An Investigation of Tri-Cum Biserial Queuing Model Connected with Three Servers

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Abstract: ***This article presents a complex network queuing model having three servers arranged paralleled in a tri-cum biserial way. The arrival and service pattern is anticipated to follow the Poisson law. Various statistical tools, generating function technique and the law of calculus have been employed to investigate the various queuing characteristics such as Queue length, variance and average waiting time. The present queuing model find vast applications in various industries such as in banking sector, food industries and production lines etc.***

Keywords: Queuing model, variance, probabilities, Poisson law.

I. INTRODUCTION

Several investigations have been performed in the last several decades which dealt with the characteristics of queuing models. Jackson [1] took the initiative in this field and investigated the behavior of queuing model with phase type service. Maggu [2] further investigated the various queuing characteristics of phase type service queues with two servers connected in bi-serieal way. Singh [3] performed the Steady-state behavior of serial queuing processes with impatient customers. Singh et al. [4] investigated the transient queuing model characteristics of parallel biserial queues. Agrawal and Singh [5, 6] examined the various queuing model parameters of tri-cum bi series models.

II. PRACTICAL ENACTMENT OF THE MODEL

In many daily life routines, person has to be seek the best option among several options available in the situation. For example, in a bank, several counters are available such as cash deposit counter, cash withdrawal counter, demand draft issue counter, other add on services i.e. internet banking, mobile banking , pass book completion, fixed deposit facility activation counter and bank loan counter etc. Once the person entered in to the bank, he/she can go to any counter and avail the facility available on that particular counter. That person can also perform multi task in a single visit. For example , he/she first deposit cash on cash counter then move to the other counter to activate the internet banking or mobile banking. In such cases, the developed queuing model can be apply to make the effortless visit.

III. MATHEMATICAL DESCRIPTION OF THE MODEL

In the present work, a queue model consist of three servers (Sr_a, Sr_b and Sr_c,) are connected in parallel in tri cum biserial way. Further, each server is connected with other servers (Sr_u, Sr_v and Sr_w,) in series. Let Q_a, Q_b, Q_c, Q_u, Q_v and Q_w are the queue length associated with servers Sr_a, Sr_b Sr_c, Sr_u, Sr_v and Sr_w respectively. In the present model, let the number of customers (n_a) coming at mean arrival rate λ_a , after completion of service at server Sr_a, the customers can avail the facility at server Sr_b or Sr_c (either of two or both) with the probabilities p_{ab} and p_{ac}. In addition to this, customer can leave the system after completion of intended work at a particular server by using the other server Sr_u with probability p_{au} , such that $p_{ab} + p_{ac} + p_{au} = 1$. The same criterion will be applicable to those customers who entered in servers Sr_b and Sr_c. The graphical representation of the considered problem is demonstrated in Figure 1.

Let us assume that Sr_a, Sr_b and Sr_c show the cash deposit counter, demand draft issue counter and add-on facility (internet banking/ mobile banking etc.) counter in a bank respectively whereas Sr_u Sr_v and Sr_w show the final receipt counter, it can be treated as a pass-book completion counter, demand draft collection or add-on facility activation letter counters respectively. Suppose, customer enter in a bank to first deposit the cash. Therefore he/she go to the server Sr_a. Once the customer deposited the cash at counter Sr_a, he/she can go to the server Sr_u to get the final receipt if he/she don't have any other work in the bank. Otherwise, customer can go to the server Sr_b or Sr_c to issue the demand draft or to activate the add-on facilities. Again, he/she directly move to the server Sr_v or Sr_w to have the final receipt. The various combinations of activities through which a customer can perform different activities are shown below.

$$Sr_a \to Sr_u,\ \ Sr_a \to Sr_b \to Sr_v,\ \ Sr_a \to Sr_c \to Sr_w,\ \ Sr_a \to Sr_b \to Sr_c \to Sr_w,\ \ Sr_a \to Sr_c \to Sr_b \to Sr_v$$

$$Sr_b \to Sr_v,\ \ Sr_b \to Sr_a \to Sr_u,\ \ Sr_b \to Sr_c \to Sr_w,\ \ Sr_b \to Sr_c \to Sr_a \to Sr_u,\ \ Sr_b \to Sr_a \to Sr_c \to Sr_w$$

$$Sr_c \to Sr_w,\ \ Sr_c \to Sr_b \to Sr_v,\ \ Sr_c \to Sr_a \to Sr_u,\ \ Sr_c \to Sr_b \to Sr_a \to Sr_u,\ \ Sr_c \to Sr_a \to Sr_b \to Sr_v$$

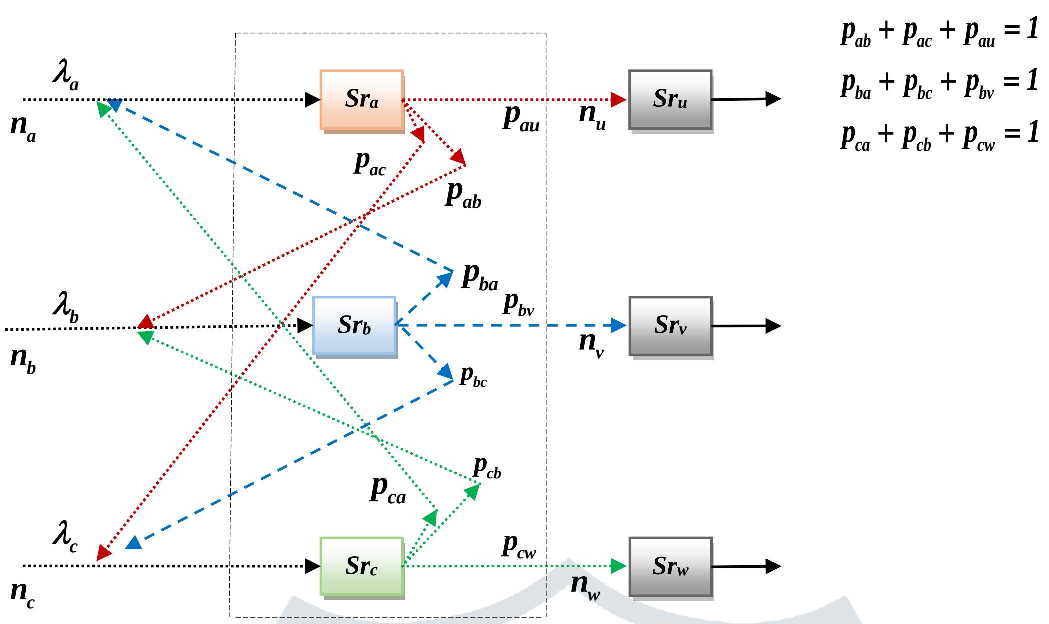


Fig 1: Tri-cum Biserial queuing network

Differential difference equation in steady (transient) state of the model is

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_c+\mu_u+\mu_v+\mu_w)P_{n_a,n_b,n_c,n_u,n_v,n_w} &= \lambda_a P_{n_a-1,n_b,n_c,n_u,n_v,n_w} + \lambda_b P_{n_a,n_b-1,n_c,n_u,n_v,n_w} \\ &+ \lambda_c P_{n_a,n_b,n_c-1,n_u,n_v,n_w} + \mu_a p_{ab} P_{n_a+1,n_b-1,n_c,n_u,n_v,n_w} + \mu_a p_{ac} P_{n_a+1,n_b,n_c-1,n_u,n_v,n_w} + \mu_a p_{au} P_{n_a+1,n_b,n_c,n_u-1,n_v,n_w} \\ &+ \mu_b p_{ba} P_{n_a-1,n_b+1,n_c,n_u,n_v,n_w} + \mu_b p_{bc} P_{n_a,n_b+1,n_c-1,n_u,n_v,n_w} + \mu_b p_{bv} P_{n_a,n_b+1,n_c,n_u,n_v-1,n_w} \\ &+ \mu_c p_{ca} P_{n_a-1,n_b,n_c+1,n_u,n_v,n_w} + \mu_c p_{cb} P_{n_a,n_b-1,n_c+1,n_u,n_v,n_w} + \mu_c p_{cw} P_{n_a,n_b,n_c+1,n_u,n_v,n_w-1} \\ &+ \mu_u P_{n_a,n_b,n_c,n_u+1,n_v,n_w} + \mu_v P_{n_a,n_b,n_c,n_u,n_v+1,n_w} + \mu_w P_{n_a,n_b,n_c,n_u,n_v,n_w+1}\end{aligned} \quad (1)$$

Considering $n_a = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_c+\mu_u+\mu_v+\mu_w)P_{0,n_b,n_c,n_u,n_v,n_w} &= \lambda_b P_{0,n_b-1,n_c,n_u,n_v,n_w} + \lambda_c P_{0,n_b,n_c-1,n_u,n_v,n_w} \\ &+ \mu_a p_{ab} P_{1,n_b-1,n_c,n_u,n_v,n_w} + \mu_a p_{ac} P_{1,n_b,n_c-1,n_u,n_v,n_w} + \mu_a p_{au} P_{1,n_b,n_c,n_u-1,n_v,n_w} + \mu_b p_{bc} P_{0,n_b+1,n_c-1,n_u,n_v,n_w} \\ &+ \mu_b p_{bv} P_{0,n_b+1,n_c,n_u,n_v-1,n_w} + \mu_c p_{cb} P_{0,n_b-1,n_c+1,n_u,n_v,n_w} + \mu_c p_{cw} P_{0,n_b,n_c+1,n_u,n_v,n_w-1} \\ &+ \mu_u P_{0,n_b,n_c,n_u+1,n_v,n_w} + \mu_v P_{0,n_b,n_c,n_u,n_v+1,n_w} + \mu_w P_{0,n_b,n_c,n_u,n_v,n_w+1}\end{aligned} \quad (2)$$

Considering $n_b = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_c+\mu_u+\mu_v+\mu_w)P_{n_a,0,n_c,n_u,n_v,n_w} &= \lambda_a P_{n_a-1,0,n_c,n_u,n_v,n_w} + \lambda_c P_{n_a,0,n_c-1,n_u,n_v,n_w} \\ &+ \mu_a p_{ac} P_{n_a+1,0,n_c-1,n_u,n_v,n_w} + \mu_a p_{au} P_{n_a+1,0,n_c,n_u-1,n_v,n_w} + \mu_b p_{ba} P_{n_a-1,1,n_c,n_u,n_v,n_w} + \mu_b p_{bc} P_{n_a,1,n_c-1,n_u,n_v,n_w} \\ &+ \mu_b p_{bv} P_{n_a,1,n_c,n_u,n_v-1,n_w} + \mu_c p_{ca} P_{n_a-1,0,n_c+1,n_u,n_v,n_w} + \mu_c p_{cw} P_{n_a,0,n_c+1,n_u,n_v,n_w-1} + \mu_u P_{n_a,0,n_c,n_u+1,n_v,n_w} \\ &+ \mu_v P_{n_a,0,n_c,n_u,n_v+1,n_w} + \mu_w P_{n_a,0,n_c,n_u,n_v,n_w+1}\end{aligned} \quad (3)$$

Considering $n_c = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_u+\mu_v+\mu_w)P_{n_a,n_b,0,n_u,n_v,n_w} &= \lambda_a P_{n_a-1,n_b,0,n_u,n_v,n_w} + \lambda_b P_{n_a,n_b-1,0,n_u,n_v,n_w} \\ &+ \mu_a p_{ab} P_{n_a+1,n_b-1,0,n_u,n_v,n_w} + \mu_a p_{au} P_{n_a+1,n_b,0,n_u-1,n_v,n_w} + \mu_b p_{ba} P_{n_a-1,n_b+1,0,n_u,n_v,n_w} \\ &+ \mu_b p_{bv} P_{n_a,n_b+1,0,n_u,n_v-1,n_w} + \mu_c p_{ca} P_{n_a-1,n_b,1,n_u,n_v,n_w} + \mu_c p_{cb} P_{n_a,n_b-1,1,n_u,n_v,n_w} + \mu_c p_{cw} P_{n_a,n_b,1,n_u,n_v,n_w-1} \\ &+ \mu_u P_{n_a,n_b,0,n_u+1,n_v,n_w} + \mu_v P_{n_a,n_b,0,n_u,n_v+1,n_w} + \mu_w P_{n_a,n_b,0,n_u,n_v,n_w+1}\end{aligned} \quad (4)$$

Considering $n_u = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_c+\mu_v+\mu_w)P_{n_a,n_b,n_c,0,n_v,n_w} &= \lambda_a P_{n_a-1,n_b,n_c,0,n_v,n_w}+\lambda_b P_{n_a,n_b-1,n_c,0,n_v,n_w}\\ &+\lambda_c P_{n_a,n_b,n_c-1,0,n_v,n_w}+\mu_a p_{ab}P_{n_a+1,n_b-1,n_c,0,n_v,n_w}+\mu_a p_{ac}P_{n_a+1,n_b,n_c-1,0,n_v,n_w}+\mu_b p_{ba}P_{n_a-1,n_b+1,n_c,0,n_v,n_w}\\ &+\mu_b p_{bc}P_{n_a,n_b+1,n_c-1,0,n_v,n_w}+\mu_b p_{bv}P_{n_a,n_b+1,n_c,0,n_v-1,n_w}+\mu_c p_{ca}P_{n_a-1,n_b,n_c+1,0,n_v,n_w}\\ &+\mu_c p_{cb}P_{n_a,n_b-1,n_c+1,0,n_v,n_w}+\mu_c p_{cw}P_{n_a,n_b,n_c+1,0,n_v,n_w-1}+\mu_u P_{n_a,n_b,n_c,1,n_v,n_w}+\mu_v P_{n_a,n_b,n_c,0,n_v+1,n_w}\\ &+\mu_w P_{n_a,n_b,n_c,0,n_v,n_w+1}\end{aligned} \tag{5}$$

Considering $n_v=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_c+\mu_u+\mu_w)P_{n_a,n_b,n_c,n_u,0,n_w} &= \lambda_a P_{n_a-1,n_b,n_c,n_u,0,n_w}+\lambda_b P_{n_a,n_b-1,n_c,n_u,0,n_w}\\ &+\lambda_c P_{n_a,n_b,n_c-1,n_u,0,n_w}+\mu_a p_{ab}P_{n_a+1,n_b-1,n_c,n_u,0,n_w}+\mu_a p_{ac}P_{n_a+1,n_b,n_c-1,n_u,0,n_w}+\mu_a p_{au}P_{n_a+1,n_b,n_c,n_u-1,0,n_w}\\ &+\mu_b p_{ba}P_{n_a-1,n_b+1,n_c,n_u,0,n_w}+\mu_b p_{bc}P_{n_a,n_b+1,n_c-1,n_u,0,n_w}+\mu_c p_{ca}P_{n_a-1,n_b,n_c+1,n_u,0,n_w}\\ &+\mu_c p_{cb}P_{n_a,n_b-1,n_c+1,n_u,0,n_w}+\mu_c p_{cw}P_{n_a,n_b,n_c+1,n_u,0,n_w-1}+\mu_u P_{n_a,n_b,n_c,n_u+1,0,n_w}+\mu_v P_{n_a,n_b,n_c,n_u,1,n_w}\\ &+\mu_w P_{n_a,n_b,n_c,n_u,0,n_w+1}\end{aligned} \tag{6}$$

Considering $n_w=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_c+\mu_u+\mu_v)P_{n_a,n_b,n_c,n_u,n_v,0} &= \lambda_a P_{n_a-1,n_b,n_c,n_u,n_v,0}+\lambda_b P_{n_a,n_b-1,n_c,n_u,n_v,0}\\ &+\lambda_c P_{n_a,n_b,n_c-1,n_u,n_v,0}+\mu_a p_{ab}P_{n_a+1,n_b-1,n_c,n_u,n_v,0}+\mu_a p_{ac}P_{n_a+1,n_b,n_c-1,n_u,n_v,0}+\mu_a p_{au}P_{n_a+1,n_b,n_c,n_u-1,n_v,0}\\ &+\mu_b p_{ba}P_{n_a-1,n_b+1,n_c,n_u,n_v,0}+\mu_b p_{bc}P_{n_a,n_b+1,n_c-1,n_u,n_v,0}+\mu_b p_{bv}P_{n_a,n_b+1,n_c,n_u,n_v-1,0}\\ &+\mu_c p_{ca}P_{n_a-1,n_b,n_c+1,n_u,n_v,0}+\mu_c p_{cb}P_{n_a,n_b-1,n_c+1,n_u,n_v,0}+\mu_u P_{n_a,n_b,n_c,n_u+1,n_v,0}+\mu_v P_{n_a,n_b,n_c,n_u,n_v+1,0}\\ &+\mu_w P_{n_a,n_b,n_c,n_u,n_v,1}\end{aligned} \tag{7}$$

Considering $n_a=0,\ n_b=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_c+\mu_u+\mu_v+\mu_w)P_{0,0,n_c,n_u,n_v,n_w} &= \lambda_c P_{0,0,n_c-1,n_u,n_v,n_w}+\mu_a p_{ac}P_{1,0,n_c-1,n_u,n_v,n_w}\\ &+\mu_a p_{au}P_{1,0,n_c,n_u-1,n_v,n_w}+\mu_b p_{bc}P_{0,1,n_c-1,n_u,n_v,n_w}+\mu_b p_{bv}P_{0,1,n_c,n_u,n_v-1,n_w}+\mu_c p_{cw}P_{0,0,n_c+1,n_u,n_v,n_w-1}\\ &+\mu_u P_{0,0,n_c,n_u+1,n_v,n_w}+\mu_v P_{0,0,n_c,n_u,n_v+1,n_w}+\mu_w P_{0,0,n_c,n_u,n_v,n_w+1}\end{aligned} \tag{8}$$

Considering $n_a=0,\ n_c=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_u+\mu_v+\mu_w)P_{0,n_b,0,n_u,n_v,n_w} &= \lambda_b P_{0,n_b-1,0,n_u,n_v,n_w}+\mu_a p_{ab}P_{1,n_b-1,0,n_u,n_v,n_w}\\ &+\mu_a p_{au}P_{1,n_b,0,n_u-1,n_v,n_w}+\mu_b p_{bv}P_{0,n_b+1,0,n_u,n_v-1,n_w}+\mu_c p_{cb}P_{0,n_b-1,1,n_u,n_v,n_w}+\mu_c p_{cw}P_{0,n_b,1,n_u,n_v,n_w-1}\\ &+\mu_u P_{0,n_b,0,n_u+1,n_v,n_w}+\mu_v P_{0,n_b,0,n_u,n_v+1,n_w}+\mu_w P_{0,n_b,0,n_u,n_v,n_w+1}\end{aligned} \tag{9}$$

Considering $n_a=0,\ n_u=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_c+\mu_v+\mu_w)P_{0,n_b,n_c,0,n_v,n_w} &= \lambda_b P_{0,n_b-1,n_c,0,n_v,n_w}+\lambda_c P_{0,n_b,n_c-1,0,n_v,n_w}\\ &+\mu_a p_{ab}P_{1,n_b-1,n_c,0,n_v,n_w}+\mu_a p_{ac}P_{1,n_b,n_c-1,0,n_v,n_w}+\mu_b p_{bc}P_{0,n_b+1,n_c-1,0,n_v,n_w}+\mu_b p_{bv}P_{0,n_b+1,n_c,0,n_v-1,n_w}\\ &+\mu_c p_{cb}P_{0,n_b-1,n_c+1,0,n_v,n_w}+\mu_c p_{cw}P_{0,n_b,n_c+1,0,n_v,n_w-1}+\mu_u P_{0,n_b,n_c,1,n_v,n_w}+\mu_v P_{0,n_b,n_c,0,n_v+1,n_w}\\ &+\mu_w P_{0,n_b,n_c,0,n_v,n_w+1}\end{aligned} \tag{10}$$

Considering $n_a=0,\ n_v=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_c+\mu_u+\mu_w)P_{0,n_b,n_c,n_u,0,n_w} &= \lambda_b P_{0,n_b-1,n_c,n_u,0,n_w}+\lambda_c P_{0,n_b,n_c-1,n_u,0,n_w}\\ &+\mu_a p_{ab}P_{1,n_b-1,n_c,n_u,0,n_w}+\mu_a p_{ac}P_{1,n_b,n_c-1,n_u,0,n_w}+\mu_a p_{au}P_{1,n_b,n_c,n_u-1,0,n_w}+\mu_b p_{bc}P_{0,n_b+1,n_c-1,n_u,0,n_w}\\ &+\mu_c p_{cb}P_{0,n_b-1,n_c+1,n_u,0,n_w}+\mu_c p_{cw}P_{0,n_b,n_c+1,n_u,0,n_w-1}+\mu_u P_{0,n_b,n_c,n_u+1,0,n_w}+\mu_v P_{0,n_b,n_c,n_u,1,n_w}\\ &+\mu_w P_{0,n_b,n_c,n_u,0,n_w+1}\end{aligned} \tag{11}$$

Considering $n_a = 0,\ n_w = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_c+\mu_u+\mu_v)P_{0,n_b,n_c,n_u,n_v,0} &= \lambda_b P_{0,n_b-1,n_c,n_u,n_v,0}+\lambda_c P_{0,n_b,n_c-1,n_u,n_v,0}\\ &+\mu_a p_{ab}P_{1,n_b-1,n_c,n_u,n_v,0}+\mu_a p_{ac}P_{1,n_b,n_c-1,n_u,n_v,0}+\mu_a p_{au}P_{1,n_b,n_c,n_u-1,n_v,0}+\mu_b p_{bc}P_{0,n_b+1,n_c-1,n_u,n_v,0}\\ &+\mu_b p_{bv}P_{0,n_b+1,n_c,n_u,n_v-1,0}+\mu_c p_{cb}P_{0,n_b-1,n_c+1,n_u,n_v,0}+\mu_u P_{0,n_b,n_c,n_u+1,n_v,0}+\mu_v P_{0,n_b,n_c,n_u,n_v+1,0}\\ &+\mu_w P_{0,n_b,n_c,n_u,n_v,1}\end{aligned} \tag{12}$$

Considering $n_b = 0,\ n_c = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_u+\mu_v+\mu_w)P_{n_a,0,0,n_u,n_v,n_w} &= \lambda_a P_{n_a-1,0,0,n_u,n_v,n_w}+\mu_a p_{au}P_{n_a+1,0,0,n_u-1,n_v,n_w}\\ &+\mu_b p_{ba}P_{n_a-1,1,0,n_u,n_v,n_w}+\mu_b p_{bv}P_{n_a,1,0,n_u,n_v-1,n_w}+\mu_c p_{ca}P_{n_a-1,0,1,n_u,n_v,n_w}+\mu_c p_{cw}P_{n_a,0,1,n_u,n_v,n_w-1}\\ &+\mu_u P_{n_a,0,0,n_u+1,n_v,n_w}+\mu_v P_{n_a,0,0,n_u,n_v+1,n_w}+\mu_w P_{n_a,0,0,n_u,n_v,n_w+1}\end{aligned} \tag{13}$$

Considering $n_b = 0,\ n_u = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_c+\mu_v+\mu_w)P_{n_a,0,n_c,0,n_v,n_w} &= \lambda_a P_{n_a-1,0,n_c,0,n_v,n_w}+\lambda_c P_{n_a,0,n_c-1,0,n_v,n_w}\\ &+\mu_a p_{ac}P_{n_a+1,0,n_c-1,0,n_v,n_w}+\mu_b p_{ba}P_{n_a-1,1,n_c,0,n_v,n_w}+\mu_b p_{bc}P_{n_a,1,n_c-1,0,n_v,n_w}+\mu_b p_{bv}P_{n_a,1,n_c,0,n_v-1,n_w}\\ &+\mu_c p_{ca}P_{n_a-1,0,n_c+1,0,n_v,n_w}+\mu_c p_{cw}P_{n_a,0,n_c+1,0,n_v,n_w-1}+\mu_u P_{n_a,0,n_c,1,n_v,n_w}+\mu_v P_{n_a,0,n_c,0,n_v+1,n_w}\\ &+\mu_w P_{n_a,0,n_c,0,n_v,n_w+1}\end{aligned} \tag{14}$$

Considering $n_b = 0,\ n_v = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_c+\mu_u+\mu_w)P_{n_a,0,n_c,n_u,0,n_w} &= \lambda_a P_{n_a-1,0,n_c,n_u,0,n_w}+\lambda_c P_{n_a,0,n_c-1,n_u,0,n_w}\\ &+\mu_a p_{ac}P_{n_a+1,0,n_c-1,n_u,0,n_w}+\mu_a p_{au}P_{n_a+1,0,n_c,n_u-1,0,n_w}+\mu_b p_{ba}P_{n_a-1,1,n_c,n_u,0,n_w}+\mu_b p_{bc}P_{n_a,1,n_c-1,n_u,0,n_w}\\ &+\mu_c p_{ca}P_{n_a-1,0,n_c+1,n_u,0,n_w}+\mu_c p_{cw}P_{n_a,0,n_c+1,n_u,0,n_w-1}+\mu_u P_{n_a,0,n_c,n_u+1,0,n_w}+\mu_v P_{n_a,0,n_c,n_u,1,n_w}\\ &+\mu_w P_{n_a,0,n_c,n_u,0,n_w+1}\end{aligned} \tag{15}$$

Considering $n_b = 0,\ n_w = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_c+\mu_u+\mu_v)P_{n_a,0,n_c,n_u,n_v,0} &= \lambda_a P_{n_a-1,0,n_c,n_u,n_v,0}+\lambda_c P_{n_a,0,n_c-1,n_u,n_v,0}\\ &+\mu_a p_{ac}P_{n_a+1,0,n_c-1,n_u,n_v,0}+\mu_a p_{au}P_{n_a+1,0,n_c,n_u-1,n_v,0}+\mu_b p_{ba}P_{n_a-1,1,n_c,n_u,n_v,0}+\mu_b p_{bc}P_{n_a,1,n_c-1,n_u,n_v,0}\\ &+\mu_b p_{bv}P_{n_a,1,n_c,n_u,n_v-1,0}+\mu_c p_{ca}P_{n_a-1,0,n_c+1,n_u,n_v,0}+\mu_u P_{n_a,0,n_c,n_u+1,n_v,0}+\mu_v P_{n_a,0,n_c,n_u,n_v+1,0}\\ &+\mu_w P_{n_a,0,n_c,n_u,n_v,1}\end{aligned} \tag{16}$$

Considering $n_c = 0,\ n_u = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_v+\mu_w)P_{n_a,n_b,0,0,n_v,n_w} &= \lambda_a P_{n_a-1,n_b,0,0,n_v,n_w}+\lambda_b P_{n_a,n_b-1,0,0,n_v,n_w}\\ &+\mu_a p_{ab}P_{n_a+1,n_b-1,0,0,n_v,n_w}+\mu_b p_{ba}P_{n_a-1,n_b+1,0,0,n_v,n_w}+\mu_b p_{bv}P_{n_a,n_b+1,0,0,n_v-1,n_w}+\mu_c p_{ca}P_{n_a-1,n_b,1,0,n_v,n_w}\\ &+\mu_c p_{cb}P_{n_a,n_b-1,1,0,n_v,n_w}+\mu_c p_{cw}P_{n_a,n_b,1,0,n_v,n_w-1}+\mu_u P_{n_a,n_b,0,1,n_v,n_w}+\mu_v P_{n_a,n_b,0,0,n_v+1,n_w}\\ &+\mu_w P_{n_a,n_b,0,0,n_v,n_w+1}\end{aligned} \tag{17}$$

Considering $n_c = 0,\ n_v = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_u+\mu_w)P_{n_a,n_b,0,n_u,0,n_w} &= \lambda_a P_{n_a-1,n_b,0,n_u,0,n_w}+\lambda_b P_{n_a,n_b-1,0,n_u,0,n_w}\\ &+\mu_a p_{ab}P_{n_a+1,n_b-1,0,n_u,0,n_w}+\mu_a p_{au}P_{n_a+1,n_b,0,n_u-1,0,n_w}+\mu_b p_{ba}P_{n_a-1,n_b+1,0,n_u,0,n_w}+\mu_c p_{ca}P_{n_a-1,n_b,1,n_u,0,n_w}\\ &+\mu_c p_{cb}P_{n_a,n_b-1,1,n_u,0,n_w}+\mu_c p_{cw}P_{n_a,n_b,1,n_u,0,n_w-1}+\mu_u P_{n_a,n_b,0,n_u+1,0,n_w}+\mu_v P_{n_a,n_b,0,n_u,1,n_w}\\ &+\mu_w P_{n_a,n_b,0,n_u,0,n_w+1}\end{aligned} \tag{18}$$

Considering $n_c = 0,\ n_w = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_u+\mu_v)P_{n_a,n_b,0,n_u,n_v,0} &= \lambda_a P_{n_a-1,n_b,0,n_u,n_v,0} + \lambda_b P_{n_a,n_b-1,0,n_u,n_v,0} \\ &+ \mu_a p_{ab} P_{n_a+1,n_b-1,0,n_u,n_v,0} + \mu_a p_{au} P_{n_a+1,n_b,0,n_u-1,n_v,0} + \mu_b p_{ba} P_{n_a-1,n_b+1,0,n_u,n_v,0} + \mu_b p_{bv} P_{n_a,n_b+1,0,n_u,n_v-1,0} \\ &+ \mu_c p_{ca} P_{n_a-1,n_b,1,n_u,n_v,0} + \mu_c p_{cb} P_{n_a,n_b-1,1,n_u,n_v,0} + \mu_u P_{n_a,n_b,0,n_u+1,n_v,0} + \mu_v P_{n_a,n_b,0,n_u,n_v+1,0} \\ &+ \mu_w P_{n_a,n_b,0,n_u,n_v,1}\end{aligned} \tag{19}$$

Considering $n_u = 0,\ n_v = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_c+\mu_w)P_{n_a,n_b,n_c,0,0,n_w} &= \lambda_a P_{n_a-1,n_b,n_c,0,0,n_w} + \lambda_b P_{n_a,n_b-1,n_c,0,0,n_w} \\ &+ \lambda_c P_{n_a,n_b,n_c-1,0,0,n_w} + \mu_a p_{ab} P_{n_a+1,n_b-1,n_c,0,0,n_w} + \mu_a p_{ac} P_{n_a+1,n_b,n_c-1,0,0,n_w} + \mu_b p_{ba} P_{n_a-1,n_b+1,n_c,0,0,n_w} \\ &+ \mu_b p_{bc} P_{n_a,n_b+1,n_c-1,0,0,n_w} + \mu_c p_{ca} P_{n_a-1,n_b,n_c+1,0,0,n_w} + \mu_c p_{cb} P_{n_a,n_b-1,n_c+1,0,0,n_w} \\ &+ \mu_c p_{cw} P_{n_a,n_b,n_c+1,0,0,n_w-1} + \mu_u P_{n_a,n_b,n_c,1,0,n_w} + \mu_v P_{n_a,n_b,n_c,0,1,n_w} + \mu_w P_{n_a,n_b,n_c,0,0,n_w+1}\end{aligned} \tag{20}$$

Considering $n_u = 0,\ n_w = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_c+\mu_v)P_{n_a,n_b,n_c,0,n_v,0} &= \lambda_a P_{n_a-1,n_b,n_c,0,n_v,0} + \lambda_b P_{n_a,n_b-1,n_c,0,n_v,0} \\ &+ \lambda_c P_{n_a,n_b,n_c-1,0,n_v,0} + \mu_a p_{ab} P_{n_a+1,n_b-1,n_c,0,n_v,0} + \mu_a p_{ac} P_{n_a+1,n_b,n_c-1,0,n_v,0} + \mu_b p_{ba} P_{n_a-1,n_b+1,n_c,0,n_v,0} \\ &+ \mu_b p_{bc} P_{n_a,n_b+1,n_c-1,0,n_v,0} + \mu_b p_{bv} P_{n_a,n_b+1,n_c,0,n_v-1,0} + \mu_c p_{ca} P_{n_a-1,n_b,n_c+1,0,n_v,0} \\ &+ \mu_c p_{cb} P_{n_a,n_b-1,n_c+1,0,n_v,0} + \mu_u P_{n_a,n_b,n_c,1,n_v,0} + \mu_v P_{n_a,n_b,n_c,0,n_v+1,0} + \mu_w P_{n_a,n_b,n_c,0,n_v,1}\end{aligned} \tag{21}$$

Considering $n_v = 0,\ n_w = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_c+\mu_u)P_{n_a,n_b,n_c,n_u,0,0} &= \lambda_a P_{n_a-1,n_b,n_c,n_u,0,0} + \lambda_b P_{n_a,n_b-1,n_c,n_u,0,0} \\ &+ \lambda_c P_{n_a,n_b,n_c-1,n_u,0,0} + \mu_a p_{ab} P_{n_a+1,n_b-1,n_c,n_u,0,0} + \mu_a p_{ac} P_{n_a+1,n_b,n_c-1,n_u,0,0} + \mu_a p_{au} P_{n_a+1,n_b,n_c,n_u-1,0,0} \\ &+ \mu_b p_{ba} P_{n_a-1,n_b+1,n_c,n_u,0,0} + \mu_b p_{bc} P_{n_a,n_b+1,n_c-1,n_u,0,0} + \mu_c p_{ca} P_{n_a-1,n_b,n_c+1,n_u,0,0} \\ &+ \mu_c p_{cb} P_{n_a,n_b-1,n_c+1,n_u,0,0} + \mu_u P_{n_a,n_b,n_c,n_u+1,0,0} + \mu_v P_{n_a,n_b,n_c,n_u,1,0} + \mu_w P_{n_a,n_b,n_c,n_u,0,1}\end{aligned} \tag{22}$$

Considering $n_a = 0,\ n_b = 0,\ n_c = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_u+\mu_v+\mu_w)P_{0,0,0,n_u,n_v,n_w} &= \mu_a p_{au} P_{1,0,0,n_u-1,n_v,n_w} + \mu_b p_{bv} P_{0,1,0,n_u,n_v-1,n_w} \\ &+ \mu_c p_{cw} P_{0,0,1,n_u,n_v,n_w-1} + \mu_u P_{0,0,0,n_u+1,n_v,n_w} + \mu_v P_{0,0,0,n_u,n_v+1,n_w} + \mu_w P_{0,0,0,n_u,n_v,n_w+1}\end{aligned} \tag{23}$$

Considering $n_a = 0,\ n_b = 0,\ n_u = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_c+\mu_v+\mu_w)P_{0,0,n_c,0,n_v,n_w} &= \lambda_c P_{0,0,n_c-1,0,n_v,n_w} + \mu_a p_{ac} P_{1,0,n_c-1,0,n_v,n_w} \\ &+ \mu_b p_{bc} P_{0,1,n_c-1,0,n_v,n_w} + \mu_b p_{bv} P_{0,1,n_c,0,n_v-1,n_w} + \mu_c p_{cw} P_{0,0,n_c+1,0,n_v,n_w-1} + \mu_u P_{0,0,n_c,1,n_v,n_w} \\ &+ \mu_v P_{0,0,n_c,0,n_v+1,n_w} + \mu_w P_{0,0,n_c,0,n_v,n_w+1}\end{aligned} \tag{24}$$

Considering $n_a = 0,\ n_b = 0,\ n_v = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_c+\mu_u+\mu_w)P_{0,0,n_c,n_u,0,n_w} &= \lambda_c P_{0,0,n_c-1,n_u,0,n_w} + \mu_a p_{ac} P_{1,0,n_c-1,n_u,0,n_w} \\ &+ \mu_a p_{au} P_{1,0,n_c,n_u-1,0,n_w} + \mu_b p_{bc} P_{0,1,n_c-1,n_u,0,n_w} + \mu_c p_{cw} P_{0,0,n_c+1,n_u,0,n_w-1} + \mu_u P_{0,0,n_c,n_u+1,0,n_w} \\ &+ \mu_v P_{0,0,n_c,n_u,1,n_w} + \mu_w P_{0,0,n_c,n_u,0,n_w+1}\end{aligned} \tag{25}$$

Considering $n_a = 0,\ n_b = 0,\ n_w = 0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_c+\mu_u+\mu_v)P_{0,0,n_c,n_u,n_v,0} &= \lambda_c P_{0,0,n_c-1,n_u,n_v,0}+\mu_a p_{ac}P_{1,0,n_c-1,n_u,n_v,0}\\ &+\mu_a p_{au}P_{1,0,n_c,n_u-1,n_v,0}+\mu_b p_{bc}P_{0,1,n_c-1,n_u,n_v,0}+\mu_b p_{bv}P_{0,1,n_c,n_u,n_v-1,0}+\mu_u P_{0,0,n_c,n_u+1,n_v,0}\\ &+\mu_v P_{0,0,n_c,n_u,n_v+1,0}+\mu_w P_{0,0,n_c,n_u,n_v,1}\end{aligned} \tag{26}$$

Considering $n_a=0,\ n_c=0,\ n_u=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_v+\mu_w)P_{0,n_b,0,0,n_v,n_w} &= \lambda_b P_{0,n_b-1,0,0,n_v,n_w}+\mu_a p_{ab}P_{1,n_b-1,0,0,n_v,n_w}\\ &+\mu_b p_{bv}P_{0,n_b+1,0,0,n_v-1,n_w}+\mu_c p_{cb}P_{0,n_b-1,1,0,n_v,n_w}+\mu_c p_{cw}P_{0,n_b,1,0,n_v,n_w-1}+\mu_u P_{0,n_b,0,1,n_v,n_w}\\ &+\mu_v P_{0,n_b,0,0,n_v+1,n_w}+\mu_w P_{0,n_b,0,0,n_v,n_w+1}\end{aligned} \tag{27}$$

Considering $n_a=0,\ n_c=0,\ n_v=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_u+\mu_w)P_{0,n_b,0,n_u,0,n_w} &= \lambda_b P_{0,n_b-1,0,n_u,0,n_w}+\mu_a p_{ab}P_{1,n_b-1,0,n_u,0,n_w}\\ &+\mu_a p_{au}P_{1,n_b,0,n_u-1,0,n_w}+\mu_c p_{cb}P_{0,n_b-1,1,n_u,0,n_w}+\mu_c p_{cw}P_{0,n_b,1,n_u,0,n_w-1}+\mu_u P_{0,n_b,0,n_u+1,0,n_w}\\ &+\mu_v P_{0,n_b,0,n_u,1,n_w}+\mu_w P_{0,n_b,0,n_u,0,n_w+1}\end{aligned} \tag{28}$$

Considering $n_a=0,\ n_c=0,\ n_w=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_u+\mu_v)P_{0,n_b,0,n_u,n_v,0} &= \lambda_b P_{0,n_b-1,0,n_u,n_v,0}+\mu_a p_{ab}P_{1,n_b-1,0,n_u,n_v,0}\\ &+\mu_a p_{au}P_{1,n_b,0,n_u-1,n_v,0}+\mu_b p_{bv}P_{0,n_b+1,0,n_u,n_v-1,0}+\mu_c p_{cb}P_{0,n_b-1,1,n_u,n_v,0}+\mu_u P_{0,n_b,0,n_u+1,n_v,0}\\ &+\mu_v P_{0,n_b,0,n_u,n_v+1,0}+\mu_w P_{0,n_b,0,n_u,n_v,1}\end{aligned} \tag{29}$$

Considering $n_a=0,\ n_u=0,\ n_v=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_c+\mu_w)P_{0,n_b,n_c,0,0,n_w} &= \lambda_b P_{0,n_b-1,n_c,0,0,n_w}+\lambda_c P_{0,n_b,n_c-1,0,0,n_w}+\mu_a p_{ab}P_{1,n_b-1,n_c,0,0,n_w}\\ &+\mu_a p_{ac}P_{1,n_b,n_c-1,0,0,n_w}+\mu_b p_{bc}P_{0,n_b+1,n_c-1,0,0,n_w}+\mu_c p_{cb}P_{0,n_b-1,n_c+1,0,0,n_w}+\mu_c p_{cw}P_{0,n_b,n_c+1,0,0,n_w-1}\\ &+\mu_u P_{0,n_b,n_c,1,0,n_w}+\mu_v P_{0,n_b,n_c,0,1,n_w}+\mu_w P_{0,n_b,n_c,0,0,n_w+1}\end{aligned} \tag{30}$$

Considering $n_a=0,\ n_u=0,\ n_w=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_c+\mu_v)P_{0,n_b,n_c,0,n_v,0} &= \lambda_b P_{0,n_b-1,n_c,0,n_v,0}+\lambda_c P_{0,n_b,n_c-1,0,n_v,0}+\mu_a p_{ab}P_{1,n_b-1,n_c,0,n_v,0}\\ &+\mu_a p_{ac}P_{1,n_b,n_c-1,0,n_v,0}+\mu_b p_{bc}P_{0,n_b+1,n_c-1,0,n_v,0}+\mu_b p_{bv}P_{0,n_b+1,n_c,0,n_v-1,0}+\mu_c p_{cb}P_{0,n_b-1,n_c+1,0,n_v,0}\\ &+\mu_u P_{0,n_b,n_c,1,n_v,0}+\mu_v P_{0,n_b,n_c,0,n_v+1,0}+\mu_w P_{0,n_b,n_c,0,n_v,1}\end{aligned} \tag{31}$$

Considering $n_a=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_c+\mu_u)P_{0,n_b,n_c,n_u,0,0} &= \lambda_b P_{0,n_b-1,n_c,n_u,0,0}+\lambda_c P_{0,n_b,n_c-1,n_u,0,0}+\mu_a p_{ab}P_{1,n_b-1,n_c,n_u,0,0}\\ &+\mu_a p_{ac}P_{1,n_b,n_c-1,n_u,0,0}+\mu_a p_{au}P_{1,n_b,n_c,n_u-1,0,0}+\mu_b p_{bc}P_{0,n_b+1,n_c-1,n_u,0,0}+\mu_c p_{cb}P_{0,n_b-1,n_c+1,n_u,0,0}\\ &+\mu_u P_{0,n_b,n_c,n_u+1,0,0}+\mu_v P_{0,n_b,n_c,n_u,1,0}+\mu_w P_{0,n_b,n_c,n_u,0,1}\end{aligned} \tag{32}$$

Considering $n_b=0,\ n_c=0,\ n_u=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_v+\mu_w)P_{n_a,0,0,0,n_v,n_w} &= \lambda_a P_{n_a-1,0,0,0,n_v,n_w}+\mu_b p_{ba}P_{n_a-1,1,0,0,n_v,n_w}\\ &+\mu_b p_{bv}P_{n_a,1,0,0,n_v-1,n_w}+\mu_c p_{ca}P_{n_a-1,0,1,0,n_v,n_w}+\mu_c p_{cw}P_{n_a,0,1,0,n_v,n_w-1}+\mu_u P_{n_a,0,0,1,n_v,n_w}\\ &+\mu_v P_{n_a,0,0,0,n_v+1,n_w}+\mu_w P_{n_a,0,0,0,n_v,n_w+1}\end{aligned} \tag{33}$$

Considering $n_b=0,\ n_c=0,\ n_v=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_u+\mu_w)P_{n_a,0,0,n_u,0,n_w}=\lambda_a P_{n_a-1,0,0,n_u,0,n_w}+\mu_a p_{au}P_{n_a+1,0,0,n_u-1,0,n_w}$$
$$+\mu_b p_{ba}P_{n_a-1,1,0,n_u,0,n_w}+\mu_c p_{ca}P_{n_a-1,0,1,n_u,0,n_w}+\mu_c p_{cw}P_{n_a,0,1,n_u,0,n_w-1}+\mu_u P_{n_a,0,0,n_u+1,0,n_w} \quad (34)$$
$$+\mu_v P_{n_a,0,0,n_u,1,n_w}+\mu_w P_{n_a,0,0,n_u,0,n_w+1}$$

Considering $n_b=0,\ n_c=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_u+\mu_v)P_{n_a,0,0,n_u,n_v,0}=\lambda_a P_{n_a-1,0,0,n_u,n_v,0}+\mu_a p_{au}P_{n_a+1,0,0,n_u-1,n_v,0}$$
$$+\mu_b p_{ba}P_{n_a-1,1,0,n_u,n_v,0}+\mu_b p_{bv}P_{n_a,1,0,n_u,n_v-1,0}+\mu_c p_{ca}P_{n_a-1,0,1,n_u,n_v,0}+\mu_u P_{n_a,0,0,n_u+1,n_v,0} \quad (35)$$
$$+\mu_v P_{n_a,0,0,n_u,n_v+1,0}+\mu_w P_{n_a,0,0,n_u,n_v,1}$$

Considering $n_b=0,\ n_u=0,\ n_v=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_c+\mu_w)P_{n_a,0,n_c,0,0,n_w}=\lambda_a P_{n_a-1,0,n_c,0,0,n_w}+\lambda_c P_{n_a,0,n_c-1,0,0,n_w}$$
$$+\mu_a p_{ac}P_{n_a+1,0,n_c-1,0,0,n_w}+\mu_b p_{ba}P_{n_a-1,1,n_c,0,0,n_w}+\mu_b p_{bc}P_{n_a,1,n_c-1,0,0,n_w}+\mu_c p_{ca}P_{n_a-1,0,n_c+1,0,0,n_w} \quad (36)$$
$$+\mu_c p_{cw}P_{n_a,0,n_c+1,0,0,n_w-1}+\mu_u P_{n_a,0,n_c,1,0,n_w}+\mu_v P_{n_a,0,n_c,0,1,n_w}+\mu_w P_{n_a,0,n_c,0,0,n_w+1}$$

Considering $n_b=0,\ n_u=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_c+\mu_v)P_{n_a,0,n_c,0,n_v,0}=\lambda_a P_{n_a-1,0,n_c,0,n_v,0}+\lambda_c P_{n_a,0,n_c-1,0,n_v,0}+\mu_a p_{ac}P_{n_a+1,0,n_c-1,0,n_v,0}$$
$$+\mu_b p_{ba}P_{n_a-1,1,n_c,0,n_v,0}+\mu_b p_{bc}P_{n_a,1,n_c-1,0,n_v,0}+\mu_b p_{bv}P_{n_a,1,n_c,0,n_v-1,0}+\mu_c p_{ca}P_{n_a-1,0,n_c+1,0,n_v,0} \quad (37)$$
$$+\mu_u P_{n_a,0,n_c,1,n_v,0}+\mu_v P_{n_a,0,n_c,0,n_v+1,0}+\mu_w P_{n_a,0,n_c,0,n_v,1}$$

Considering $n_b=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_c+\mu_u)P_{n_a,0,n_c,n_u,0,0}=\lambda_a P_{n_a-1,0,n_c,n_u,0,0}+\lambda_c P_{n_a,0,n_c-1,n_u,0,0}$$
$$+\mu_a p_{ac}P_{n_a+1,0,n_c-1,n_u,0,0}+\mu_a p_{au}P_{n_a+1,0,n_c,n_u-1,0,0}+\mu_b p_{ba}P_{n_a-1,1,n_c,n_u,0,0}+\mu_b p_{bc}P_{n_a,1,n_c-1,n_u,0,0} \quad (38)$$
$$+\mu_c p_{ca}P_{n_a-1,0,n_c+1,n_u,0,0}+\mu_u P_{n_a,0,n_c,n_u+1,0,0}+\mu_v P_{n_a,0,n_c,n_u,1,0}+\mu_w P_{n_a,0,n_c,n_u,0,1}$$

Considering $n_c=0,\ n_u=0,\ n_v=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_w)P_{n_a,n_b,0,0,0,n_w}=\lambda_a P_{n_a-1,n_b,0,0,0,n_w}+\lambda_b P_{n_a,n_b-1,0,0,0,n_w}$$
$$+\mu_a p_{ab}P_{n_a+1,n_b-1,0,0,0,n_w}+\mu_b p_{ba}P_{n_a-1,n_b+1,0,0,0,n_w}+\mu_c p_{ca}P_{n_a-1,n_b,1,0,0,n_w}+\mu_c p_{cb}P_{n_a,n_b-1,1,0,0,n_w} \quad (39)$$
$$+\mu_c p_{cw}P_{n_a,n_b,1,0,0,n_w-1}+\mu_u P_{n_a,n_b,0,1,0,n_w}+\mu_v P_{n_a,n_b,0,0,1,n_w}+\mu_w P_{n_a,n_b,0,0,0,n_w+1}$$

Considering $n_c=0,\ n_u=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_v)P_{n_a,n_b,0,0,n_v,0}=\lambda_a P_{n_a-1,n_b,0,0,n_v,0}+\lambda_b P_{n_a,n_b-1,0,0,n_v,0}$$
$$+\mu_a p_{ab}P_{n_a+1,n_b-1,0,0,n_v,0}+\mu_b p_{ba}P_{n_a-1,n_b+1,0,0,n_v,0}+\mu_b p_{bv}P_{n_a,n_b+1,0,0,n_v-1,0} \quad (40)$$
$$+\mu_c p_{ca}P_{n_a-1,n_b,1,0,n_v,0}+\mu_c p_{cb}P_{n_a,n_b-1,1,0,n_v,0}+\mu_u P_{n_a,n_b,0,1,n_v,0}+\mu_v P_{n_a,n_b,0,0,n_v+1,0}+\mu_w P_{n_a,n_b,0,0,n_v,1}$$

Considering $n_c=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_u)P_{n_a,n_b,0,n_u,0,0}=\lambda_a P_{n_a-1,n_b,0,n_u,0,0}+\lambda_b P_{n_a,n_b-1,0,n_u,0,0}$$
$$+\mu_a p_{ab}P_{n_a+1,n_b-1,0,n_u,0,0}+\mu_a p_{au}P_{n_a+1,n_b,0,n_u-1,0,0}+\mu_b p_{ba}P_{n_a-1,n_b+1,0,n_u,0,0}+\mu_c p_{ca}P_{n_a-1,n_b,1,n_u,0,0} \quad (41)$$
$$+\mu_c p_{cb}P_{n_a,n_b-1,1,n_u,0,0}+\mu_u P_{n_a,n_b,0,n_u+1,0,0}+\mu_v P_{n_a,n_b,0,n_u,1,0}+\mu_w P_{n_a,n_b,0,n_u,0,1}$$

Considering $n_u=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_c)P_{n_a,n_b,n_c,0,0,0}=\lambda_a P_{n_a-1,n_b,n_c,0,0,0}+\lambda_b P_{n_a,n_b-1,n_c,0,0,0}+\lambda_c P_{n_a,n_b,n_c-1,0,0,0}$$
$$+\mu_a p_{ab}P_{n_a+1,n_b-1,n_c,0,0,0}+\mu_a p_{ac}P_{n_a+1,n_b,n_c-1,0,0,0}+\mu_b p_{ba}P_{n_a-1,n_b+1,n_c,0,0,0}+\mu_b p_{bc}P_{n_a,n_b+1,n_c-1,0,0,0} \quad (42)$$
$$+\mu_c p_{ca}P_{n_a-1,n_b,n_c+1,0,0,0}+\mu_c p_{cb}P_{n_a,n_b-1,n_c+1,0,0,0}+\mu_u P_{n_a,n_b,n_c,1,0,0}+\mu_v P_{n_a,n_b,n_c,0,1,0}+\mu_w P_{n_a,n_b,n_c,0,0,1}$$

Considering $n_a=0,\ n_b=0,\ n_c=0,\ n_u=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_v+\mu_w)P_{0,0,0,0,n_v,n_w}=\mu_b p_{bv}P_{0,1,0,0,n_v-1,n_w}+\mu_c p_{cw}P_{0,0,1,0,n_v,n_w-1}+\mu_u P_{0,0,0,1,n_v,n_w}$$
$$+\mu_v P_{0,0,0,0,n_v+1,n_w}+\mu_w P_{0,0,0,0,n_v,n_w+1} \quad (43)$$

Considering $n_a=0,\ n_b=0,\ n_c=0,\ n_v=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_u+\mu_w)P_{0,0,0,n_u,0,n_w}=\mu_a p_{au}P_{1,0,0,n_u-1,0,n_w}+\mu_c p_{cw}P_{0,0,1,n_u,0,n_w-1}+\mu_u P_{0,0,0,n_u+1,0,n_w}$$
$$+\mu_v P_{0,0,0,n_u,1,n_w}+\mu_w P_{0,0,0,n_u,0,n_w+1} \quad (44)$$

Considering $n_a=0,\ n_b=0,\ n_c=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_u+\mu_v)P_{0,0,0,n_u,n_v,0}=\mu_a p_{au}P_{1,0,0,n_u-1,n_v,0}+\mu_b p_{bv}P_{0,1,0,n_u,n_v-1,0}+\mu_u P_{0,0,0,n_u+1,n_v,0}$$
$$+\mu_v P_{0,0,0,n_u,n_v+1,0}+\mu_w P_{0,0,0,n_u,n_v,1} \quad (45)$$

Considering $n_a=0,\ n_b=0,\ n_u=0,\ n_v=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_c+\mu_w)P_{0,0,n_c,0,0,n_w}=\lambda_c P_{0,0,n_c-1,0,0,n_w}+\mu_a p_{ac}P_{1,0,n_c-1,0,0,n_w}+\mu_b p_{bc}P_{0,1,n_c-1,0,0,n_w}$$
$$+\mu_c p_{cw}P_{0,0,n_c+1,0,0,n_w-1}+\mu_u P_{0,0,n_c,1,0,n_w}+\mu_v P_{0,0,n_c,0,1,n_w}+\mu_w P_{0,0,n_c,0,0,n_w+1} \quad (46)$$

Considering $n_a=0,\ n_b=0,\ n_u=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_c+\mu_v)P_{0,0,n_c,0,n_v,0}=\lambda_c P_{0,0,n_c-1,0,n_v,0}+\mu_a p_{ac}P_{1,0,n_c-1,0,n_v,0}+\mu_b p_{bc}P_{0,1,n_c-1,0,n_v,0}$$
$$+\mu_b p_{bv}P_{0,1,n_c,0,n_v-1,.0}+\mu_u P_{0,0,n_c,1,n_v,0}+\mu_v P_{0,0,n_c,0,n_v+1,0}+\mu_w P_{0,0,n_c,0,n_v,1} \quad (47)$$

Considering $n_a=0,\ n_b=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_c+\mu_u)P_{0,0,n_c,n_u,0,0}=\lambda_c P_{0,0,n_c-1,n_u,0,0}+\mu_a p_{ac}P_{1,0,n_c-1,n_u,0,0}+\mu_a p_{au}P_{1,0,n_c,n_u-1,0,0}$$
$$+\mu_b p_{bc}P_{0,1,n_c-1,n_u,0,0}+\mu_u P_{0,0,n_c,n_u+1,0,0}+\mu_v P_{0,0,n_c,n_u,1,0}+\mu_w P_{0,0,n_c,n_u,0,1} \quad (48)$$

Considering $n_a=0,\ n_c=0,\ n_u=0,\ n_v=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_w)P_{0,n_b,0,0,0,n_w}=\lambda_b P_{0,n_b-1,0,0,0,n_w}+\mu_a p_{ab}P_{1,n_b-1,0,0,0,n_w}+\mu_c p_{cb}P_{0,n_b-1,1,0,0,n_w}$$
$$+\mu_c p_{cw}P_{0,n_b,1,0,0,n_w-1}+\mu_u P_{0,n_b,0,1,0,n_w}+\mu_v P_{0,n_b,0,0,1,n_w}+\mu_w P_{0,n_b,0,0,0,n_w+1} \quad (49)$$

Considering $n_a=0,\ n_c=0,\ n_u=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_v)P_{0,n_b,0,0,n_v,0}=\lambda_b P_{0,n_b-1,0,0,n_v,0}+\mu_a p_{ab}P_{1,n_b-1,0,0,n_v,0}+\mu_b p_{bv}P_{0,n_b+1,0,0,n_v-1,0}$$
$$+\mu_c p_{cb}P_{0,n_b-1,1,0,n_v,0}+\mu_u P_{0,n_b,0,1,n_v,0}+\mu_v P_{0,n_b,0,0,n_v+1,0}+\mu_w P_{0,n_b,0,0,n_v,1} \quad (50)$$

Considering $n_a=0,\ n_c=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_u)P_{0,n_b,0,n_u,0,0}=\lambda_b P_{0,n_b-1,0,n_u,0,0}+\mu_a p_{ab}P_{1,n_b-1,0,n_u,0,0}+\mu_a p_{au}P_{1,n_b,0,n_u-1,0,0}$$
$$+\mu_c p_{cb}P_{0,n_b-1,1,n_u,0,0}+\mu_u P_{0,n_b,0,n_u+1,0,0}+\mu_v P_{0,n_b,0,n_u,1,0}+\mu_w P_{0,n_b,0,n_u,0,1} \quad (51)$$

Considering $n_a=0,\ n_u=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_b+\mu_c)P_{0,n_b,n_c,0,0,0} &= \lambda_b P_{0,n_b-1,n_c,0,0,0} + \lambda_c P_{0,n_b,n_c-1,0,0,0} + \mu_a p_{ab} P_{1,n_b-1,n_c,0,0,0} \\ &+ \mu_a p_{ac} P_{1,n_b,n_c-1,0,0,0} + \mu_b p_{bc} P_{0,n_b+1,n_c-1,0,0,0} + \mu_c p_{cb} P_{0,n_b-1,n_c+1,0,0,0} + \mu_u P_{0,n_b,n_c,1,0,0} \\ &+ \mu_v P_{0,n_b,n_c,0,1,0} + \mu_w P_{0,n_b,n_c,0,0,1}\end{aligned} \tag{52}$$

Considering $n_b=0,\ n_c=0,\ n_u=0,\ n_v=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_w)P_{n_a,0,0,0,0,n_w} &= \lambda_a P_{n_a-1,0,0,0,0,n_w} + \mu_b p_{ba} P_{n_a-1,1,0,0,0,n_w} + \mu_c p_{ca} P_{n_a-1,0,1,0,0,n_w} \\ &+ \mu_c p_{cw} P_{n_a,0,1,0,0,n_w-1} + \mu_u P_{n_a,0,0,1,0,n_w} + \mu_v P_{n_a,0,0,0,1,n_w} + \mu_w P_{n_a,0,0,0,0,n_w+1}\end{aligned} \tag{53}$$

Considering $n_b=0,\ n_c=0,\ n_u=0,\ n_w=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_v+\mu_w)P_{n_a,0,0,0,n_v,0} &= \lambda_a P_{n_a-1,0,0,0,n_v,0} + \mu_b p_{ba} P_{n_a-1,1,0,0,n_v,0} \\ &+ \mu_b p_{bv} P_{n_a,1,0,0,n_v-1,0} + \mu_c p_{ca} P_{n_a-1,0,1,0,n_v,0} + \mu_u P_{n_a,0,0,1,n_v,0} + \mu_v P_{n_a,0,0,0,n_v+1,0} + \mu_w P_{n_a,0,0,0,n_v,1}\end{aligned} \tag{54}$$

Considering $n_b=0,\ n_c=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_u)P_{n_a,0,0,n_u,0,0} &= \lambda_a P_{n_a-1,0,0,n_u,0,0} + \mu_a p_{au} P_{n_a+1,0,0,n_u-1,0,0} + \mu_b p_{ba} P_{n_a-1,1,0,n_u,0,0} \\ &+ \mu_c p_{ca} P_{n_a-1,0,1,n_u,0,0} + \mu_u P_{n_a,0,0,n_u+1,0,0} + \mu_v P_{n_a,0,0,n_u,1,0} + \mu_w P_{n_a,0,0,n_u,0,1}\end{aligned} \tag{55}$$

Considering $n_b=0,\ n_u=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_c)P_{n_a,0,n_c,0,0,0} &= \lambda_a P_{n_a-1,0,n_c,0,0,0} + \lambda_c P_{n_a,0,n_c-1,0,0,0} + \mu_a p_{ac} P_{n_a+1,0,n_c-1,0,0,0} \\ &+ \mu_b p_{ba} P_{n_a-1,1,n_c,0,0,0} + \mu_b p_{bc} P_{n_a,1,n_c-1,0,0,0} + \mu_c p_{ca} P_{n_a-1,0,n_c+1,0,0,0} + \mu_u P_{n_a,0,n_c,1,0,0} \\ &+ \mu_v P_{n_a,0,n_c,0,1,0} + \mu_w P_{n_a,0,n_c,0,0,1}\end{aligned} \tag{56}$$

Considering $n_c=0,\ n_u=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b)P_{n_a,n_b,0,0,0,0} &= \lambda_a P_{n_a-1,n_b,0,0,0,0} + \lambda_b P_{n_a,n_b-1,0,0,0,0} + \mu_a p_{ab} P_{n_a+1,n_b-1,0,0,0,0} \\ &+ \mu_b p_{ba} P_{n_a-1,n_b+1,0,0,0,0} + \mu_c p_{ca} P_{n_a-1,n_b,1,0,0,0} + \mu_c p_{cb} P_{n_a,n_b-1,1,0,0,0} + \mu_u P_{n_a,n_b,0,1,0,0} \\ &+ \mu_v P_{n_a,n_b,0,0,1,0} + \mu_w P_{n_a,n_b,0,0,0,1}\end{aligned} \tag{57}$$

Considering $n_a=0,\ n_b=0,\ n_c=0,\ n_u=0,\ n_v=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_w)P_{0,0,0,0,0,n_w} &= \mu_c p_{cw} P_{0,0,1,0,0,n_w-1} + \mu_u P_{0,0,0,1,0,n_w} + \mu_v P_{0,0,0,0,1,n_w} \\ &+ \mu_w P_{0,0,0,0,0,n_w+1}\end{aligned} \tag{58}$$

Considering $n_a=0,\ n_b=0,\ n_c=0,\ n_u=0,\ n_w=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_v+\mu_w)P_{0,0,0,0,n_v,0} &= \mu_b p_{bv} P_{0,1,0,0,n_v-1,0} + \mu_u P_{0,0,0,1,n_v,0} + \mu_v P_{0,0,0,0,n_v+1,0} \\ &+ \mu_w P_{0,0,0,0,n_v,1}\end{aligned} \tag{59}$$

Considering $n_a=0,\ n_b=0,\ n_c=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_u)P_{0,0,0,n_u,0,0} &= \mu_a p_{au} P_{1,0,0,n_u-1,0,0} + \mu_u P_{0,0,0,n_u+1,0,0} + \mu_v P_{0,0,0,n_u,1,0} \\ &+ \mu_w P_{0,0,0,n_u,0,1}\end{aligned} \tag{60}$$

Considering $n_a=0,\ n_b=0,\ n_u=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$\begin{aligned}(\lambda_a+\lambda_b+\lambda_c+\mu_c)P_{0,0,n_c,0,0,0} &= \lambda_c P_{0,0,n_c-1,0,0,0} + \mu_a p_{ac} P_{1,0,n_c-1,0,0,0} + \mu_b p_{bc} P_{0,1,n_c-1,0,0,0} \\ &+ \mu_u P_{0,0,n_c,1,0,0} + \mu_v P_{0,0,n_c,0,1,0} + \mu_w P_{0,0,n_c,0,0,1}\end{aligned} \tag{61}$$

Considering $n_a=0,\ n_c=0,\ n_u=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_b)P_{0,n_b,0,0,0,0}=\lambda_b P_{0,n_b-1,0,0,0,0}+\mu_a p_{ab}P_{1,n_b-1,0,0,0,0}+\mu_c p_{cb}P_{0,n_b-1,1,0,0,0} + \mu_u P_{0,n_b,0,1,0,0}+\mu_v P_{0,n_b,0,0,1,0}+\mu_w P_{0,n_b,0,0,0,1} \quad (62)$$

Considering $n_b=0,\ n_c=0,\ n_u=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c+\mu_a)P_{n_a,0,0,0,0,0}=\lambda_a P_{n_a-1,0,0,0,0,0}+\mu_b p_{ba}P_{n_a-1,1,0,0,0,0}+\mu_c p_{ca}P_{n_a-1,0,1,0,0,0} + \mu_u P_{n_a,0,0,1,0,0}+\mu_v P_{n_a,0,0,0,1,0}+\mu_w P_{n_a,0,0,0,0,1} \quad (63)$$

Considering $n_a=0,\ n_b=0,\ n_c=0,\ n_u=0,\ n_v=0,\ n_w=0$; the Eq (1) can be given as

$$(\lambda_a+\lambda_b+\lambda_c)P_{0,0,0,0,0,0}=\mu_u P_{0,0,0,1,0,0}+\mu_v P_{0,0,0,0,1,0}+\mu_w P_{0,0,0,0,0,1} \quad (64)$$

IV. GOVERNING EQUATIONS AND SOLUTION METHODOLOGY

To solve the governing Equations, Generating function is assumed as

$$F(X_1,X_2,X_3,X_4,X_5,X_6)=\sum_{n_a=0}^{\infty}\sum_{n_b=0}^{\infty}\sum_{n_c=0}^{\infty}\sum_{n_u=0}^{\infty}\sum_{n_v=0}^{\infty}\sum_{n_w=0}^{\infty} P_{n_a,n_b,n_c,n_u,n_v,n_w} X_1^{n_a} X_2^{n_b} X_3^{n_c} X_4^{n_u} X_5^{n_v} X_6^{n_w} \quad (65)$$

such that $|X_1|=|X_2|=|X_3|=|X_4|=|X_5|=|X_6|\le 1$

Also, taking partial generating function as

$$F_{n_b,n_c,n_u,n_v,n_w}(X_1)=\sum_{n_a=0}^{\infty} P_{n_a,n_b,n_c,n_u,n_v,n_w}.X_1^{n_a} \quad (66)$$

$$F_{n_c,n_u,n_v,n_w}(X_1,X_2)=\sum_{n_b=0}^{\infty} F_{n_b,n_c,n_u,n_v,n_w}(X_1).X_2^{n_b} \quad (67)$$

$$F_{n_u,n_v,n_w}(X_1,X_2,X_3)=\sum_{n_c=0}^{\infty} F_{n_c,n_u,n_v,n_w}(X_1,X_2).X_3^{n_c} \quad (68)$$

$$F_{n_v,n_w}(X_1,X_2,X_3,X_4)=\sum_{n_u=0}^{\infty} F_{n_u,n_v,n_w}(X_1,X_2,X_3).X_4^{n_u} \quad (69)$$

$$F_{n_w}(X_1,X_2,X_3,X_4,X_5)=\sum_{n_v=0}^{\infty} F_{n_v,n_w}(X_1,X_2,X_3,X_4).X_5^{n_v} \quad (70)$$

$$F(X_1,X_2,X_3,X_4,X_5,X_6)=\sum_{n_w=0}^{\infty} F_{n_w}(X_1,X_2,X_3,X_4,X_5).X_6^{n_w} \quad (71)$$

On solving equations from (1) to (64) with the assistance of (65) to (71) by using the technique in [2, 4], we get

$$\begin{aligned}
&(\lambda_a+\lambda_b+\lambda_c+\mu_a+\mu_b+\mu_c+\mu_u+\mu_v+\mu_w)F(X_1,X_2,X_3,X_4,X_5,X_6)\\
&-\mu_w F_0(X_1,X_2,X_3,X_4,X_5)-\mu_v F_0(X_1,X_2,X_3,X_4,X_6)-\mu_u F_0(X_1,X_2,X_3,X_5,X_6)\\
&-\mu_c F_0(X_1,X_2,X_4,X_5,X_6)-\mu_b F_0(X_1,X_3,X_4,X_5,X_6)-\mu_a F_0(X_2,X_3,X_4,X_5,X_6)\\
&\quad=\lambda_a X_1 F(X_1,X_2,X_3,X_4,X_5,X_6)+\lambda_b X_2 F(X_1,X_2,X_3,X_4,X_5,X_6)\\
&\quad+\lambda_c X_3 F(X_1,X_2,X_3,X_4,X_5,X_6)\\
&\quad+\frac{\mu_a p_{ab}}{X_1}X_2\begin{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6)\\-F_0(X_2,X_3,X_4,X_5,X_6)\end{bmatrix}+\frac{\mu_a p_{ac}}{X_1}X_3\begin{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6)\\-F_0(X_2,X_3,X_4,X_5,X_6)\end{bmatrix}\\
&\quad+\frac{\mu_a p_{au}}{X_1}X_4\begin{bmatrix}F(X_1,X_2X_3,X_4,X_5,X_6)\\-F_0(X_2,X_3,X_4,X_5,X_6)\end{bmatrix}+\frac{\mu_b p_{ba}}{X_2}X_1\begin{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6)\\-F_0(X_1,X_3,X_4,X_5,X_6)\end{bmatrix}\\
&\quad+\frac{\mu_b p_{bc}}{X_2}X_3\begin{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6)\\-F_0(X_1,X_3,X_4,X_5,X_6)\end{bmatrix}+\frac{\mu_b p_{bv}}{X_2}X_5\begin{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6)\\-F_0(X_1,X_3,X_4,X_5,X_6)\end{bmatrix}\\
&\quad+\frac{\mu_c p_{ca}}{X_3}X_1\begin{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6)\\-F_0(X_1,X_2,X_4,X_5,X_6)\end{bmatrix}+\frac{\mu_c p_{cb}}{X_3}X_2\begin{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6)\\-F_0(X_1,X_2,X_4,X_5,X_6)\end{bmatrix}\\
&\quad+\frac{\mu_c p_{cw}}{X_3}X_6\begin{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6)\\-F_0(X_1,X_2,X_4,X_5,X_6)\end{bmatrix}+\frac{\mu_u}{X_4}\begin{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6)\\-F_0(X_1,X_2,X_3,X_5,X_6)\end{bmatrix}\\
&\quad+\frac{\mu_v}{X_5}\begin{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6)\\-F_0(X_1,X_2,X_3,X_4,X_6)\end{bmatrix}+\frac{\mu_w}{X_6}\begin{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6)\\-F_0(X_1,X_2,X_3,X_4,X_5)\end{bmatrix}
\end{aligned} \tag{72}$$

on simplify (72), we get

$$\begin{aligned}
&\begin{bmatrix}\lambda_a(1-X_1)+\lambda_b(1-X_2)+\lambda_c(1-X_3)+\mu_a\left\{1-\frac{p_{ab}}{X_1}X_2-\frac{p_{ac}}{X_1}X_3-\frac{p_{au}}{X_1}X_4\right\}\\
+\mu_b\left\{1-\frac{p_{ba}}{X_2}X_1-\frac{p_{bc}}{X_2}X_3-\frac{p_{bv}}{X_2}X_5\right\}+\mu_c\left\{1-\frac{p_{ca}}{X_3}X_1-\frac{p_{cb}}{X_3}X_2-\frac{p_{cw}}{X_3}X_6\right\}\\
+\mu_u\left\{1-\frac{1}{X_4}\right\}+\mu_v\left\{1-\frac{1}{X_5}\right\}+\mu_w\left\{1-\frac{1}{X_6}\right\}\end{bmatrix}F(X_1,X_2,X_3,X_4,X_5,X_6)\\
&\quad=\mu_a\left\{1-\frac{p_{ab}}{X_1}X_2-\frac{p_{ac}}{X_1}X_3-\frac{p_{au}}{X_1}X_4\right\}F_0(X_2,X_3,X_4,X_5,X_6)\\
&\quad+\mu_b\left\{1-\frac{p_{ba}}{X_2}X_1-\frac{p_{bc}}{X_2}X_3-\frac{p_{bv}}{X_2}X_5\right\}F_0(X_1,X_3,X_4,X_5,X_6)\\
&\quad+\mu_c\left\{1-\frac{p_{ca}}{X_3}X_1-\frac{p_{cb}}{X_3}X_2-\frac{p_{cw}}{X_3}X_6\right\}F_0(X_1,X_2,X_4,X_5,X_6)\\
&\quad+\mu_u\left\{1-\frac{1}{X_4}\right\}F_0(X_1,X_2,X_3,X_5,X_6)\\
&\quad+\mu_v\left\{1-\frac{1}{X_5}\right\}F_0(X_1,X_2,X_3,X_4,X_6)\\
&\quad+\mu_w\left\{1-\frac{1}{X_6}\right\}F_0(X_1,X_2,X_3,X_4,X_5)
\end{aligned}$$

Assuming

$$F_0(X_2,X_3,X_4,X_5,X_6)=F_a,\;\; F_0(X_1,X_3,X_4,X_5,X_6)=F_b\;,\; F_0(X_1,X_2,X_4,X_5,X_6)=F_c$$
$$F_0(X_1,X_2,X_3,X_5,X_6)=F_u,\; F_0(X_1,X_2,X_3,X_4,X_6)=F_v\;,\; F_0(X_1,X_2,X_3,X_4,X_5)=F_w$$

We get,

$$F(X_1,X_2,X_3,X_4,X_5,X_6)=\frac{\left[\begin{array}{l}\mu_a\left\{1-\frac{p_{ab}}{X_1}X_2-\frac{p_{ac}}{X_1}X_3-\frac{p_{au}}{X_1}X_4\right\}F_a+\mu_b\left\{1-\frac{p_{ba}}{X_2}X_1-\frac{p_{bc}}{X_2}X_3-\frac{p_{bv}}{X_2}X_5\right\}F_b\\+\mu_c\left\{1-\frac{p_{ca}}{X_3}X_1-\frac{p_{cb}}{X_3}X_2-\frac{p_{cw}}{X_3}X_6\right\}F_c+\mu_u\left\{1-\frac{1}{X_4}\right\}F_u+\mu_v\left\{1-\frac{1}{X_5}\right\}F_v\\+\mu_w\left\{1-\frac{1}{X_6}\right\}F_w\end{array}\right]}{\left[\begin{array}{l}\lambda_a(1-X_1)+\lambda_b(1-X_2)+\lambda_c(1-X_3)+\mu_a\left\{1-\frac{p_{ab}}{X_1}X_2-\frac{p_{ac}}{X_1}X_3-\frac{p_{au}}{X_1}X_4\right\}\\+\mu_b\left\{1-\frac{p_{ba}}{X_2}X_1-\frac{p_{bc}}{X_2}X_3-\frac{p_{bv}}{X_2}X_5\right\}+\mu_c\left\{1-\frac{p_{ca}}{X_3}X_1-\frac{p_{cb}}{X_3}X_2-\frac{p_{cw}}{X_3}X_6\right\}\\+\mu_u\left\{1-\frac{1}{X_4}\right\}+\mu_v\left\{1-\frac{1}{X_5}\right\}+\mu_w\left\{1-\frac{1}{X_6}\right\}\end{array}\right]} \quad (73)$$

Since $F(1,1,1,1,1,1) = 1$, the total probability.

Considering $X_1 = 1$ as $X_2 \to 1,\ X_3 \to 1,\ X_4 \to 1,\ X_5 \to 1,\ X_6 \to 1$,

$F(X_1,X_2,X_3,X_4,X_5,X_6)$ is of (0/0) form, which is indeterminate. Therefore, using L- Hospital rule, differentiating numerator and denominator separately w.r.t. X_1, we get

$$1=\frac{\mu_a(p_{ab}+p_{ac}+p_{au})F_a+\mu_b(-p_{ba})F_b+\mu_c(-p_{ca})F_c}{-\lambda_a+\mu_a(p_{ab}+p_{ac}+p_{au})+\mu_b(-p_{ba})+\mu_c(-p_{ca})}$$

where $p_{ab}+p_{ac}+p_{au}=1$

$$\mu_a F_a-\mu_b p_{ba}F_b-\mu_c p_{ca}F_c=-\lambda_a+\mu_a-\mu_b p_{ba}-\mu_c p_{ca} \quad (74)$$

Again differentiating numerator and denominator separately w.r.t. X_2 by taking $X_2 = 1$ as $X_1 \to 1,\ X_3 \to 1,\ X_4 \to 1,\ X_5 \to 1,\ X_6 \to 1$, we get

$$1=\frac{\mu_a(-p_{ab})F_a+\mu_b(p_{ba}+p_{bc}+p_{bv})F_b+\mu_c(-p_{cb})F_c}{-\lambda_b+\mu_a(-p_{ab})+\mu_b(p_{ba}+p_{bc}+p_{bv})+\mu_c(-p_{cb})}$$

where $p_{ba}+p_{bc}+p_{bv}=1$

$$-\mu_a p_{ab}F_a+\mu_b F_b-\mu_c p_{cb}F_c=-\lambda_b-\mu_a p_{ab}+\mu_b-\mu_c p_{cb} \quad (75)$$

Again differentiating numerator and denominator separately w.r.t. X_3 by taking $X_3 = 1$ as $X_1 \to 1,\ X_2 \to 1,\ X_4 \to 1,\ X_5 \to 1,\ X_6 \to 1$, we get

$$1=\frac{\mu_a(-p_{ac})F_a+\mu_b(-p_{bc})F_b+\mu_c(p_{ca}+p_{cb}+p_{cw})F_c}{-\lambda_c+\mu_a(-p_{ac})+\mu_b(-p_{bc})+\mu_c(p_{ca}+p_{cb}+p_{cw})}$$

where $p_{ca}+p_{cb}+p_{cw}=1$

$$-\mu_a p_{ac}F_a-\mu_b p_{bc}F_b+\mu_c F_c=-\lambda_c-\mu_a p_{ac}-\mu_b p_{bc}+\mu_c \quad (76)$$

Again differentiating numerator and denominator separately w.r.t. X_4 by taking $X_4 = 1$ as $X_1 \to 1,\ X_2 \to 1,\ X_3 \to 1,\ X_5 \to 1,\ X_6 \to 1$, we get

$$1=\frac{\mu_a(-p_{au})F_a+\mu_u F_u}{\mu_a(-p_{au})+\mu_u}$$

$$\mu_u F_u=-\mu_a p_{au}(1-F_a)+\mu_u \quad (77)$$

Again differentiating numerator and denominator separately w.r.t. X_5 by taking $X_5 = 1$ as $X_1 \to 1,\ X_2 \to 1,\ X_3 \to 1,\ X_4 \to 1,\ X_6 \to 1$, we get

$$1=\frac{\mu_b(-p_{bv})F_b+\mu_v F_v}{\mu_b(-p_{bv})+\mu_v}$$

$$\mu_v F_v=-\mu_b p_{bv}(1-F_b)+\mu_v \quad (78)$$

Again differentiating numerator and denominator separately w.r.t. X_6 by taking $X_6 = 1$ as $X_1 \to 1,\ X_2 \to 1,\ X_3 \to 1,\ X_4 \to 1,\ X_5 \to 1$, we get

$$1 = \frac{\mu_c(-p_{cw})F_c + \mu_w F_w}{\mu_c(-p_{cw}) + \mu_w}$$

$$\mu_w F_w = -\mu_c p_{cw}(1-F_c) + \mu_w \tag{79}$$

on solving (74), (75), (76), (77), (78) & (79) using [5,6], we get

$$F_a = 1 - \frac{\lambda_a(1-p_{cb}p_{bc}) + \lambda_b\{p_{ba}(1-p_{cb}p_{bc}) + p_{bc}(p_{ca}+p_{cb}p_{ba})\} + \lambda_c(p_{ca}+p_{cb}p_{ba})}{\mu_a\{(1-p_{ab}p_{ba})(1-p_{cb}p_{bc}) - (p_{ac}+p_{ab}p_{bc})(p_{ca}+p_{cb}p_{ba})\}}$$

$$F_b = 1 - \frac{\lambda_a(p_{ab}+p_{ac}p_{cb}) + \lambda_b(1-p_{ac}p_{ca}) + \lambda_c\{p_{ca}(p_{ab}+p_{ac}p_{cb}) + p_{cb}(1-p_{ac}p_{ca})\}}{\mu_b\{(1-p_{bc}p_{cb})(1-p_{ac}p_{ca}) - (p_{ba}+p_{bc}p_{ca})(p_{ab}+p_{ac}p_{cb})\}}$$

$$F_c = 1 - \frac{\lambda_a\{p_{ab}(p_{bc}+p_{ba}p_{ac}) + p_{ac}(1-p_{ab}p_{ba})\} + \lambda_b(p_{bc}+p_{ba}p_{ac}) + \lambda_c(1-p_{ab}p_{ba})}{\mu_c\{(1-p_{ac}p_{ca})(1-p_{ab}p_{ba}) - (p_{cb}+p_{ab}p_{ca})(p_{bc}+p_{ba}p_{ac})\}}$$

$$F_u = 1 - \frac{\mu_a p_{au}}{\mu_u}(1-F_a)$$

$$F_v = 1 - \frac{\mu_b p_{bv}}{\mu_v}(1-F_b)$$

$$F_w = 1 - \frac{\mu_c p_{cw}}{\mu_w}(1-F_c)$$

The solution (Joint Probability) of the model is written as

$$P_{n_a,n_b,n_c,n_u,n_v,n_w} = (1-F_a)^{n_a}(1-F_b)^{n_b}(1-F_c)^{n_c}(1-F_u)^{n_u}(1-F_v)^{n_v}(1-F_w)^{n_w}F_aF_bF_cF_uF_vF_w \tag{80}$$

$$P_{n_a,n_b,n_c,n_u,n_v,n_w} = \rho_a^{n_a}\ \rho_b^{n_b}\ \rho_c^{n_c}\ \rho_u^{n_u}\ \rho_v^{n_v}\ \rho_w^{n_w}\ (1-\rho_a)(1-\rho_b)(1-\rho_c)(1-\rho_u)(1-\rho_v)(1-\rho_w)$$

where $\rho_a = 1-F_a,\ \rho_b = 1-F_b,\ \rho_c = 1-F_c,\ \rho_u = 1-F_u,\ \rho_v = 1-F_v,\ \rho_w = 1-F_w$

$$\rho_a = \frac{\lambda_a(1-p_{cb}p_{bc}) + \lambda_b\{p_{ba}(1-p_{cb}p_{bc}) + p_{bc}(p_{ca}+p_{cb}p_{ba})\} + \lambda_c(p_{ca}+p_{cb}p_{ba})}{\mu_a\{(1-p_{ab}p_{ba})(1-p_{cb}p_{bc}) - (p_{ac}+p_{ab}p_{bc})(p_{ca}+p_{cb}p_{ba})\}}$$

$$\rho_b = \frac{\lambda_a(p_{ab}+p_{ac}p_{cb}) + \lambda_b(1-p_{ac}p_{ca}) + \lambda_c\{p_{ca}(p_{ab}+p_{ac}p_{cb}) + p_{cb}(1-p_{ac}p_{ca})\}}{\mu_b\{(1-p_{bc}p_{cb})(1-p_{ac}p_{ca}) - (p_{ba}+p_{bc}p_{ca})(p_{ab}+p_{ac}p_{cb})\}}$$

$$\rho_c = \frac{\lambda_a\{p_{ab}(p_{bc}+p_{ba}p_{ac}) + p_{ac}(1-p_{ab}p_{ba})\} + \lambda_b(p_{bc}+p_{ba}p_{ac}) + \lambda_c(1-p_{ab}p_{ba})}{\mu_c\{(1-p_{ac}p_{ca})(1-p_{ab}p_{ba}) - (p_{cb}+p_{ab}p_{ca})(p_{bc}+p_{ba}p_{ac})\}}$$

$$\rho_u = \frac{\mu_a p_{au}}{\mu_u}\left[\frac{\lambda_a(1-p_{cb}p_{bc}) + \lambda_b\{p_{ba}(1-p_{cb}p_{bc}) + p_{bc}(p_{ca}+p_{cb}p_{ba})\} + \lambda_c(p_{ca}+p_{cb}p_{ba})}{\mu_a\{(1-p_{ab}p_{ba})(1-p_{cb}p_{bc}) - (p_{ac}+p_{ab}p_{bc})(p_{ca}+p_{cb}p_{ba})\}}\right]$$

$$\rho_v = \frac{\mu_b p_{bv}}{\mu_v}\left[\frac{\lambda_a(p_{ab}+p_{ac}p_{cb}) + \lambda_b(1-p_{ac}p_{ca}) + \lambda_c\{p_{ca}(p_{ab}+p_{ac}p_{cb}) + p_{cb}(1-p_{ac}p_{ca})\}}{\mu_b\{(1-p_{bc}p_{cb})(1-p_{ac}p_{ca}) - (p_{ba}+p_{bc}p_{ca})(p_{ab}+p_{ac}p_{cb})\}}\right]$$

$$\rho_w = \frac{\mu_c p_{cw}}{\mu_w}\left[\frac{\lambda_a\{p_{ab}(p_{bc}+p_{ba}p_{ac}) + p_{ac}(1-p_{ab}p_{ba})\} + \lambda_b(p_{bc}+p_{ba}p_{ac}) + \lambda_c(1-p_{ab}p_{ba})}{\mu_c\{(1-p_{ac}p_{ca})(1-p_{ab}p_{ba}) - (p_{cb}+p_{ab}p_{ca})(p_{bc}+p_{ba}p_{ac})\}}\right]$$

The solution of this model exists if $\rho_a,\ \rho_b,\ \rho_c,\ \rho_u,\ \rho_v,\ \rho_w < 1$ (81)

V. QUEUING MODEL CHARACTERSTICS

(i) ***Mean queue length (average number of customers)***

$$L_Q = L_a + L_b + L_c + L_u + L_v + L_w$$

$$L_Q = \frac{\rho_a}{1-\rho_a} + \frac{\rho_b}{1-\rho_b} + \frac{\rho_c}{1-\rho_c} + \frac{\rho_u}{1-\rho_u} + \frac{\rho_v}{1-\rho_v} + \frac{\rho_w}{1-\rho_w}$$

Where $L_a = \frac{\rho_a}{1-\rho_a}$, $L_b = \frac{\rho_b}{1-\rho_b}$, $L_c = \frac{\rho_c}{1-\rho_c}$, $L_u = \frac{\rho_u}{1-\rho_u}$, $L_v = \frac{\rho_v}{1-\rho_v}$, $L_w = \frac{\rho_w}{1-\rho_w}$

(ii) ***Fluctuation (Variance) in queue length***

$$V_{ar} = V_a + V_b + V_c + V_u + V_v + V_w$$

$$V_{ar} = \frac{\rho_a}{(1-\rho_a)^2} + \frac{\rho_b}{(1-\rho_b)^2} + \frac{\rho_c}{(1-\rho_c)^2} + \frac{\rho_u}{(1-\rho_u)^2} + \frac{\rho_v}{(1-\rho_v)^2} + \frac{\rho_w}{(1-\rho_w)^2}$$

Where $V_a = \frac{\rho_a}{(1-\rho_a)^2}, V_b = \frac{\rho_b}{(1-\rho_b)^2}, V_c = \frac{\rho_c}{(1-\rho_c)^2}, V_u = \frac{\rho_u}{(1-\rho_u)^2}, V_v = \frac{\rho_v}{(1-\rho_v)^2}, V_w = \frac{\rho_w}{(1-\rho_w)^2}$

(iii) ***Average waiting time for customer***

$$E_{wt} = L_Q / \lambda \ , \quad where \quad \lambda = \lambda_a + \lambda_b + \lambda_c$$

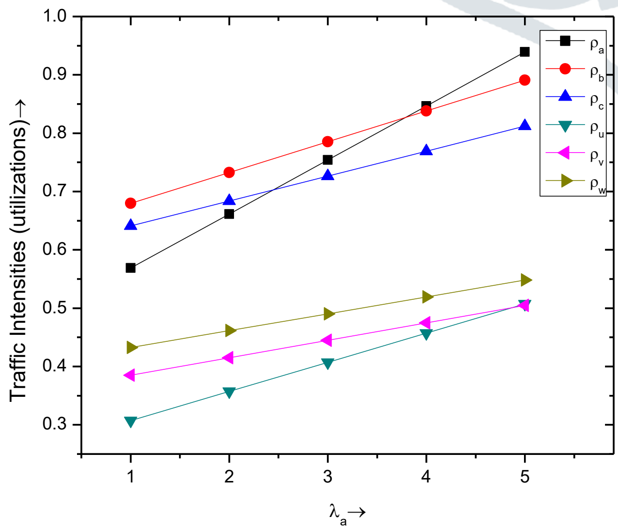
VI. RESULTS AND DISCUSSION

In the present work the complex queuing model with three servers connected in tri-cum biserial way have been examined. These servers individually connected to the three severs in series. In section 3 and 4, the detailed description of the mathematical model and associated governing equations have been shown. In the present study, six queues Q_a, Q_b, Q_c, Q_u, Q_v and Q_w associated with servers Sr_a, Sr_b , Sr_c, Sr_u, Sr_v, and Sr_w have been considered. The details of various input parameters used to compute the various characteristics such as traffic intensities (utilizations of the servers), queue lengths and average waiting time are presented in Table 1.

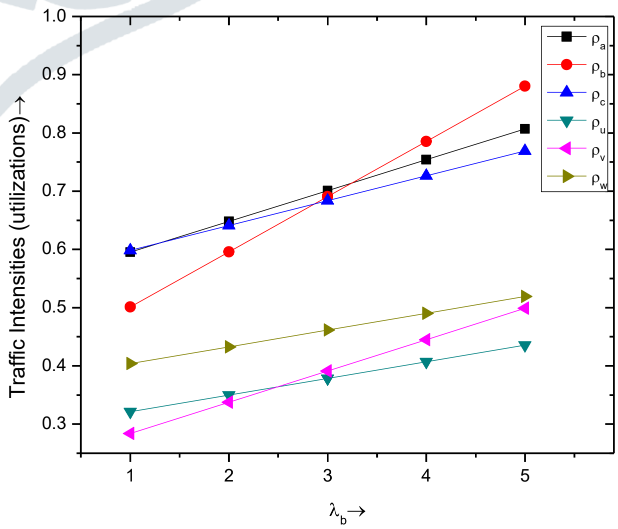
Figure 2 [(a), (b), (c)] shows the utilization of servers (traffic intensities) for various mean arrival rates (λ_a , λ_b and λ_c). It is clear from the results that as the mean arrival rate increases from 1 to 5, the traffic intensity increases. It is also found from the results that the values of traffic intensities is less than 1 which also confirms the basic conditions given in Eq (81). In Figure 3, 4 and 5 [(a), (b), (c)] the queue lengths, variances and average waiting times are plotted against mean arrival rates. It is evident from the results that on increasing the mean arrival rate, these parameters increases gradually.

Table 1: Various input parameters considered in computation of results

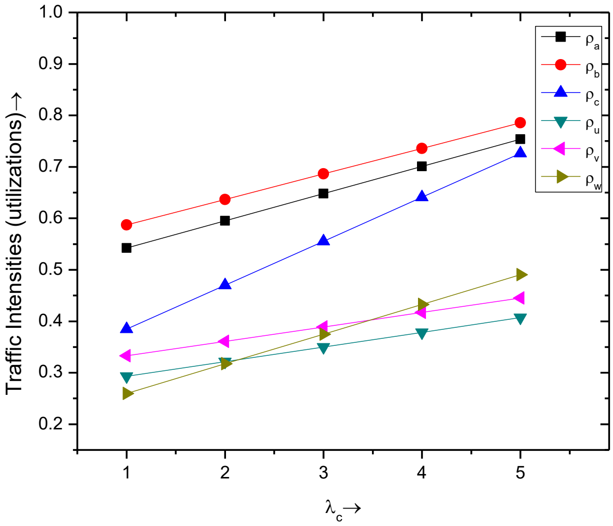
μ_a	μ_b	μ_c	μ_u	μ_v	μ_w	p_{ab}	p_{ac}	p_{au}	p_{ba}	p_{bc}	p_{bv}	p_{ca}	p_{cb}	p_{cw}
18	17	18	10	9	8	0.4	0.3	0.3	0.4	0.3	0.3	0.4	0.3	0.3



(a) $\lambda_b = 4, \ \lambda_c = 5$

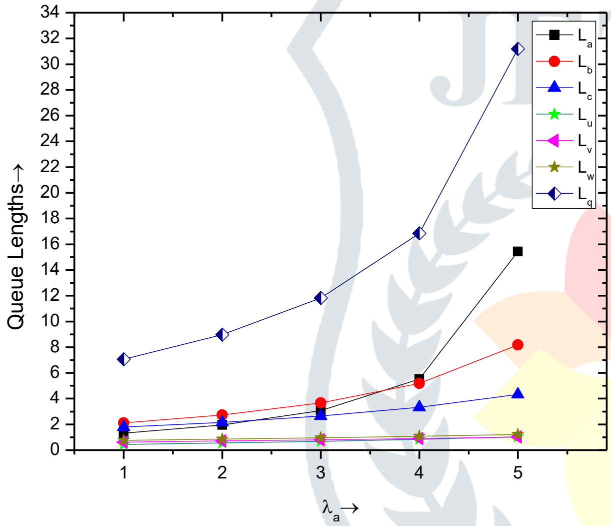


(b) $\lambda_a = 3, \ \lambda_c = 5$

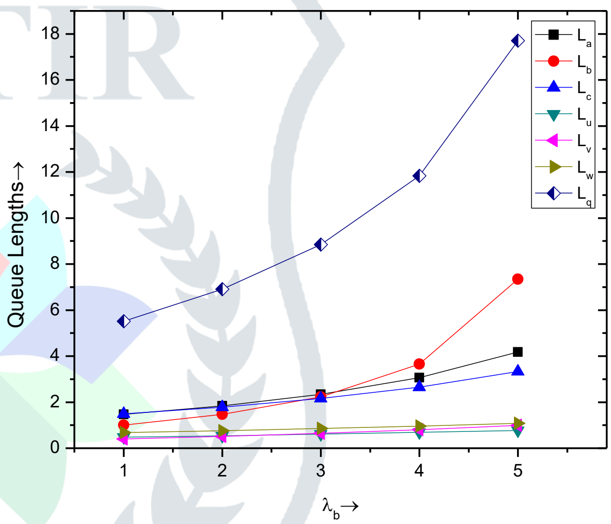


(c) $\lambda_a = 3,\ \lambda_b = 4$

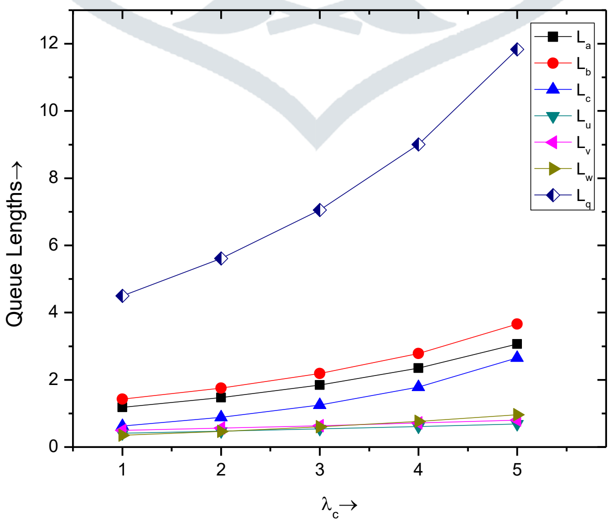
Fig 2 [(a), (b), (c)]: Utilization of servers for various mean arrival rates



(a) $\lambda_b = 4,\ \lambda_c = 5$

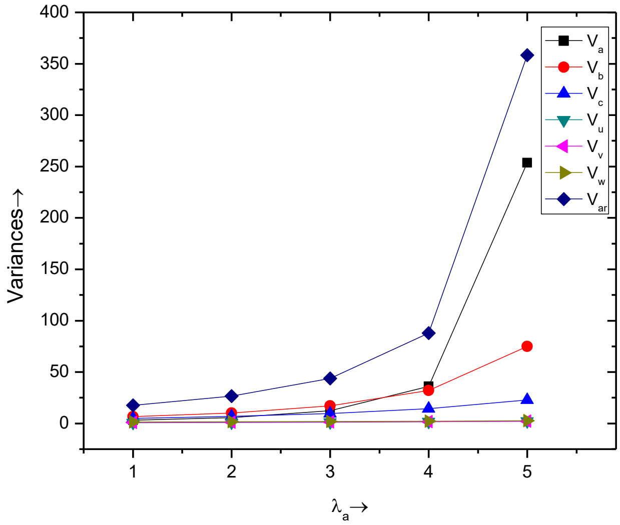


(b) $\lambda_a = 3,\ \lambda_c = 5$

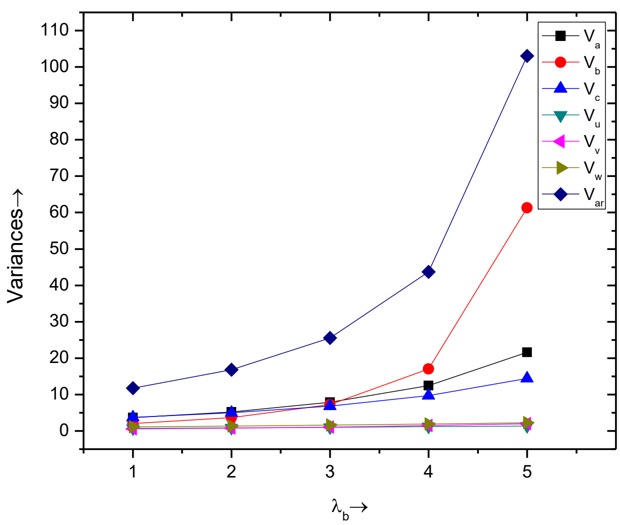


(c) $\lambda_a = 3,\ \lambda_b = 4$

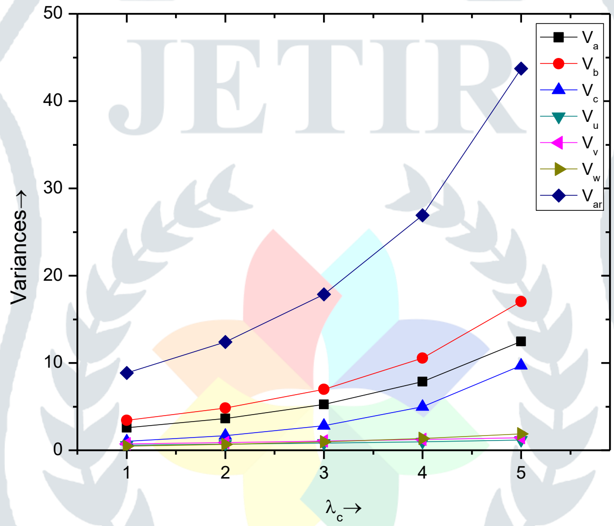
Fig 3 [(a), (b), (c)]: Queue lengths for various mean arrival rates



(a) $\lambda_b = 4,\ \lambda_c = 5$

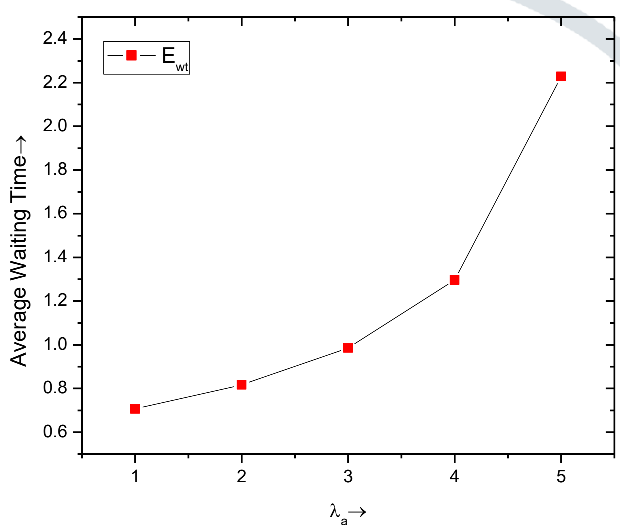


(b) $\lambda_a = 3,\ \lambda_c = 5$

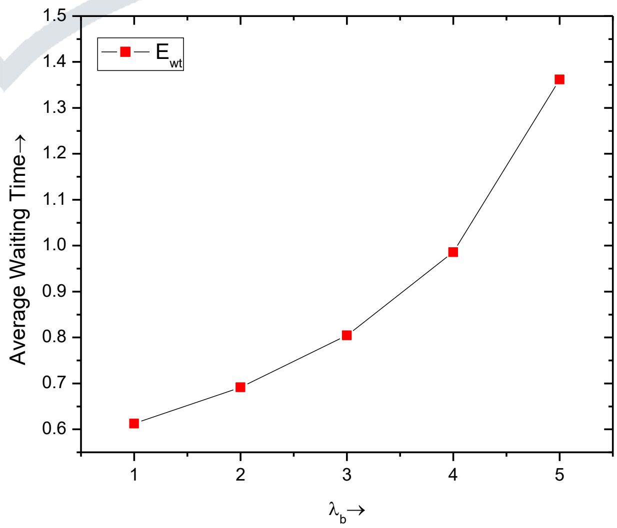


(c) $\lambda_a = 3,\ \lambda_b = 4$

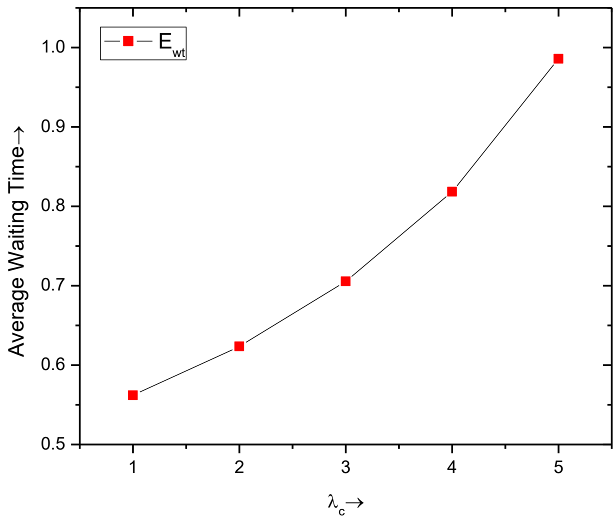
Fig 4 [(a), (b), (c)]: Variances for various mean arrival rates



(a) $\lambda_b = 4,\ \lambda_c = 5$



(b) $\lambda_a = 3,\ \lambda_c = 5$



(c) $\lambda_a = 3, \ \lambda_b = 4$

Fig 5 [(a), (b), (c)]: Average waiting Time for various mean arrival rates

VII. CONCLUSION

The present work dealt with the investigation of various queuing model characteristics of complex queue model having servers connected in tri-cum bi series way. The practical implementation of the developed model has been carried out by using realistic example in banking sector. Various queuing characteristics such as traffic intensities, queue lengths, variances, and average waiting time for customers have been computed using proposed mathematical model. It is concluded that the present model can be efficiently implemented in such a complex problems to provide the better service facilities.

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Appendix

Symbol	Notations
Servers	$Sr_a, Sr_b, Sr_c, Sr_u, Sr_v, Sr_w$
Joint Probability	$P_{n_a, n_b, n_c, n_u, n_v, n_w}$
Mean arrival rates	$\lambda_a, \lambda_b, \lambda_c$
Mean Service Rates	$\mu_a, \mu_b, \mu_c, \mu_u, \mu_v, \mu_w$
probabilities	$p_{ab}, p_{ac}, p_{au}, p_{ba}, p_{bc}, p_{bv}, p_{ca}, p_{cb}, p_{cw}$
No. of Customers	$n_a, n_b, n_c, n_u, n_v, n_w$
Traffic intensity or utilization of servers	$\rho_a, \rho_b, \rho_c, \rho_u, \rho_v, \rho_w$
Length of queues	$L_a, L_b, L_c, L_u, L_v, L_w, L_Q$
Variances	$V_a, V_b, V_c, V_u, V_v, V_w, V_{ar}$
Average waiting time for customer	E_{wt}