APPLICATIONS OF SLIGHTLY m-PRECONTINUOUS MULTIFUNCTIONS IN REAL LIFE

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Abstract : ***In this paper, we introduce a new class of functions namely slightly m- precontinuous multifunction. Further we obtain characterizations and relationships between upper/ lower slightly m- precontinuous multifunctions and other related multifunctions.***

IndexTerms - Upper slightly precontinuous, lower slightly precontinuous, upper slightly m-precontinuous, lower slightly m-precontinuous, upper m-precontinuous, lower m-precontinuous.

1. INTRODUCTION

The notion of slightly continuous functions was introduced by R.C. Jain [2]. Nour [12] defined slightly semi-continuous functions as a weak form of slight continuous functions and obtained their properties. Further this concept is extended to develop the class of slightly m-continuous functions. Popa [14] and Smithson [23] studied the concept of weakly continuous multifunctions. In this paper, we introduce the notion of slightly m-precontinuous multifunction and investigate the relationships among m-precontinuity, weak m-precontinuity and slight m-precontinuity for multifunctions.

2. P RELIMINARIES

Throughout this paper, all spaces (X,τ) and (Y,σ) are always topological spaces. A subset A of a space X is said to be **regular open (resp.regular closed)**, if A=Int(Cl(A))(resp. A=Cl (Int(A))), where Cl(A) and Int(A) denote the closure and interior of A . A subset A of a space X is called **preopen** if $A\subset Int(Cl(A))$.The complement of a preopen set is said to be **preclosed**. The family of all regular open (resp. regular closed, preopen, preclosed, clopen) sets of X is denoted by RO(X) (resp. RC(X), PO(X), PC(X), CO(X)). A subset A of a space X is said to be **semi-open** if $A\subset Cl(Int(A))$.The complement of a semi open set is called **semi-closed.** The union of all pre open sets of X contained in A is called the **pre-interior** of A and is denoted by pInt(A).A subset A of a space X is said to be **clopen** if it is both open and closed.

Throughout this paper, the spaces (X,τ) and (Y,σ)(or simply X and Y) denote topological spaces and $F:(X,\tau)\to(Y,\sigma)$ represents a multivalued function. For a multifunction $F:(X,\tau)\to(Y,\sigma)$, we shall denote the **upper** and **lower inverse** of a set B of a space Y by $F^+(B)$ and $F^-(B)$ respectively.

i.e. $F^+(B) = \{x\in X: F(x)\subset B\}$,$F^-(B) = \{x\in X: F(x)\cap B\neq\phi\}$.

Definition: 2.1[20]

A subfamily m_X of the power set P(X) of a nonempty set X is called a **minimal structure** (briefly m-structure) on X, if $\phi\in m_X$ and $X\in m_X$.

Definition: 2.2 [20]

By (X, m_X), we denote a nonempty set X with a minimal structure m_X on X and call it an **m-space**. Each member of m_X is said to be **m_X -open** (or briefly m-open). The complement of an m_X-open set is said to be **m_X-closed** (or briefly m-closed).

Remark: 2.3 [20]

Let (X,τ) be a topological space. Then the families τ, SO(X), PO(X), α (X), β(X), are all m-structures on X.

Definition: 2.4 [20]

A function $f:(X,\tau)\to(Y,\sigma)$ is said to be **slightly continuous** if for each point $x\in X$ and each clopen set V containing f(x), there exists an open set U containing x such that $f(U)\subset V$.

Definition: 2.5 [20]

A topological space (X,τ) is said to be **extremally disconnected** (briefly E.D), if the closure of each open set of X is open in X.

SLIGHTLY m-PRECONTINUOUS MULTIFUNCTIONS

Definition:3.1

A multifunction $F:(X,\tau)\to(Y,\sigma)$ is said to be

(a) **Upper slightly precontinuous** (resp. upper slightly continuous, upper slightly semi-continuous or upper faintly precontinuous, upper slightly β-continuous), if for each point $x\in X$ and each clopen set V of Y containing F(x), there exists an preopen (resp. open, semi-open, β-open) set U of X containing x such that $F(U)\subset V$.

(b) **Lower slightly precontinuous** (resp. lower slightly continuous, lower slightly semi-continuous or lower faintly precontinuous, lower slightly β-continuous), if for each point $x\in X$ and each clopen set V of Y such that $F(x)\cap V\neq\phi$, there exists an preopen (resp. open, semi-open, β-open) set U of X containing x such that $F(u)\cap V\neq\phi$, for each $u\in U$.

Definition: 3.2

A function $f:(X,m_X)\to(Y,\sigma)$, where (X,m_X) is a nonempty X with an minimal structure m_X and (Y,σ) is a topological space, is said to be **slightly m-pre continuous**, if for each $x\in X$ and each clopen set V of Y containing f(x), there exist a preopen set $U\in m_X$ containing x such that $f(U)\subset V$.

Definition: 3.3

A multifunction $F:(X,m_X)\to (Y,\sigma)$ is said to be

(a) **Upper m-precontinuous**(resp. upper almost m-precontinuous, upper weakly m-precontinuous), if for each point $x\in X$ and each open set V of Y containing F(x), there exists a preopen set $U\in m_X$ containing x such that $F(U)\subset V$(resp. $F(U)\subset Int(Cl(V))$, $F(U)\subset Cl(V)$).

(b) **Lower m-precontinuous** (lower almost m-precontinuous, lower weakly m-precontinuous), if for each point $x\in X$ and each open set V of Y such that $F(x)\cap V\neq\phi$, there exists a pre open set $U\in m_X$ containing x such that $F(u)\cap V\neq\phi$(resp. $F(u)\cap Int(Cl(V))\neq\phi$, $F(u)\cap Cl(V)\neq\phi$) for each $u\in U$.

Definition: 3.4

Let X be a nonempty set and m_X an m-structure on X. For a subset A of X, the m_X-preclosure of A and the m_X-preinterior of A are defined as follows.

1. $m_X\text{-}pCl(A) = \cap\{F: A\subset F,\ X-F\in m_X\}$.
2. $m_X\text{-}pInt(A) = \cup\{U: U\subset A,\ U\in m_X\}$.

Remark: 3.5

Let (X,τ) be a topological space and A be a subset of X. If $m_X = \tau$(resp.SO(X), PO(X), α(X)), then we have

1. $m_X\text{-}Cl(A) = Cl(A)$(resp.sCl(A),pCl(A), αCl(A)).
2. $m_X\text{-}Int(A) = Int(A)$(resp. sInt(A), pInt(A), αInt(A)).

Definition: 3.6

A multifunction $F:(X,m_X)\rightarrow(Y,\sigma)$, is said to be

(a) **Upper slightly m-precontinuous**, if for each point $x\in X$ and each colpen set V of Y containing F(x), there exist preopen set $U\in m_X$ containing x such that $F(U)\subset V$.

(b) **Lower slightly m-precontinuous,** if for each point $x\in X$ and each colpen set V of Y such that $F(x)\cap V\neq\phi$ there exists a preopen set $U\in m_X$ containing x such that $F(u)\cap V\neq\phi$ for each $u\in U$

Theorem: 3.7

For a multifunction $F:(X, m_X)\rightarrow(Y,\sigma)$ the following are equivalent.

1) F is upper slightly m-precontinuous
2) $F^+(V) = m_X\text{-}pInt(F^+(V))$ for each $V\in CO(Y)$
3) $F^-(V) = m_X\text{-}pCl(F^-(V))$ for each $V\in CO(Y)$

Proof:

(1)$\Rightarrow$(2): Let V be any clopen set of Y and $x\in F^+(V)$ then $F(x)\in V$ $\rightarrow$ (1).By (1), there exists a preopen set $U\in m_X$ containing x such that $F(U)\subset V$. Thus $x\in U\subset F^+(V)$ $\Rightarrow$ $x\in m_X\text{-}pInt(F^+(V))$ $\rightarrow$ (2).From eqn (1) & (2) we get,$F^+(V)\subset m_X\text{-}pInt(F^+(V))$.i.e. $m_X\text{-}pInt(F^+(V))\subset F^+(V)$, by lemma (3.1) of [20] $F^+(V) = m_X\text{-}pInt(F^+(V))$ for each $V\in CO(Y)$.

(2) $\Rightarrow$ (3): Let K be any clopen set of Y. Then Y-K is clopen in Y.By (2), $F^+(V) = m_X\text{-}pInt(F^+(V))$.By lemma (3.1) of [20] we have, $X-F^-(K) = F^+(Y-K) = m_X\text{-}pInt(F^+(Y-K)) = X-[m_X\text{-}pCl(F^-(K))]$.Therefore $F^-(K) = m_X\text{-}pCl(F^-K))$.

(3) $\Rightarrow$ (2): Let B be any clopen set of Y. Then Y- B is clopen in Y.By (3) & lemma (3.1) of [20] we have, $X-F^+(B) = F^-(Y-B) = m_X\text{-}pCl(F^+(V)) = X-[m_X\text{-}pInt(F^+(V))$.Therefore $F^+(B) = m_X\text{-}pInt(F^+(V))$.

(2) $\Rightarrow$ (1): Let $x\in X$ and V be any clopen set of Y containing F(x). Then $x\in F^+(V) = m_X\text{-}pInt(F^+(V))$,there exists a preopen set $U\in m_X$ containing x such that $x\in U\subset F^+(V)$. Therefore we have, $x\in U$ and $U\in m_X$ and $f(U)\subset V$. Hence F is upper slightly m-precontinuous.

Theorem: 3.8

For a multi function $F:(X, m_X)\rightarrow(Y,\sigma)$ the following are equivalent.

1) F is lower slightly m-precontinuous.
2) $F^-(V) = m_X\text{-}pInt((F^-(V))$ for each $V\in CO(Y)$.
3) $F^+(V) = m_X\text{-}pCl(F^+(V))$ for each $V\in CO(Y)$

Proof:

(1) $\Rightarrow$ (2): Let $V\in CO(Y)$ and $x\in F^-(V)$. Then $F(x)\cap V\neq\phi$.By (1), there exists a preopen set $U\in m_X$ containing x such that $F(u)\cap V\neq\phi$ for each $u\in U$.Therefore

we have, $U\subset F^-(V)\Rightarrow$ $x\in U\subset m_X\text{-}pInt(F^-(V))\Rightarrow x\in m_X\text{-}pInt(F^-(V))$ $\Rightarrow$ $F^-(V)\subset m_X\text{-}pInt(F^-(V))$ and by lemma (3.1) of [20] we have $F^-(V) = m_X\text{-}pInt(F^-(V))$.

(2) $\Rightarrow$ (3): Let $V\in CO(Y)$.Then $Y-V\in CO(Y)$,by (2), we have,$X-F^+(V) = F^-(V) =$ $F^-(Y-V) = m_X\text{-}pInt(F^-(Y-V)) = X-[m_X\text{-}pCl(F^+(V))$.Hence $F^+(V) = m_X\text{-}pCl(F^+(V))$

(3)$\Rightarrow$ (1): Let $x\in X$ and $V\in CO(Y)$ such that $F(x)\cap V\neq\phi$,then $x\in F^-(V)$ and $x\notin X-F^-(V) = F^+(Y-V)$.By (3) we have,$x\notin m_X\text{-}pCl(F^+(V))$.By lemma (3.2) of [20], there exists preopen set $U\in m_X$ containing x such that $U\cap F^+(Y-V) = \phi$.Thus $U\subset$ $F^-(V)$.Therefore $F(u)\cap V\neq\phi$ for each $u\in U$. Hence F is lower slightly m-precontinuous.

Theorem: 3.9

Let (Y,σ) be E.D. For a multifunction $F:(X, m_X)\rightarrow(Y,\sigma)$, the following are equivalent.

1) F is upper slightly m-precontinuous.
2) $m_X\text{-}pCl(F^-(V))\subset F^-(Cl(V))$ for every open set V of (Y,σ)
3) $F^+(Int(C))\subset m_X\text{-}pInt(F^+(C))$ for every closed set C of (Y,σ)

Proof:

(1) $\Rightarrow$ (2): Let V be any open set of Y. Then $Cl(V)\in CO(Y)$,by theorem (3.7), we have,$F^-(Cl(V)) = m_X\text{-}pCl(F^-(Cl(V)))$ and $F^-(V)\subset F^-(Cl(V))$,by lemma (3.1) of [20], we have, $m_X\text{-}pCl(F^-(V))\subset m_X\text{-}pCl(F^-(Cl(V))) = F^-(Cl(V))\Rightarrow m_X\text{-}pCl(F^-(V))\subset$ $F^-(Cl(V))$

(2) $\Rightarrow$ (3): Let C be any closed set of (Y,σ) and V = Y-C. Then V is open in (Y,σ). By lemma (3.1) of [20], we have, $X-[m_X\text{-}pInt(F^+(C))] = m_X\text{-}pCl(x-F^+(C)) = m_X\text{-}pCl(F^-(Y-C))\subset F^-(Y-Int(C))$. Therefore $X-[m_X\text{-}pInt(F^+(C))] = X-F^+(Int(C))\Rightarrow F^+(Int(C))\subset m_X\text{-}pInt(F^+(C))$

(3) $\Rightarrow$ (1): Let $x\in X$ and let $V\in CO(Y)$ containing F(x). Then By (3) we have, $x\in F^+(V) = F^+(Int(V))\subset m_X\text{-}pInt(F^+(V))$.There exists a preopen set $U\in m_X$ such that $x\in U\subset F^+(V)$. Thus $x\in U$, $U\in m_X$ and $F(U)\subset V$. Hence F is upper slightly m-precontinuous.

Theorem: 3.10

Let (Y,σ) be E .D. For a multifunction $F:(X, m_X) \to (Y,\sigma)$, the following are equivalent.
1) F is lower slightly m-precontinuous.
2) m_X-pCl($F^+(V)$)$\subset F^+(Cl(V))$ for every open set V of (Y,σ)
3) $F^-(Int(C)) \subset m_X$-pInt($F^-(C)$) for every closed set C of (Y,σ)

Proof:

(1) $\Rightarrow$ (2): Let V be any open set of Y. Then $Cl(V) \in CO(Y)$.By theorem (3.8), $F^+(Cl(V)) = m_X$-pCl($F^+(Cl(V))$) and $F^+(V) \subset (F^+(Cl(V)))$.By lemma (3.1) of [20], we have m_X-pCl($F^+(V)$)$\subset m_X$-pCl($F^+(Cl(V))$)$=F^+(Cl(V)) \Rightarrow m_X$-pCl($F^+(V)$)$\subset F^+(Cl(V))$

(2) $\Rightarrow$ (3): Let C be any closed set of (Y,σ) and V= Y-C, then V is open in (Y,σ)
By lemma (3.1) of [20], we have, X- [m_X-pInt ($F^-(C)$)] = m_X-pCl(X-($F^-(C)$))= m_X-pCl($F^+(Y-C)$)$\subset$ $F^+(Cl(Y-C))$ (by given (2))=F^+(Y-Int(C)).Therefore X- [m_X-pInt
($F^-(C)$)] = X- $F^-(Int(C)) \Rightarrow F^-(Int(C)) \subset m_X$-pInt($F^-(C)$)

(3) $\Rightarrow$ (1): Let $x \in X$ and $V \in CO(Y)$ containing $F(x) \cap V \neq \phi$.Let V = Y-C is open in Y. Let $x \in F^+(C)$ and $x \notin X- F^+(C) \notin F^-(Y-C)$.By (3) we have, $x \notin m_X$-pInt ($F^-(Y-C)$),by theorem (3.8) we have, $x \notin m_X$-pCl($F^+(Y-C)$).By lemma (3.2) of [20], there exists a preopen set $U \in m_X$ containing x such that $U \cap (F^+(Y-C)) = \phi$.Hence $U \subset$
$F^-(V)$and $F(x) \cap V \neq \phi$for each $u \in U$. Hence F is lower slightly m-precontinuous.

Slight m-precontinuity and other forms m-precontinuity.

Theorem: 4.1

If multifunction $F:(X, m_X) \to (Y,\sigma)$ is upper weakly m-precontinuous, then it is upper slightly m-precontinuous.

Proof:

Let $x \in X$ and $V \in CO(Y)$ containing F(x).Since F is upper weakly m-precontinuous. There exists a preopen set $U \in m_X$ containing x such that $F(U) \subset Cl(V) = V$. Hence F is upper slightly m-precontinuous.

Theorem: 4.2

If a multi function $F:(X, m_X) \to (Y,\sigma)$ is lower weakly m-precontinuous then it is lower slightly m-precontinuous.

Proof:

Let $x \in X$ and $V \in CO(Y)$ such that $F(x) \cap V \neq \phi$.Since F is lower weakly m-precontinuous there exist a preopen set $U \in m_X$ containing x such that $F(U) \cap Cl(V) \neq \phi$, for each $u \in U \Rightarrow$ $F(U) \cap (V) \neq \phi$, for each $u \in U$. Hence F is lower slightly m-precontinuous.

Lemma: 4.3

A multifunction $F:(X, m_X) \to (Y,\sigma)$ is upper almost m-precontinuous (resp.lower almost m-precontinuous) iff for each regular open set V containing F(x) (resp. meeting F(x)), there exist a preopen set $U \in m_X$ containing x such that $F(U) \subset V$ (resp. $F(u) \cap (V) \neq \phi$, for every $u \in U$)

Proof:

Let $x \in X$ and V be regular open set of Y containing F(x).Since F is upper almost m-precontinuous there exist a preopen set $U \in m_X$ containing x such that $F(U) \subset Int(Cl(V)) = V$. Hence for each regular open set V containing F(x), there exist a preopen set $U \in m_X$ containing x such that un $F(U) \subset V$. Conversely, there exist a preopen set $U \in m_X$ containing x such that $F(U) \subset V \subset Cl(V) \Rightarrow IntF(U) \subset Int(Cl(V)) \Rightarrow F(U) \subset Int(Cl(V))$.Hence F is upper almost m-precontinuous.

Theorem: **4.4**

If a multi function $F:(X, m_X) \to (Y,\sigma)$ is upper slightly m- precontinuous and (Y,σ) is E.D, then F is upper almost m- precontinuous.

Proof:

Let $x \in X$ and V be any regular open set of (Y,σ) containing F (x). Then By lemma 5.6 of [13] we have,$V \in CO(X)$.Since (Y,σ) is E.D and F is upper almost m- precontinuous, there exist a preopen set $U \in m_X$ containing x such that $F(U) \subset V$. By lemma (4.3), F is upper almost m- precontinuous.

Theorem: **4.5**

If a multi function F: $(X, m_X) \to (Y,\sigma)$ is lower slightly m- precontinuous and (Y,σ) is E.D , then F is lower almost m-precontinuous.

Proof:

Let $x \in X$ and V be any regular open set of (Y,σ) containing F(x). Then By lemma 5.6 of [13] we have, $V \in CO(X)$.Since (Y,σ) is E.D. Since F is lower slightly m- precontinuous, there exist a pre open set $U \in m_X$ containing x such that $F(U) \cap V \neq \phi$ for each $u \in U$. By lemma (4.4), F is lower almost m- pre continuous.

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