A STUDY GENERAL THEOREM ON THE CONDITIONAL CONVERGENCE OF TRIGONOMETRIC SERIES

M. Umapathi[1], Dr.N.Srinivasan
1 Research shcolar and Assistant professor
2 Head of professor, Department of mathematics
St. Peter's Institute of Higher Education and Research[1],Avadi, Chennai-54.

***Abstract :** The reason for this paper is to set up, paralleling an outstanding outcome for clear integrals, the contingent convergence of a group of trigonometric sine arrangement. The principal thought is to gather properly the terms of the arrangement keeping in mind the end goal to indicate outright dissimilarity of the arrangement, given the entrenched outcome that the arrangement the way things are is convergent.*

IndexTerms - Conditionally Convergent; Steadily Decreasing Sequence; Euler's Formula

I. INTRODUCTION

It is notable that the group of ill-advised trigonometric sine integrals whose coefficients unite relentlessly to zero shape an arrangement of restrictively focalized integrals at whatever point the basic of the coefficients themselves divergences (see [1]). Be that as it may, it is additionally intriguing to watch that there is a parallel outcome for unending arrangement. The discrete issue requires an altogether unique and novel approach, which is exhibited in this paper. The curiosity lives in a point by point comprehension of the best possible ties of the best whole number capacity as it identifies with convergence and in the utilization of Euler's capacity to find out an appropriate uniform lower bound. There is likewise a notable number hypothesis result which ends up being valuable. The accompanying hypothesis gives the proper generalization:

Theorem.2

Assume that one has a relentlessly diminishing arrangement of numbers $f(n), 0 \leq n < \infty$ (where n is a non-negative whole number), with the end goal that $f(n)$ watches out for 0 as n keeps an eye on vastness. Assume additionally that the aggregate of the $f(n)$'s is interminable. At that point

$$\sum_{n=1}^{\infty} f(n) abs(\sin n) \quad (1)$$

is in like manner unbounded, where abs remains for "outright esteem". At the end of the day the arrangement

$$\sum_{n=1}^{\infty} f(n)\ (\sin n) \quad (2)$$

is restrictively concurrent.

Proof.

Above all else it is notable that the arrangement (2) is joined (see [2]). Next let us watch that $sin 1, sin 2$, and $sin 3$ (points being communicated in radians) are all positive; at that point $sin 4$, $sin 5$, and $sin 6$ are for the most part negative, and so on., the signs exchanging basically in gatherings of 3 (or maybe 4 now and again). Truth be told we might demonstrate that we have arrangements$([k\pi] + 1 \leq n \leq [(k+1)\pi])$, $0 \leq k < \infty$, with sinn being of consistent sign in each succession and with the sections meaning the best whole number capacity. Without a doubt we see that

$$[k\pi] + 3 < k\pi + 3 < k\pi + \pi = (k+1)\pi$$

so that

$$[k\pi] + 3 \leq (k+1)\pi \quad (3)$$

It takes after that both $[k\pi] + 1$ 1nd $[k\pi] + 2$ are estimations of n whose sines are inside the kth succession. Additionally, $[k\pi] + 4$ might be an estimation of n whose sine is inside that succession, however such an occasion will acquire for a vast number of estimations of k (see [3]). Then again, we can demonstrate that $[k\pi] + 5\pi] + 5$ an estimation of n whose sine is in the kth arrangement. Truth be told

$$[k\pi] + 5 > k\pi - 1 + 5 \quad (4)$$

$= k\pi + 4 > (k+1)\pi > [(k+1)\pi\ ]$π+4>k+1π >k+1π

So the kth succession unquestionably has either three or four individuals. In any occasion unmistakably

$$abs\ (\sin([k\pi] + 1) + \sin([k\pi] + 2) + \sin([k\pi] + 3))$$
$$= abs(\sin([k\pi] + 1)) + abs(\sin([k\pi] + 2)) + abs(\sin([k\pi] + 3)) \quad (5)$$

where, as some time recently, abs signifies "supreme esteem".

Watch now that, just on the off chance that wrongdoing $([k\pi] + 4)$ would show up in a gathering, Equation (5) would surely master vide a lower bound on the total of the outright esteems inside that gathering.

Our following stage is to utilize Euler's recipe to acquire a shut frame articulation for Equation (5). To be sure we have

$$exp\big(([k\pi] + 1)i\big) + exp\big(([k\pi] + 2)i\big) + exp\big(([k\pi] + 3)i\big)$$
$$= exp\big(([k\pi] + 1)i\big)(1 + \exp(i) + \exp(2i))$$
$$= \frac{i\ exp\big(([k\pi] + 1)i\big)\ \Big(exp\big(-i/2\big) - exp\big(5i/2\big)\Big)}{2\sin(1/2)} \quad (6)$$

where exp remains for the exponential capacity. Along these lines, so as to decide Equation (5), we require the fanciful piece of the correct individual from Equation (6), which is observed to be $(\sin(3/2))(\sin([k\pi]+2))/(\sin(1/2))$ (7)

We see that our shut frame articulation for Equation (5) is the supreme estimation of Expression (7), which is simply

$$(\sin(3/2))abs(\sin([k\pi]+2))/(\sin(1/2)) \quad (8)$$

Next let us decide a uniform positive lower destined for Expression (8), i.e., for all k. Watch that

$$k\pi+1=k\pi-1+2<[k\pi]+2<k\pi+2 \quad (9)$$

From Expression (9) it takes after that $abs(sin([k\pi]+2))$ lies amongst sin1 and sin2, sin1 being the littler of the two. Consequently our positive lower headed for Quantity (8) (for all k) is $C=(\sin(3/2))(\sin 1)/(\sin(1/2))$ (10)

Along these lines, since $\{f(n)\}$ is a consistently diminishing sequence, we attest that

$$\sum_{n=1}^{\infty} f(n)abs(\sin n) \geq C\sum_{n=1}^{\infty} f([k\pi]+4) \quad (11)$$

Our last errand is to demonstrate that the entirety on the correct side of Inequality (11) is boundless. Despite what might be expected accept that the total is limited. Give us a chance to inspect the aggregates

$$\sum_{n=1}^{\infty} f([k\pi]+i), 0 \leq i \leq 3. \quad (12)$$

For instance assume that $i = 2$ or 3. Presently

$$[(k-1)\pi]+4<(k-1)\pi+4$$
$$= k\pi-1+4(\pi-1)<[k\pi]+5-\pi$$
$$<[k\pi]+5<[k\pi]+3. \quad (13)$$

In any case, at that point

$$f([k-1]\pi+4)>f([k\pi]+i), i=2,3, \quad (14)$$

so that, by strength, (12) focalizes for $i = 2$ and 3. Assume next that $i = 0$ or 1. In a manner like the advancement of Expression (13), one has

$$[(k-2)\pi]+4<(k-2)\pi+4$$
$$= k\pi-1+5-2\pi<[k\pi]+5-2\pi$$
$$<[k\pi]<[k\pi]+1. \quad (15)$$

Along these lines

$$f([k-2]\pi+4)>f([k\pi]+i), i=0,1, \quad (16)$$

It takes after that Quantities (12) join, and hence

$$\sum_{n=1}^{\infty} f(n)<\infty. \quad (17)$$

in logical inconsistency to the speculation of the hypothesis. There-fore, the arrangement on the correct side of Inequality (11) diverges, and the hypothesis is demonstrated.

Illustration.

Consider

$$\sum_{n=2}^{\infty}(\sin nx)/\log n \quad (18)$$

at the point when $x = 1$. Unmistakably $f(n)=1/\log n$ is an entirely diminishing capacity of n and keeps an eye on 0 as n tends to ∞. Likewise, since $1/\log n > 1/n$ and the symphonious arrangement is different, so is the entirety of the $f(n)'s$. As per our hypothesis, this unbounded arrangement for $x = 1$ is restrictively focalized. This in addition is a great case of a trigonometric arrangement which isn't a Fourier arrangement (see [4]). The fundamental purpose behind that conclusion is that

$$\sum_{n=2}^{\infty} 1/(n\log n)$$

is divergent, a reality which takes after from the notable essential test since

$$\int_{x=2}^{x} dx/(x\log x)=(\log(\log\infty))-(\log(\log 2))=\infty.$$

2. Conclusion

Utilizing a novel approach in the discrete case, which employs an outstanding outcome in number hypothesis together with Euler's equation, we have demonstrated a joining hypothesis for unbounded arrangement which is a consistent parallel to the corresponding vital case including a swaying integrand.

.

REFERENCES

1. D. V. Widder, "Advanced Calculus," 2nd Edition, Prentice Hall, Inc., Englewood Cliffs, 1961, pp. 333-335.

2. W. Rogosinski, "Fourier Series," 2nd Edition, Chelsea Publishing Company, New York, 1959, p. 18.

3. I.Niven, "Irrational Numbers," The Mathematical Association of America, Washington DC, 1956, pp. 72-81. E. C. Titchmarsh, "The Theory of Functions," 2nd Edition, Oxford University Press, Amen House, London, 1939, p. 420.