AN APPLICAION OF HEXAGONAL FUZZY NUMBER AND MATRICES TO SOLVE DECISION MAKING PROBLEM

S. JOHNSON SAVARIMUTHU[1] AND N. KARTHIKA[2]
Assistant Professor[1]& Research Scholar[2]
PG & Research Department of Mathematics
ST. Joseph's College of Arts and Science(Autonomous)
Cuddalore, Tamilnadu, India.

ABSTRACT:

In this paper, we use the Hexagonal fuzzy number and fuzzy matrices to choose the find best one among a set of selected alternatives of fighters. The filed of decision making are most interacting area of application fuzzy set theory.

KEY WORDS:

Hexagonal fuzzy number, Hexagonal fuzzy matrices, alpha cut, Decision makers.

INTRODUCTION:

Fuzzy set theory was introduced by L.A.Zadeh in the year 1965. Fuzzy set in any set which allows to have a members in the interval [0 ,1] and it is known membership function fuzzy environment has a potential to solve uncertainty . It involves the many field such as medicine engineering, etc.

Decision making in the fuzzy environment is means a decision process in which the goals and the constrains .Fuzzy large allows decision making under incomplete or uncertain information decision making is a tool to find the best solution or better then that

The maximizing decision is defined as a point in the space of alternatives fuzzy sets are membership function of a fuzzy decision attains its maximum value. They are various kinds of decision making. This paper have been basic definitions decision making under fuzzy the hexagonal number arithmetic operation, procedure, illustrative and finally solution.

2.Preliminaries:

Definition 1:

A *fuzzy set* is characterized by its membership function, taking values from the domain, space or universe of discourse mapped into the unit interval [0,1]. A fuzzy set A in the universal set X is defined as A = (x, μ(x);x∈X). Here, $\mu_A : A \rightarrow [0,1]$ is the grade of the membership function and $\mu_A(x)$ is the grade value of $x \subset X$ in the fuzzy set A.

Definition 2:

A fuzzy set A is called *normal* if there exists an element $x \subset X$ whose membership value is one, i.e., $\mu_A(x) = 1$.

Definition 3:

A *fuzzy number* A is a subset of real line R, with the membership function μ_A satisfying the following properties:

(i) $\mu_A(x)$ is piecewise continuous in its domain.

(ii) A is normal, i.e., there is a x0ϵA such that $\mu_A(x0) = 1$.

(iii) A is convert, i.e., $\mu_A(\lambda x1 + (1-\lambda)x2) \geq \min(\mu_A(x1), \mu_A(x2))$. For all x_1, x_2 in X.

Due to wide applications of the fuzzy number, two types of fuzzy number, namely, triangular fuzzy number and trapezoidal fuzzy number, are introduced in the field of fuzzy algebra.

Definition 4:

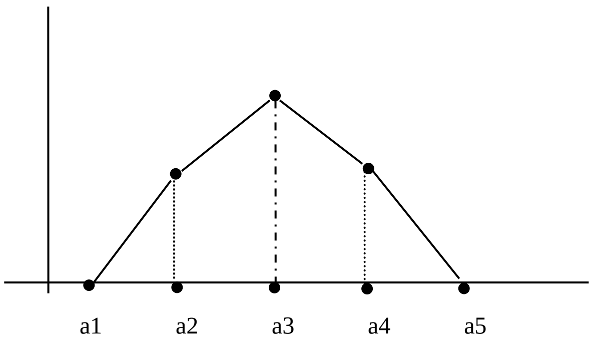
A fuzzy number A = (a_1,a_2,a_3,a_4) is said to be *trapezoidal fuzzy number* if its membership function is given by where $a_1 \leq a_2 \leq a_3 \leq a_4$

$$\mu_A(x) = \begin{cases} 0 & \text{for } x < a1 \\ \frac{(x-a_1)}{a_2-a_1} & \text{for } a1 \leq x \leq a2 \\ 1 & \text{for } a2 \leq x \leq a3 \\ \frac{(a_4-x)}{a_4-a_3} & \text{for } a2 \leq x \leq a3 \\ 0 & \text{for } x < a4 \end{cases}$$

Definition 5:

A *pentagonal fuzzy number* defied as A =(a_1,a_2,a_3,a_4,a_5) and its the membership function is given by,

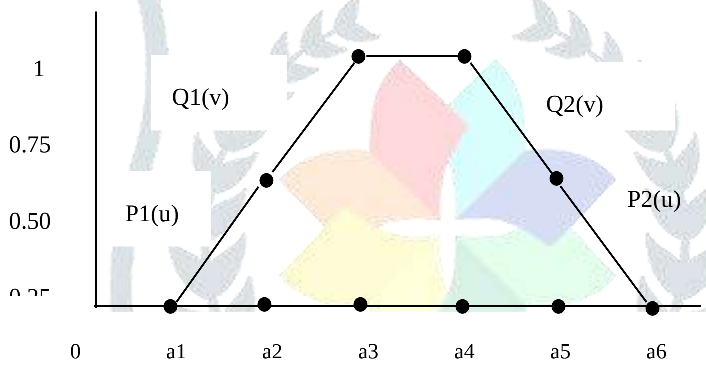
$$\mu_A(x) = \begin{cases} 0 & \text{for } x < a1 \\ \frac{(x-a_1)}{a_2-a_1} & \text{for } a1 \leq x \leq a2 \\ \frac{(x-a_2)}{a_3-a_2} & \text{for } a2 \leq x \leq a3 \\ 1 & x = a3 \\ \frac{(a_4-x)}{a_4-a_3} & \text{for } a3 \leq x \leq a4 \\ \frac{(a_4-x)}{a_4-a_3} & \text{for } a4 \leq x \leq a5 \\ 0 & \text{for } x < a5 \end{cases}$$



Definition 6:

A fuzzy number AH is a hexagonal fuzzy number denoted by AH = (a1,a2,a3,a4,a5,a6) where a1,a2,a3,a4,a5,a6 are real number and its membership function μAH(x) is given by

$$\mu A(x) = \begin{cases} 0 & \text{for } x < a1 \\ \frac{1}{2}\frac{(x-a_1)}{(a_2-a_1)} & \text{for } a1 \leq x \leq a2 \\ \frac{1}{2}+\frac{1}{2}\frac{(x-a_2)}{(a_3-a_2)} & \text{for } a2 \leq x \leq a3 \\ 1 & \text{for } a3 \leq x \leq a4 \\ 1-\frac{1}{2}\frac{(a_4-x)}{a_4-a_3} & \text{for } a4 \leq x \leq a5 \\ \frac{1}{2}\frac{(a_6-x)}{a_6-a_5} & \text{for } a5 \leq x \leq a6 \\ 0 & \text{for } x > a6 \end{cases}$$



Definition 7: pentagonal fuzzy number matrix

The element of pentagonal fuzzy number matrix is A = $(a_{ij})_{n\times n}$ $a_{ij} = (a_{ijP}, a_{ijQ}, a_{ijR}, a_{ijS}, a_{ijT})$ be the ij[th] element of A.

Definition 8: pentagonal fuzzy membership matrix

The membership function of $a_{ij} = (a_{ijP}, a_{ijQ}, a_{ijR}, a_{ijS}, a_{ijT})$ is defined as $\left(\frac{a_{ijP}}{10}, \frac{a_{ijQ}}{10}, \frac{a_{ijR}}{10}, \frac{a_{ijS}}{10}, \frac{a_{ijT}}{10}\right)$, If $0 \leq a_{ijP} \leq a_{ijQ} \leq a_{ijR} \leq a_{ijS} \leq a_{ijT} \leq 1$
where $0 \leq \frac{a_{ijP}}{10} \leq \frac{a_{ijQ}}{10} \leq \frac{a_{ijR}}{10} \leq \frac{a_{ijS}}{10} \leq \frac{a_{ijT}}{10} \leq 1$.

Definition 9: Hexagonal fuzzy number matrix

The element of Hexagonal fuzzy number matrix is A = $(a_{ij})_{n\times n}$ $a_{ij} = (a_{ijP}, a_{ijQ}, a_{ijR}, a_{ijS}, a_{ijT}, a_{ijU})$ be the ij[th] element of A.

Definition 10: Hexagonal fuzzy membership matrix

The membership function of $a_{ij} = (a_{ijP}, a_{ijQ}, a_{ijR}, a_{ijS}, a_{ijT}, a_{ijU})$ is defined as $\left(\frac{a_{ijP}}{10}, \frac{a_{ijQ}}{10}, \frac{a_{ijR}}{10}, \frac{a_{ijS}}{10}, \frac{a_{ijT}}{10}, \frac{a_{ijU}}{10}\right)$, If $0 \leq a_{ijP} \leq a_{ijQ} \leq a_{ijR} \leq a_{ijS} \leq a_{ijT} \leq a_{ijU} \leq 1$
where $0 \leq \frac{a_{ijP}}{10} \leq \frac{a_{ijQ}}{10} \leq \frac{a_{ijR}}{10} \leq \frac{a_{ijS}}{10} \leq \frac{a_{ijT}}{10} \leq \frac{a_{ijU}}{10} \leq 1$.

III. ARITHMETIC OPERATION ON HEXAGONAL FUZZY NUMBER MATRIX

Let A = $(a_{ij})_{m\times n}$ and B = $(b_{ij})_{m\times n}$ are two hexagonal fuzzy number matrices of same order n× n

11) Addition:(A+B) = $(a_{ij}+b_{ij})_{m\times n}$

Where $(a_{ij}+b_{ij}) = (a_{ijP}+b_{ijP}, a_{ijQ}+b_{ijQ}, a_{ijR}+b_{ijR}, a_{ijS}+b_{ijS}, a_{ijT}+b_{ijT}, a_{ijU}+b_{ijU})$ is the ijth element of (A + B).

12) Subtraction:(A - B) = $(a_{ij}-b_{ij})_{m\times n}$

Where $(a_{ij}-b_{ij}) = (a_{ijP}-b_{ijP}, a_{ijQ}-b_{ijQ}, a_{ijR}-b_{ijR}, a_{ijS}-b_{ijS}, a_{ijT}-b_{ijT}, a_{ijU}-b_{ijU})$ is the ijth element of (A - B).

13) Maximum operation on Hexagonal fuzzy number:

The maximum operation is given by max(A,B) = (sup{ a_{ij}, b_{ij}})

Where

sup(a_{ij}, b_{ij})=

$(sup(a_{ijP}, b_{ijP}), \sup(a_{ijQ}, b_{ijQ}), \sup(a_{ijR}, b_{ijR}), \sup(a_{ijS}, b_{ijS}), \sup(a_{ijT}, b_{ijT}), \sup(a_{ijU}, b_{ijU})$ is the ijth element of max(A,B).

14) Arithmetic Mean (AM) for Hexagonal fuzzy Number:

Let A = $(a_1,a_2,a_3,a_4,a_5,a_6)$ be Hexagonal fuzzy number

$$AM(A) = \frac{(a1+a2+a3+a4+a5+a6)}{5}$$

15) Relativity function :

Let x and y be a variables defined on a universal set X. The relativity function is defined as

$$f\left(\frac{x}{y}\right) = \left\{\frac{\mu_y(x)-\mu_x(y)}{\max\{\mu_y(x),\mu_x(y)\}}\right\}$$

where $\mu_y(x)$ is the membership function of x with respect to 'y' for hexagonal fuzzy number and $\mu_x(y)$ is the membership function of y with respect to 'x' got hexagonal fuzzy number. Here

16) Comparison Matrix:

Let A = $\{x_1, x_2, x_3, \dots\dots.. x_{i-1}, x_i, x_{i+1}, \dots\dots\dots.. x_n\}$ be a set n – variables defined on X. Form a matrix of respectively values $f\left(\frac{x_i}{x_j}\right)$ where x_i's for i = 1 to n- variables defied on a matrix x. The matrix C = (c_{ij}) is a square matrix of order n is called comparison matrix (or)

C- matrix with $f\left(\frac{x_i}{x_j}\right) = \left\{\frac{\mu_{x_j}(x_i)-\mu_{x_i}(x_j)}{\max\{\mu_{x_j}(x_i),\mu_{x_i}(x_j)\}}\right\}$ where AM represents arithmetic mean.

V.PROCEDURE

step:1

consider the hexagonal fuzzy number matrix from the imprecise estimation needed for the problem using the definition "Hexagonal fuzzy number matrix" .

step:2

convert the given matrix into membership function using the definition of "Hexagonal fuzzy membership matrix ".

Step:3

Calculate the relativity values $f\left(\frac{x_i}{x_j}\right)$ by the definition

Step:4

Calculate the comparison matrix from the values of $f\left(\frac{x_i}{x_j}\right)$ using the definition "Comparison matrix".

Step:5

Find the minimum value of each row C_i.

Step:6

The maximum value of the column C_i is the required solution.

VI . LUSTRATIVE EXAMPLE

Let us consider three students Ram, Hari, Ranjith participated in the fighter competition . They are the Top three contestants of this competition.

Vocalists will be judged based on the criteria listed below and will receive judge's brief comments immediately following their performance.

Criteria	**Possible points**
Maximum speed	10
Fight Range	10
Maximum Load	10
Purchase Expense	10
Reliability	10
Judge's overall impression	10

Let us consider the set $\{s_1,s_2,s_3,s_4,s_5,s_6\}$ as universal set, where s_1,s_2,s_3,s_4,s_5,s_6 denotes Maximum speed, Fight Range, Maximum Load, Purchase Expense, Reliability, Judge's overall impression respectively.

Choose performance based on the overall quality of performance.

The matrix A represents the scores in the form of hexagonal fuzzy number matrix.

Setp:1

$$A = \begin{array}{c} \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{x} & \mathbf{y} & \mathbf{z} \end{array} \\ \begin{pmatrix} (1,3,5,6,7,9) & (3,6,7,8,9,10) & (3,4,5,7,8,9) \\ (3,5,6,7,8,10) & (2,3,5,7,8,8) & (1,3,4,6,8,10) \\ (1,2,6,6,7,7) & (4,5,6,7,7,10) & (2,4,5,6,7,8) \end{pmatrix} \end{array}$$

Setp:2

		X	y	z	x
$(A)_{mem}$ =		(0.1,0.3,0.5,0.6,0.7,0.9)	(0.3,0.6,0.7,0.8,0.9,1.0)	(0.3,0.4,0.5,0.7,0.8,0.9)	
	y	(0.3,0.5,0.6,0.7,0.8,1.0)	(0.2,0.3,0.5,0.7,0.8,0.8)	(0.1,0.3,0.4,0.6,0.8,1.0)	
	z	(01,0.2,0.6,0.6,0.7,0.7)	(0.4,0.5,0.6,0.7,0.7,1.0)	(0.2,0.4,0.5,0.6,0.7,0.8)	

$\mu_x(x) =$ (0.1,0.3,0.5,0.6,0.7,0.9)

$\mu_y(x) =$ (0.3,0.6,0.7,0.8,0.9,1.0)

$\mu_z(x) =$ (0.3,0.4,0.5,0.7,0.8,0.9)

$\mu_x(y) =$ (0.3,0.5,0.6,0.7,0.8,1.0)

$\mu_y(y) =$ (0.2,0.3,0.5,0.7,0.8,0.8)

$\mu_z(y) =$ (0.1,0.3,0.4,0.6,0.8,1.0)

$\mu_x(z) =$ (0.1,0.2,0.6,0.6,0.7,0.7)

$\mu_y(z) =$ (0.4,0.5,0.6,0.7,0.7,1.0)

$\mu_z(z) =$ (0.2,0.4,0.5,0.6,0.7,0.8)

Step:3

$$f\left(\frac{x}{x}\right) = \left\{\frac{\mu_X(X)-\mu_X(X)}{\max\{\mu_X(X),\mu_X(X)\}}\right\}$$

$$= \frac{(0.1,0.3,0.5,0.6,0.7,0.9)-(0.1,0.3,0.5,0.6,0.7,0.9)}{\max\ (0.1,0.3,0.5,0.6,0.7,0.9),(0.1,0.3,0.5,0.6,0.7,0.9)}$$

$$= \frac{(0,0,0,0,0,0)}{(0.1,0.3,0.5,0.6,0.7,0.9)}$$

$$= 0.$$

Step:4

$$f\left(\frac{x}{y}\right) = \left\{\frac{\mu_y(X)-\mu_X(Y)}{\max\{\mu_y(X),\mu_X(Y)\}}\right\}$$

$$= \frac{(0.3,0.6,0.7,0.8,0.9,1.0)-(0.3,0.5,0.6,0.7,0.8,1.0)}{max(0.3,0.6,0.7,0.8,0.9,1.0),(0.3,0.5,0.6,0.7,0.8,1.0)}$$

$$= \frac{(0,0.1,0.1,0.1,0.1,0)}{0.3,0.6,0.7,0.8,0.9,1}$$

$$= \frac{0.4}{4.3}$$

$$= 0.093023$$

Step:5

$$f\left(\frac{x}{z}\right) = \left\{\frac{\mu_z(X)-\mu_X(Z)}{\max\{\mu_Z(X),\mu_X(Z)\}}\right\}$$

$$= \frac{(0.3,0.4,0.5,0.7,0.8,0.9) - (0.1,0.2,0.6,0.6,0.7,0.7)}{\max(0.3,0.4,0.5,0.7,0.8,0.9),(0.1,0.2,0.6,0.6,0.7,0.7)}$$

$$= \frac{(0.2,0.2,0.1,0.1,0.1,0.2)}{0.3,0.4,0.6,0.7,0.8,0.9}$$

$$= \frac{0.7}{3.7}$$

$$= 0.18918$$

Step:6

$$f\left(\frac{y}{x}\right) = \left\{\frac{\mu_X(X)-\mu_Y(X)}{\max\{\mu_X(Y),\mu_Y(X)\}}\right\}$$

$$= \frac{(0.3,0.5,0.6,0.7,0.8,1) - (0.3,0.5,0.6,0.7,0.8,1)}{\max(0.3,0.5,0.6,0.7,0.8,1),(0.3,0.6,0.7,0.8,0.9,1)}$$

$$= \frac{(0,-0.2,-0.1,-0.1,-0.1,0)}{0.3,0.6,0.7,0.8,0.9,1}$$

$$= \frac{-0.4}{4.3}$$

$$= -0.0930$$

Step:7

$$f\left(\frac{Y}{Y}\right) = \left\{\frac{\mu_Y(Y)-\mu_Y(Y)}{\max\{\mu_Y(Y),\mu_Y(Y)\}}\right\}$$

$$= \frac{(0.2,0.3,0.5,0.7,0.8,0.8)- (0.2,0.3,0.5,0.7,0.8,0.8)}{\max\ (0.2,0.3,0.5,0.7,0.8,0.8),(0.2,0.3,0.5,0.7,0.8,0.8)}$$

$$= \frac{(0,0,0,0,0,0)}{(0.2,0.3,0.5,0.7,0.8,0.8)}$$

$$= 0.$$

Step:8

$$f\left(\frac{Y}{Z}\right) = \left\{\frac{\mu_Z(Y)-\mu_Y(Z)}{\max\{\mu_Z(Y),\mu_Y(Z)\}}\right\}$$

$$= \frac{(0.1,0.3,0.4,0.6,0.8,1.0)- (0.4,0.5,0.6,0.7,0.7,1.0)}{\max\ (0.1,0.3,0.4,0.6,0.8,1.0),(0.4,0.5,0.6,0.7,0.7,1.0)}$$

$$= \frac{(-0.3,-0.2,-0.2,-0.1,0.1,0.1,0)}{(0.4,0.5,0.6,0.7,0.8,1)}$$

$$= \frac{-0.7}{4}$$

= -0.175

Step:9

$$f\left(\frac{Z}{X}\right) = \left\{\frac{\mu_X(Z)-\mu_Z(X)}{\max\{\mu_X(Z),\mu_Z(X)\}}\right\}$$

$$= \frac{(0.1,0.2,0.6,0.6,0.7,0.7)-(0.3,0.4,0.5,0.7,0.8,0.9)}{\max(0.1,0.2,0.6,0.6,0.7,0.7),(0.3,0.4,0.5,0.7,0.8,0.9)}$$

$$= \frac{(-0.2,-0.2,0.1,-0.1,0.-0.2)}{(0.3,0.4,0.6,0.7,0.8,0.9)}$$

$$= \frac{-0.5}{3.7}$$

= -0.13513

Step:10

$$f\left(\frac{Z}{Y}\right) = \left\{\frac{\mu_Y(Z)-\mu_Z(Y)}{\max\{\mu_Y(Z),\mu_Z(Y)\}}\right\}$$

$$= \frac{(0.4,0.5,0.6,0.7,0.7,1.0)-(0.1,0.3,0.4,0.6,0.8,1.0)}{\max(0.4,0.5,0.6,0.7,0.7,1.0),(0.1,0.3,0.4,0.6,0.8,1.0)}$$

$$= \frac{(0.3,0.2,0.1,0.1,-0.1,0)}{(0.4,0.5,0.6,0.7,0.8,1.0)}$$

$$= \frac{0.7}{4}$$

= 0.175

Step:11

$$f\left(\frac{Z}{Z}\right) = \left\{\frac{\mu_Z(Z)-\mu_Z(Z)}{\max\{\mu_Z(Z),\mu_Z(Z)\}}\right\}$$

$$= \frac{(0.4,0.5,0.6,0.7,0.7,1.0)-(0.4,0.5,0.6,0.7,0.7,1.0)}{\max(0.4,0.5,0.6,0.7,0.7,1.0),(0.4,0.5,0.6,0.7,0.7,1.0)}$$

$$= \frac{(0,0,0,0,0,0)}{(0.4,0.5,0.6,0.7,0.7,1.0)}$$

= 0.

C = C_{ij}

= AM $f\left(\frac{X_i}{X_j}\right)$

$$C = \begin{array}{c} \\ X \\ Y \\ Z \end{array} \begin{array}{c} \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{pmatrix} 0 & 0.0930 & 0.18918 \\ -0.0930 & 0 & -0.1750 \\ -0.1351 & 0.1750 & 0 \end{pmatrix} \end{array}$$

C_{ij} = Minimum of the i[th] row

X = 0

Y = -0.175

Z = -0.1351

Comparison:

Hexagonal fuzzy number matrix in decision making	Normal method
X is the winner	X is the winner

Step:12

'X' is the winner of the fighter contest.

ie . Ram is the winner.

VII.CONCLUSION:

The decision making process with the Hexagonal fuzzy number have been used. It solve the real life problems to solve easily.

To easily finding the participation who have performed best in the competition well based on the given criteria.

REFERENCE:

[1] Apurba Panda, Madhumangal Pal, "A study on pentagonal fuzzy number and its corresponding matrices", Pacific Science Review B: Humanities and Social Science, nov 2015, vol.l(3):131-139.

[2] Ponnivalavan K. and Pathinathan K. " Intuitionistic Pentagonal fuzzy numbers " , APPN Joounal of Engineering and applied Science, Vol. 10, No. 12, july 2015.

[3] Shayamal A.K. and Pal M., "Two new operation on fuzzy matrices", J.Appl.Math, Comput., Volume 15, issue 1, 2004, pp, 91-107.

[4] Zedah L. A., "Fuzzy sets, Information and control" 8(1965), 338-353.

[5] Bhowmik, M., pal, M., 2012. Some results on generalized interval-valued Intuitionistic fuzzy set. Int.j.Fuzzy Syst. 14(2), 193-203.

[6] Bhowmik, M., pal, M., 2008. Criculant triangular fuzzy number matrices. V.U.J. phys Sci. 29(3), 533-554.

[7] Dubois, D., Prade, H., 1980. Theory and Application . Fuzzy Set and System. Academic press, London.

[8] Dubois, D., Prade, H., 1978. Operations on fuzzy numbers, Internationl Journal of Systems Science, Vol.9, no.6.,pp.613-626.

[7] Hashimoto, H., 1983. Convergence of powers of fuzzy transitive matrix. Fuzzy Sets Syst. 9, 153-160.

[9] Hashimoto, H., 1983. Canonical form of a transitive fuzzy matrix. Fuzzy Set Syst. 11, 151 – 156.

[10] Kim, K.H., Roush, F.W., 1980. Generalized fuzzy matrices. Fuzzy Set Syst.04, 293-315.

[11] Kauffmann,A.,1980. Introduction to Fuzzy Arithmetic:Theory and Applicatons, Van Nostrand Reinhold, New York.

[12] Shayamal, A.K., Pal, M., 2007. Triangular fuzzy matrtices. Iran.J.Fuzzy Syst. 4(1), 75-87.

[13]Thomason, 1977. Convergence of powers of fuzzy matrix. J.Math. Anal.Appl.57, 476-480.