2}, \tag{6.6}$$

which vanish independently after symmetrizing over the four legs. Therefore, the $R^3$ vertex can be added with an arbitrary coefficient, which is to be expected from the covariant approaches.

**4.1.6. Gravity plus scalar matter.**

A different situation is with the scalar-tensor theory, which in addition to gravity contains a two-derivative vertex that couples the scalar field stress-tensor to gravity:

$$h_3 = \Phi^2\Phi^2\Phi^{-2}\left[\frac{\bar{\mathbb{P}}_{12}^2\beta_3^2 C^{2,2,-2}}{\beta_1^2\beta_2^2}\right] + \Phi^0\Phi^0\Phi^2\left[\frac{\bar{\mathbb{P}}_{12}^2 C^{0,0,2}}{\beta_3^2}\right] + \text{c.c.} \tag{6.7}$$

In this case the vanishing of (4.5) imposes a single constraint:

$$C^{2,2,-2} = C^{0,0,2}, \tag{6.8}$$

i.e. the scalar field coupling equals to that of the gravity—the equivalence principle.

**4.1.7. Einstein–Yang–Mills theory.**

We can also try to couple a spin-one field to gravity, i.e. to activate the $C^{2,1,-1}$ vertex:

$$h_3 = \Phi^2\Phi^2\Phi^{-2}\left[\frac{\bar{\mathbb{P}}_{12}^2\beta_3^2 C^{2,2,-2}}{\beta_1^2\beta_2^2}\right] + \Phi^1\Phi^{-1}\Phi^2\left[\frac{\bar{\mathbb{P}}_{12}^2 C^{1,-1,2}\beta_2}{\beta_1\beta_3^2}\right] + \text{c.c.} \tag{6.9}$$

As before the vanishing of (4.5) imposes a single constraint:

$$C^{2,2,-2} = C^{1,-1,2}, \tag{6.10}$$

i.e. the equivalence principle for a Maxwell field.

**4.2. Universality of gravity and Yang–Mills**

Even before attempting to look for a complete theory we can ask a simpler question: what happens if we have a higher-spin field which is coupled to gravity or the Yang–Mills theory.

Generalizing the low-spin examples above, we can take a spin-s field and a spin-one Yang–Mills field and turn on $C^{s,-s,1}$ in addition to the Yang–Mills interaction itself. Then, vanishing of the holomorphic terms in $[H_3, J_3]$ implies that all higher-spin fields couple universally to spin-one:

$$s - s - 1: \qquad C^{s,-s,1} = C^{1,1,-1} = g.$$

The same exercise for the gravitation interaction, i.e. with $C^{2,2,-2}$ and $C^{2,s,-s}$ switched on implies that all higher-spin fields couple universally to spin-two16:

$$s - s - 2: \qquad C^{s,-s,2} = C^{2,2,-2} = g\, l_p.$$

The fact that the strength of the backreaction from higher-spin fields on gravity must be the same for all spins s = 0, 1, 2, 3, 4,... is a reincarnation of the equivalence principle which, as it turns out, holds true for fields of any spin17.

The higher-spin equivalence principle also implies that there is a system made of graviton and a spin-s field with only the Einstein–Hilbert $C^{2,2,-2}$ and gravitational $C^{s,-s,2}$ vertices switched on that solves the holomorphic constraints (4.5). Therefore, this solution explicitly avoids the Aragone–Deser argument in the light-cone approach and suggests that it may be possible to put higher-spin fields on more general backgrounds. However, (4.5) is a necessary condition and an obstruction can come from the rest of the constraints (4.6) and higher orders.

It should be noted that the Weinberg low-energy theorem, if applied literally to the higher-spin case, does imply that all couplings should be equal but it simultaneously imposes a too restrictive conservation law that can only be obeyed by the scattering processes that simply permute the particles' momenta. Pessimistically, this should then be seen later in the light-cone approach too. Optimistically, the Weinberg theorem can be avoided by the light-cone approach.

## 5. CONCLUSIONS AND DISCUSSION

Finally pointed out that due to the holomorphic splitting of the Poincare algebra consistency relations there exists a complete chiral higher-spin theory in 4d flat space. Such a theory provides a counterexample to a widespread belief that higher-spin interactions are impossible in the Minkowski space. However, the theory is non-unitary.

While the chiral theory is an encouraging result, we expect the unitary higher-spin theory to exist too. Its derivation requires more efforts since the Poincare deformation procedure does not stop at the cubic order. This concpet have fixed a part of the quartic Hamiltonian that determines an infinite series of the quartic contact vertices of the scalar field. This can be thought of as the Minkowski space counterpart of the AdS result obtained recently in. In particular the flat space quartic action shares some features with its $AdS_4$ cousin: it is naively non-local in having an unbounded order in derivatives arranged into a series of positive powers of the transverse momenta. However, there are no wild non-localities of type $1/\Box$ or $1/p_i \cdot p_j$, which would trivialize the deformation procedure. Such non-localities arise in some of the covariant studies, but not in the others. Formally, the quartic scalar self-interaction drops off the Noether procedure at this order since scalar field does not feature its own gauge parameter. The equation for the quartic scalar vertex is a part of the quintic No ether consistency conditions.

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