DESIGN OF CENTRALIZED CONTROLLER FOR A COUPLED TANK SYSTEM

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Abstract: This work presents some of the design methods of centralized PI controllers for a coupled tank system. The design methods implemented is based on the transfer function matrix of the system considered. The centralized controller methods used are the Davison method, the decoupler design and the Tanttu and Lieslehto method. The performance of the closed loop system for a step change in the set point is compared. The controller designed for the coupled tank system is also evaluated through realtime study.

***IndexTerms* – Centralized controller, coupled tank, Decoupler .**

I. INTRODUCTION

Many systems in chemical and process industry are multi-input/multi-output (MIMO). In most cases cross coupling between inputs and outputs is low. Therefore conventional single input single output (SISO) controllers can be successfully applied [1]. However, if multivariable systems exhibit stronger Cross-coupling between process inputs and outputs, multivariable controllers should be applied in order to achieve satisfactory performance. Usually two types of control schemes are available to control MIMO processes. The first is decentralized control scheme where single loop (multiloop) controllers are used here the controller is a diagonal one. The second scheme is centralized (full multivariable) controller where the controller is not a diagonal one.

The decentralized control scheme is favoured over centralized control scheme because the control scheme uses a simple algorithm, which is especially important when the control calculations are implemented with analog computing equipment. A second advantage is the case of understanding by plant operating personal, which results from the simplicity of control structure. Since each controller uses only one measured controlled variable and adjusts only one manipulated variable, the actions of the controllers are relatively easy to monitor. A third advantage is that standard control designs have been developed for the common unit operations, such as furnaces, boilers, compressors, and simple distillation towers. This does not mean that a signal control design functions well for all unit operations of the same type. However, several general structures are in common use, and selection, among alternatives can be based on analysis and experience. However ,the design methods of such decentralized controllers require first pairing of input-output variables, and tuning of controllers requires trial and error steps. Only well experienced operators can tune such control loops. For strongly interacting systems, decentralized controllers will not give satisfactory responses. There are some simpler methods of tuning centralized controllers. The centralized control systems require n x n controllers for controlling n output variables using n manipulated variables, which is disadvantage if we use standard PI controllers. But if we are calculating the control action using a computer, then this problem of requiring n x n controllers does not exist. The advantage of the centralized controller is easy to tune, even with knowledge of the steady –state gain matrix alone multivariable PI controllers can be easily designed. The objective of this paper is to review and compare the existing methods of multivariable PI controller and assess their applicability to coupled tank system.

II. MULTIVARIABLE CONTROLLER TUNING METHODS

In SISO control, the primary objective is to maintain only one variable nearer to its set point, though several measured variables involved. By contrast, in MIMO control involves the objective of maintaining several controlled variables at independent set points. This can be achieved in two method they are decentralized control and centralized control. The first method is applied only the interaction is negligible but mostly the real time industrial processes are having more interactions between the variables. So the centralized controller is mostly preferable for MIMO system. The centralized controller tuning methods those which requires minimum modeling efforts and model free control design techniques are considered in this work.The free model multivariable PI tuning techniques to be studied are the Davison method, Tanttu and Lieslehto method and decoupler method**.**

2.1 DAVISON METHOD

Davison method outlined by the Davison [5] is considered in this work with the assumption on the plant being taken is open loop stable and Linear Time Invariant(LTI). Davison has proposed a multivariable PI controller where the matrices Kc and ki are expressed in terms of the steady state gain of the process parameter as

$$K_c = \delta[G(s = 0)]^{-1} \quad (1)$$

$$\boldsymbol{K_i = \varepsilon[G(s=0)]^{-1}} \qquad (2)$$

Here $[\boldsymbol{G(s=0)}]^{-1}$ is called the rough tuning matrix and δ and Ɛ are the fine tuning parameters and these parameters are tuned starting with small positive value and adjusted until the output response of closed loop for step input has the maximum speed of the response. The rough tuning matrix is the inverse of the steady state gain matrix of the system. The fine tuning parameters range is from 0 to 1.

2.2 TANTTU AND LIESLEHTO

Morari and Zafiriou have discussed the design of a PI controller tuning method based on Internal Model Control (IMC). Tanttu and Lieslehto developed a multivariable PI controller tuning based on IMC principles [6]. First, a PI controller (Kc,ij) for each of the scalar transfer functions(Gp,ij) of the process is designed based on the IMC method. Then the multivariable PI controllers can be designed by using the following equations.

$$\boldsymbol{K_c} = \begin{bmatrix} 1/k_{c,11} & \cdots & 1/k_{c,1n} \\ \vdots & \vdots & \vdots \\ 1/k_{c,n1} & \cdots & 1/k_{c,nn} \end{bmatrix}^{-1} \qquad (3)$$

$$\boldsymbol{K_I} = \begin{bmatrix} 1/k_{I,11} & \cdots & 1/k_{I,1n} \\ \vdots & \vdots & \vdots \\ 1/k_{I,n1} & \cdots & 1/k_{I,nn} \end{bmatrix}^{-1} \qquad (4)$$

In this method there is only one tuning parameter. This parameter is approximately the inverse of the dominant bandwidth of the closed loop system and it is based on the first stage deign method (IMC)

2.3 DECOUPLER DESIGN METHOD

In this method the matrix G_c (s) is given for a 2 x2 system by

$$\boldsymbol{G_c(s)} = \begin{bmatrix} G_{c,11} & G_{c,22}d_{12} \\ G_{c,11}d_{21} & G_{c,22} \end{bmatrix} \qquad (5)$$

Where $G_{c,11}$ and $G_{c,22}$ are single loop PI controllers designed for $G_{p,11}$ and $G_{p,22}$. The expressions for d_{12} and d_{22} are given by

$$\boldsymbol{d_{12}} = -\,G_{p,12}/G_{p,11} \qquad (6)$$

$$\boldsymbol{d_{21}} = -\,G_{p,21}/G_{p,22} \qquad (7)$$

The design of d_{12} and d_{21} involves only the substitution of the plant transfer functions and approximating the resulting expressions to that of P,PI or PID controller transfer function.

III. PROCESS DESCRIPTION

The schematic diagram of coupled tank process considered in the present work is as shown in Fig.1. The system consists of two identical cylindrical tanks with equal area of cross section and the tanks are coupled by an inter-connecting pipe. The measured variables are the levels of tank1 (h_1) and tank2 (h_2). Inflow of tank1 (F_{in1}) and tank2 (F_{in2}) are chosen as known input variables. The inputs are adjusted by the applied voltage to the variable speed pump. The outflow is assumed to be proportional to the square root of the difference between the heights of the liquid in the interconnected tanks. Further it is assumed that the density of the liquid is constant throughout. The experiment has two inputs (pump speeds) which can be manipulated to control the two outputs (tank levels). The system exhibits interacting multivariable dynamics because each of the pumps affects both of the outputs.

For the coupled tank system the non linear equation from Bernoulli's law

$$A\frac{dh_1}{dt} = k_1u_1 - \beta_1 a\sqrt{2gh_1} - \beta_X a\sqrt{2g(h_1-h_2)} \qquad (8)$$

$$A\frac{dh_2}{dt} = k_2u_2 + \beta_X a\sqrt{2g(h_1-h_2)} - \beta_2 a\sqrt{2g(h_2)} \qquad (9)$$

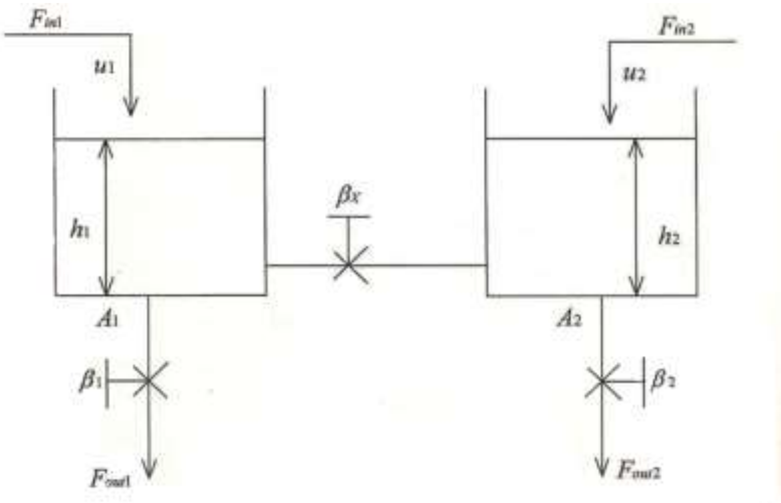


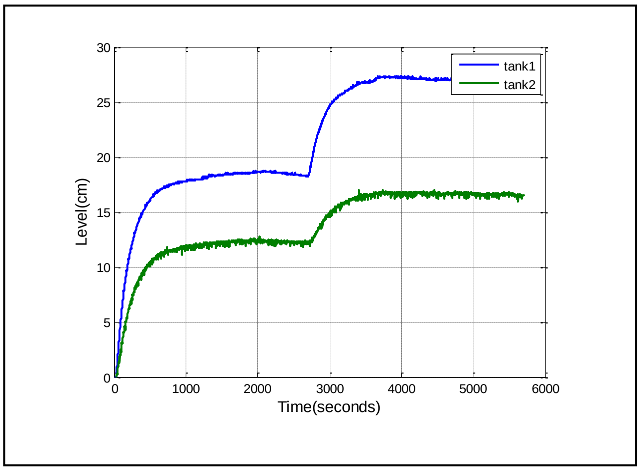
Fig.1 Schematic diagram of coupled tank system

Table 1: Parameters of coupled tank system

PARAMETERS	VALUES
A_1 , A_2 (cm^2)	154
a_2 , a_{12} (cm^2)	0.5
β_1	0.7498
β_2	0.8040
β_X	0.2245
h_1 (cm)	18.32
h_2 (cm)	12.23
u_1 (V)	2.5
u_2 (V)	2.0
g(cm^2/s)	981
k_1(cm^3/V.s)	33.336
k_2(cm^3/V.s)	25.002

The open loop model for two tank is obtained by first bringing the process to steady state with the pump1 and pump2 voltage as 2.5 and 2.0 respectively. The system attains steady state value at 18.32 and 12.23 cm. Then the pump1 voltage is changed to 3.0 voltage the reaction curve is obtained as shown in Fig.2.Similarly Fig.3 is obtained by changing the pump2 voltage to 2.5 by maintaining pump1 voltage. Based on the open loop response the transfer function model is obtained by process reaction curve method and transfer function model is

$$G(s) = \begin{bmatrix} \frac{16.99e^{-12.89s}}{1+214.035s} & \frac{6.691e^{-72.57s}}{1+204.93s} \\ \frac{9.2308e^{-35.0065s}}{1+256.4415s} & \frac{11.38e^{-25.035s}}{1+169.15s} \end{bmatrix} \quad (10)$$



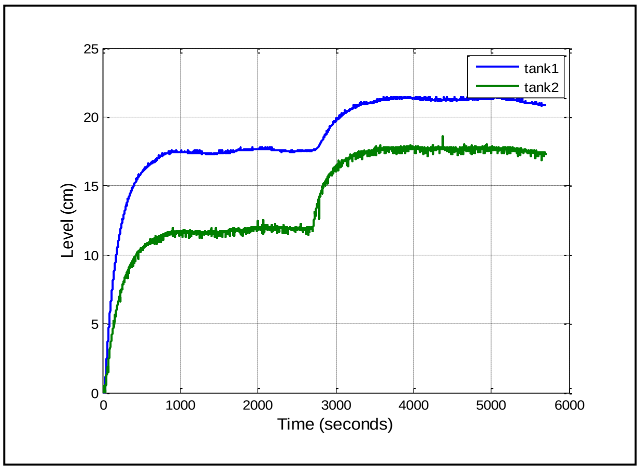


Fig.2 open loop response for input change in pump1 Fig.3 open loop response input change in pump2

IV. Results and Discussions

Based on the transfer function model of the coupled tank and the RGA value the level of tank1 is paired with pump1 and level of tank2 is paired with pump2 [4]. The multivariable controllers are designed based on the open loop transfer function model and the simulation is carried out using MATLAB software.

4.1 Davison method

In Davison method the controller gain is $K_c = \delta[G(s=0)]^{-1}$

From the two tank transfer function

$$G_p(s=0) = \begin{bmatrix} 16.99 & 6.691 \\ 9.2308 & 11.38 \end{bmatrix}$$

Hence $[G_p(s=0)]^{-1}$ is given by

$$\begin{bmatrix} 0.0865 & -0.0509 \\ -0.0702 & 0.1291 \end{bmatrix}$$

The fine tuning parameter δ and Ɛ are made to be equal for reducing the number of tuning parameters. The values for δ=Ɛ =0.25, 0.5, 1.0 are assumed. The resulting matrices of K_c and K_i are given below

For δ=Ɛ=0.25

$$K_c = K_I = \begin{bmatrix} 0.0216 & -0.0127 \\ -0.0175 & 0.0323 \end{bmatrix}$$

For δ=Ɛ=0.50

$$K_c = K_I = \begin{bmatrix} 0.0432 & -0.0254 \\ -0.0351 & 0.0646 \end{bmatrix}$$

For δ=Ɛ=0.75

$$K_c = K_I = \begin{bmatrix} 0.0649 & -0.0381 \\ -0.0526 & 0.0968 \end{bmatrix}$$

For δ=Ɛ=1.0

$$K_c = K_I = \begin{bmatrix} 0.0865 & -0.0509 \\ -0.0702 & 0.01291 \end{bmatrix}$$

4.2 Tanttu and Lieslehto method

In the Tanttu and Lieslehto tuning method, the IMC controller for the individual transfer function has to be designed. Since the transfer function for the two tank system is first order plus dead time the PI controller using IMC is given by

Time delay neglected$K_{c,ij}$

$$K_{c,ij} = \left(\frac{\tau_{p,ij}}{K_{p,ij}\lambda_{ij}}\right) \qquad T_{I,ij} = \tau_{p,ij}$$

$$T_{I,ij} = \tau_{p,ij}$$

Time delay neglected and effective time constant increased by 0.5θ

$$K_{c,ij} = \left(\frac{\tau_{p,ij+\frac{\theta}{2}}}{K_{p,ij}\lambda_{ij}}\right)$$

$$T_{I,ij} = \tau_{p,ij} + \frac{\theta_{ij}}{2}$$

Where λij is the desired closed loop time constant which is the only one tuning factor. Let us assume that $\lambda_{ij} = \alpha\tau_{pij}$ where α is a constant now it is the only single tuning factor. Hence the matrix K_c is given by

$$K_c = \left(\frac{1}{\alpha}\right)\begin{bmatrix} k_{p,11} & k_{p,12} \\ k_{p,21} & k_{p,22} \end{bmatrix}^{-1}$$

$$K_c = \left(\frac{1}{\alpha}\right)\begin{bmatrix} 0.0865 & -0.0509 \\ -0.0702 & 0.1291 \end{bmatrix}$$

Similarly, the integral gain matrix Ki is given by

$$K_I = \left(\frac{1}{\alpha}\right)\begin{bmatrix} k_{p,11}\tau_{p,11} & k_{p,12}\tau_{p,12} \\ k_{p,21}\tau_{p,21} & k_{p,22}\tau_{p,22} \end{bmatrix}^{-1}$$

$$K_I = \left(\frac{1}{\alpha}\right)\begin{bmatrix} 0.5127 & -0.3652 \\ -0.6305 & 0.9687 \end{bmatrix} 10^{-3}$$

For α=0.25

$$K_c = \begin{bmatrix} 0.3459 & -0.2034 \\ -0.2806 & 0.5165 \end{bmatrix}$$

$$K_I = \begin{bmatrix} 0.0021 & -0.0015 \\ -0.0025 & 0.0039 \end{bmatrix}$$

For α=0.50

$$K_c = \begin{bmatrix} 0.1730 & -0.1017 \\ -0.1403 & 0.2582 \end{bmatrix}$$

$$K_I = \begin{bmatrix} 0.0010 & -0.0007 \\ -0.0013 & 0.0019 \end{bmatrix}$$

For α=0.75

$$K_c = \begin{bmatrix} 0.1153 & -0.0678 \\ -0.0935 & 0.1722 \end{bmatrix}$$

$$K_I = \begin{bmatrix} 0.0007 & -0.0005 \\ -0.0008 & 0.0013 \end{bmatrix}$$

For α=1.00

$$K_c = \begin{bmatrix} 0.0865 & -0.0509 \\ -0.0702 & 0.1291 \end{bmatrix}$$

$$K_I = \begin{bmatrix} 0.5127 & -0.3652 \\ -0.6305 & 0.9687 \end{bmatrix} 10^{-3}$$

4.3 Decoupler Design Method

The single loop PI controllers for $G_{p,11}(s)$ and $G_{p,22}(s)$ are calculated using the Ziegler-Nichols tuning method

$$K_c = \frac{0.9\tau_p}{k_p\theta}$$

$$T_i = 3.33\theta$$

The static decoupler is designed in order to get simpler expression for d_{12} and d_{21}

$$d_{12} = -\frac{G_{p,12}(s)}{G_{p,11}(s)} = -0.3938$$

$$d_{21} = -\frac{G_{p,21}(s)}{G_{p,22}(s)} = -0.81$$

$$K_c = \begin{bmatrix} 0.88 & -0.21 \\ -0.713 & 0.5807 \end{bmatrix}$$

$$K_I = \begin{bmatrix} 0.0205 & -0.0025 \\ -0.0166 & 0.007 \end{bmatrix}$$

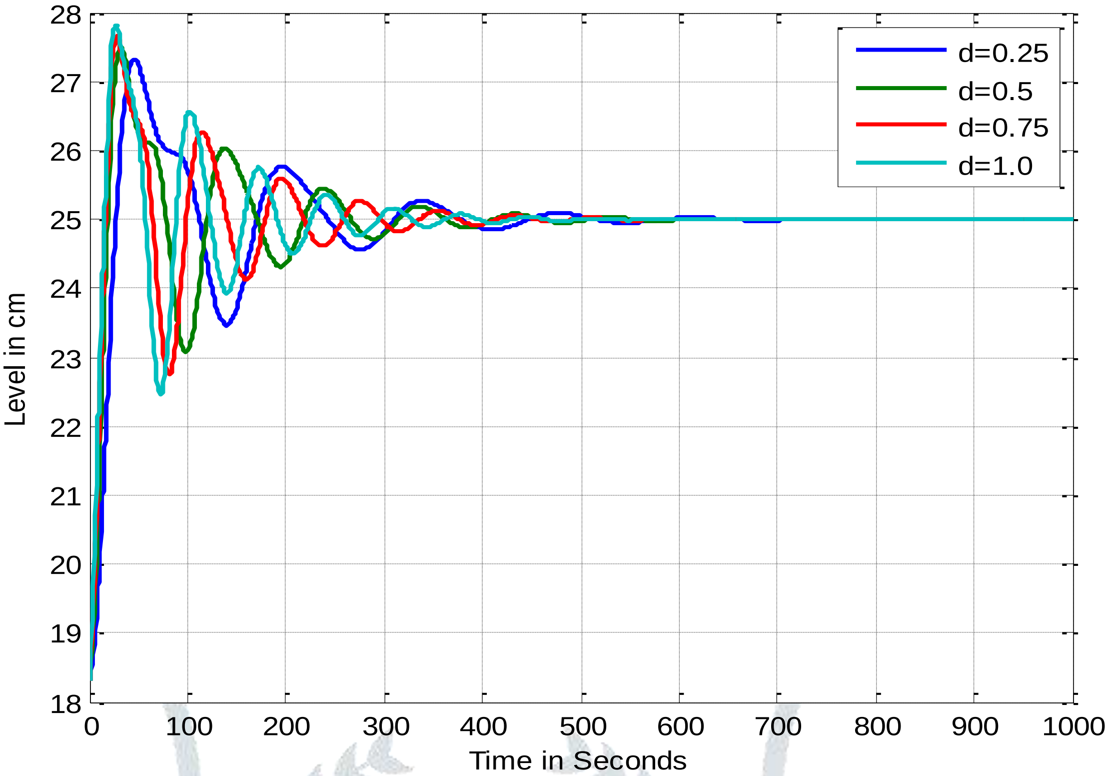


Fig.4 Closed loop response for set point change in tank1 using Davison method

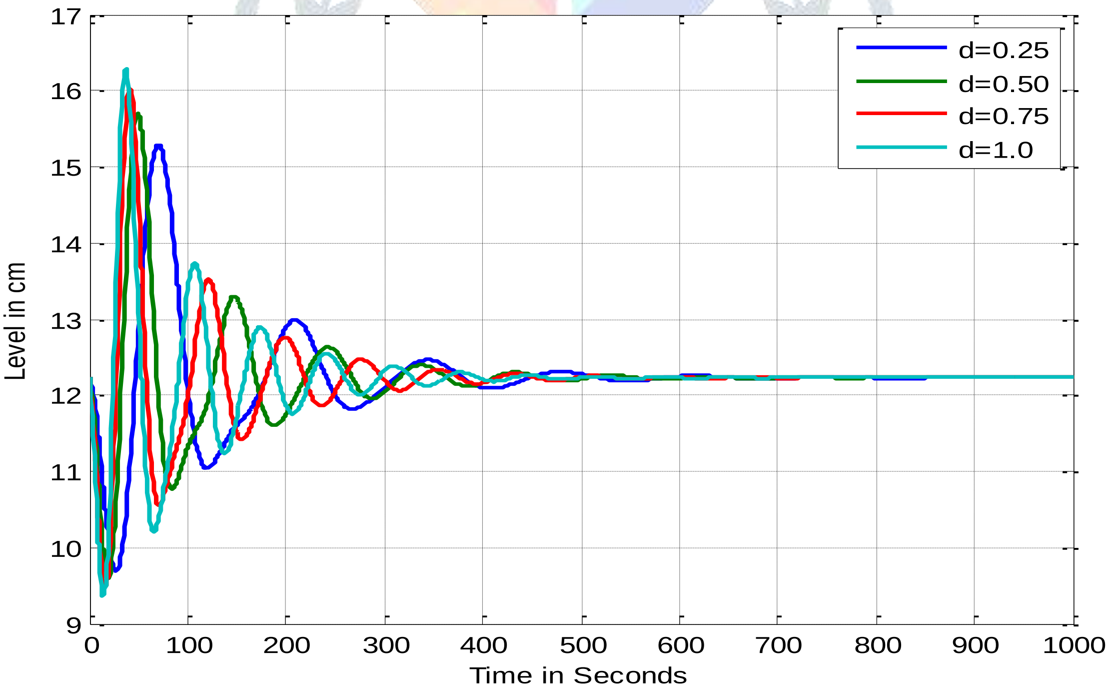


Fig. 5 Closed loop interaction response for set point change in tank1 using Davison method

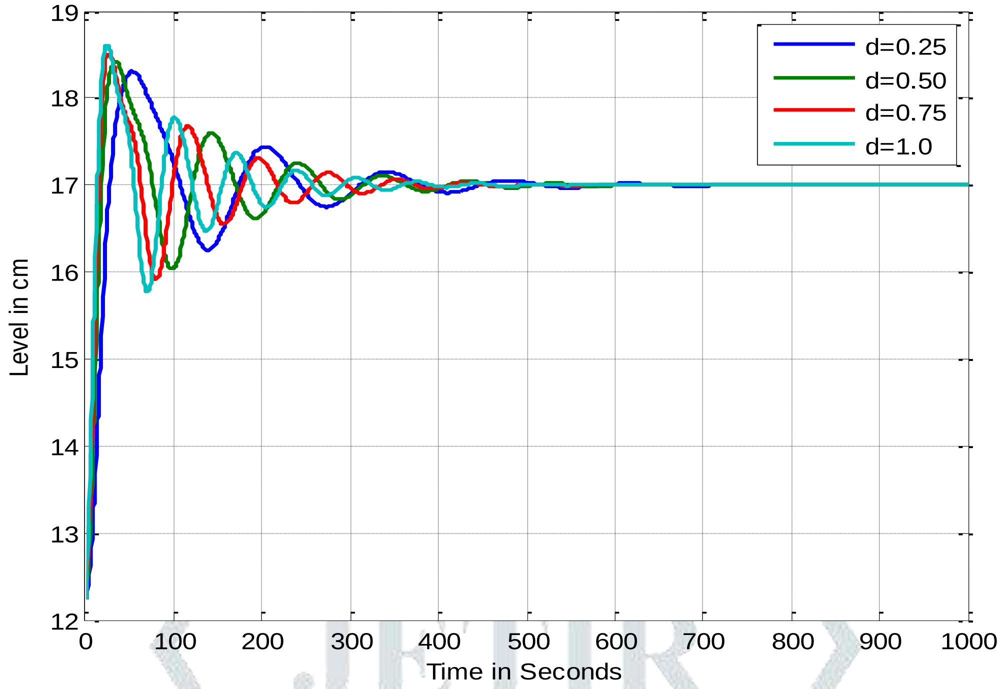


Fig.6 Closed loop response for set point change in tank2 using Davison method

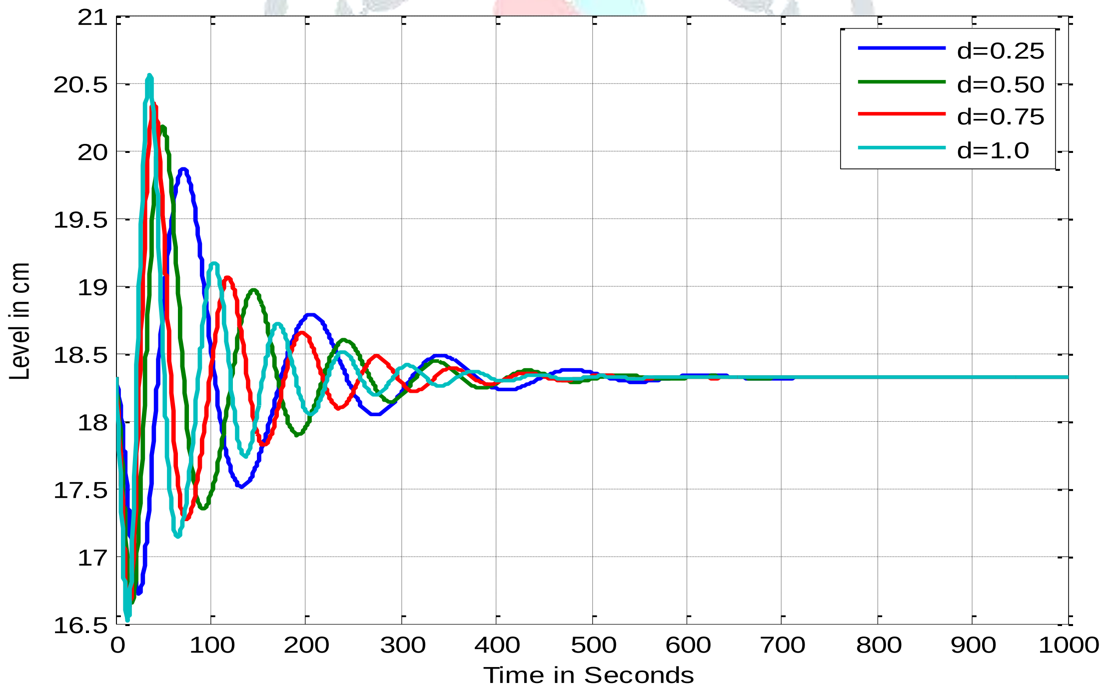


Fig.7 Closed loop interaction response for set point change in tank2 from using davison method

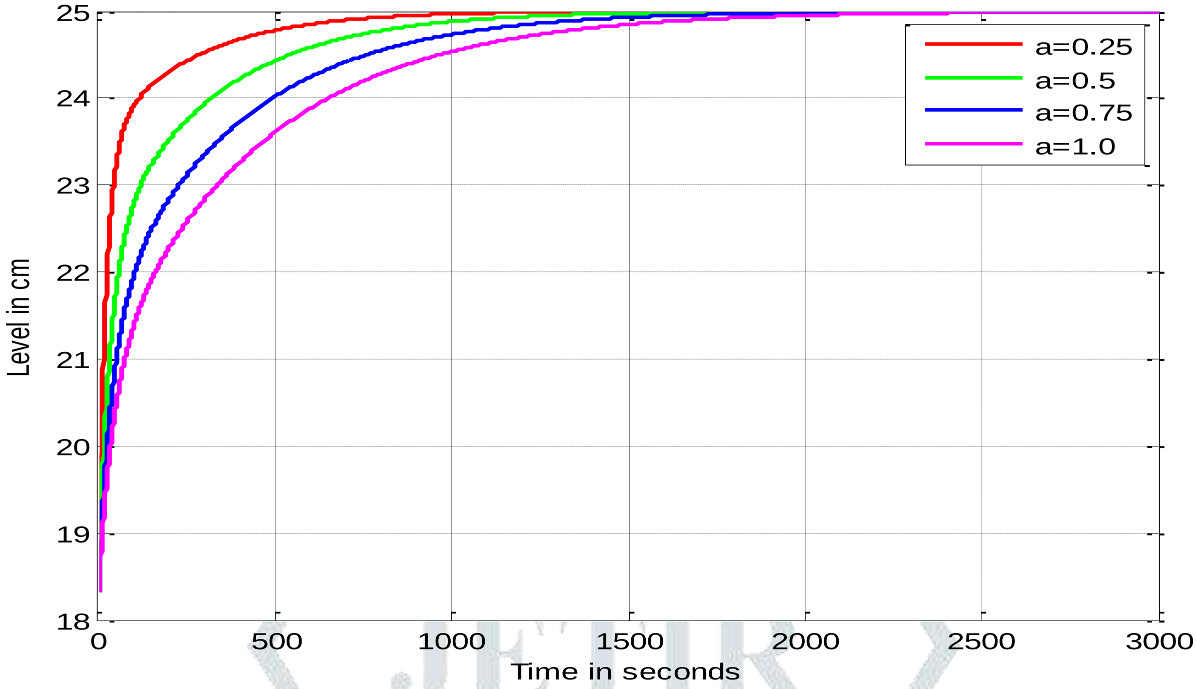


Fig.8 Closed loop respons for set point change in tank1using Tanttu and Lieslehto method

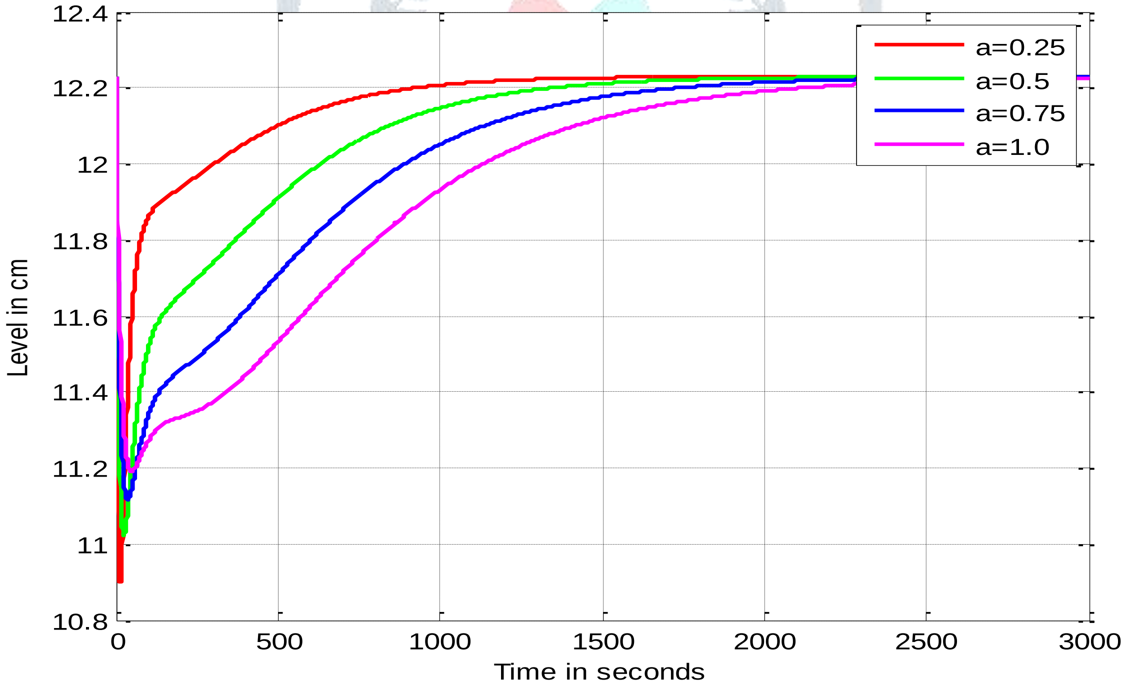


Fig.9 Closed loop response for set point change in tank1using Tanttu and Lieslehto method

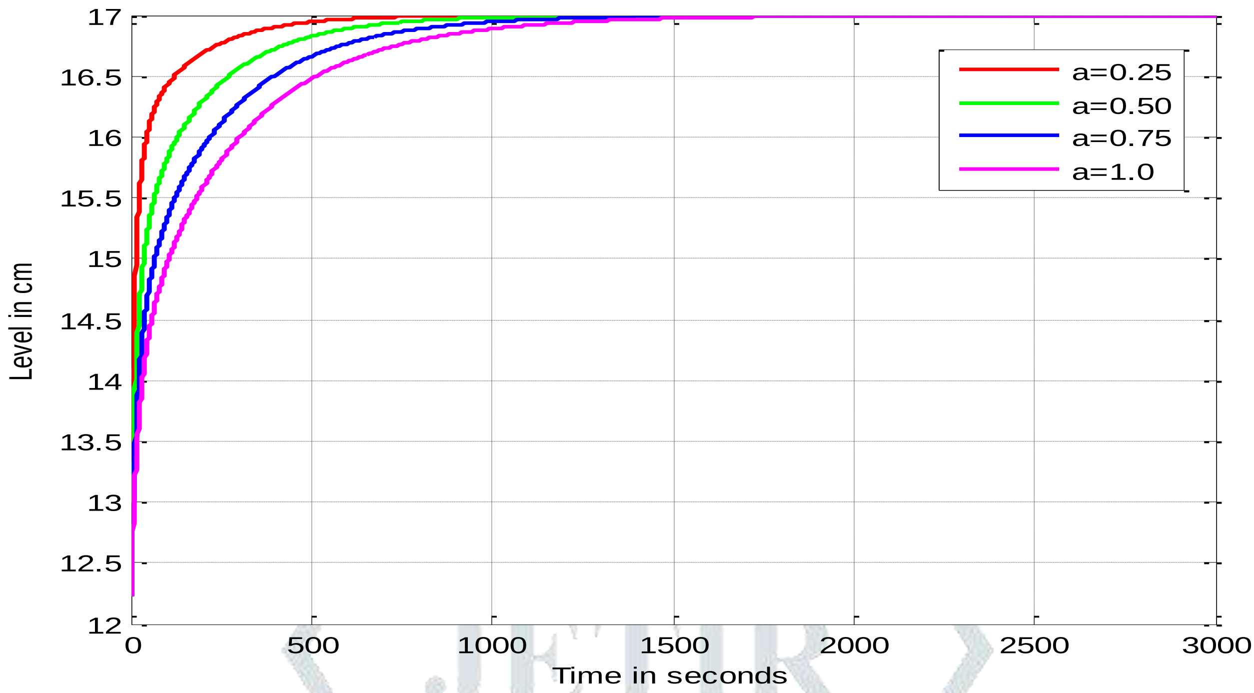


Fig.10 Closed loop response for set point change in tank2 using Tanttu and Lieslehto method

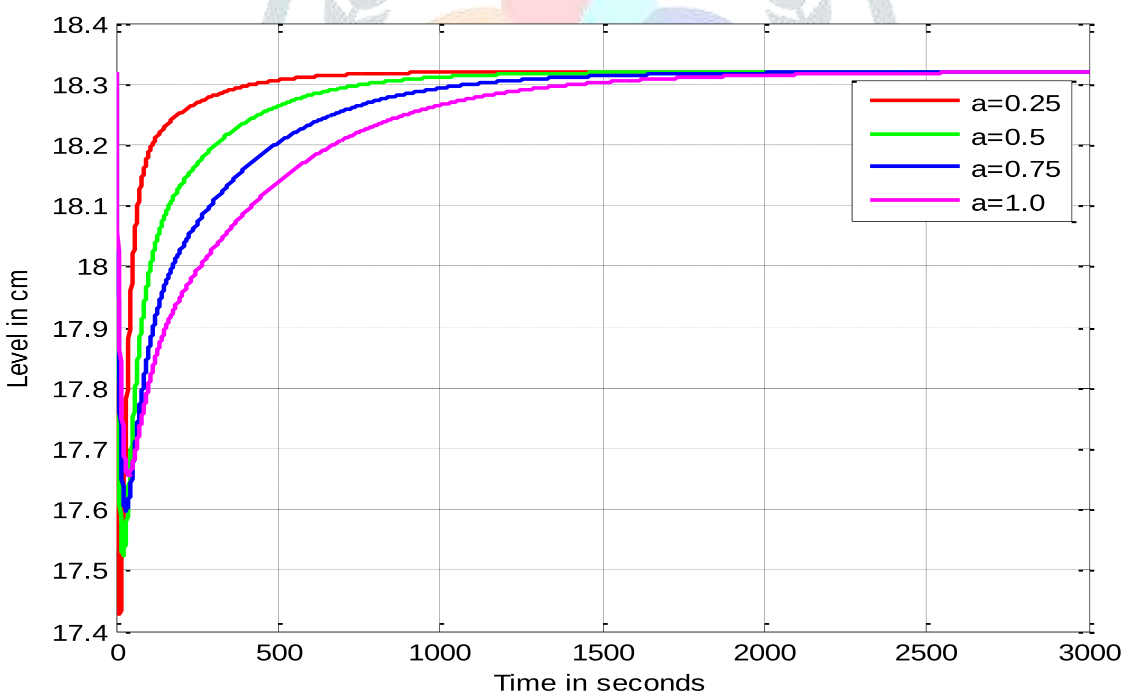


Fig. 11 Closed loop interaction response for set point change in tank1 using Tanttu and Lieslehto method

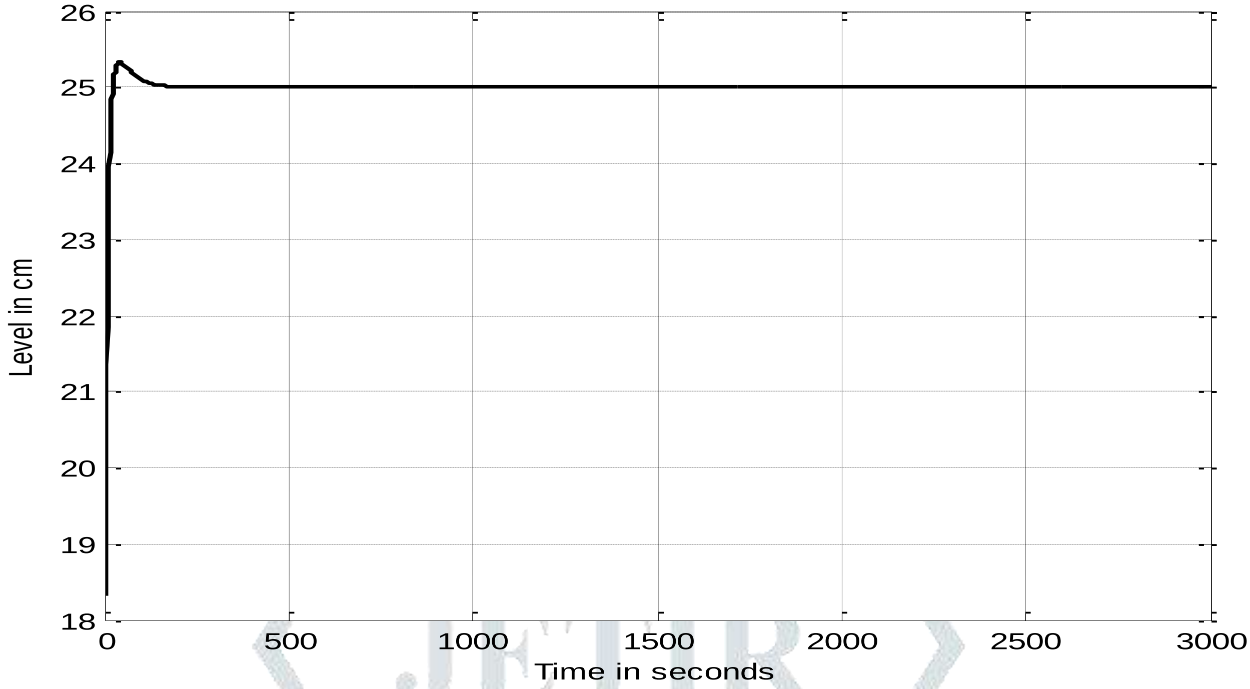


Fig.12 Closed loop response for set point change in tank1 using Decoupling method

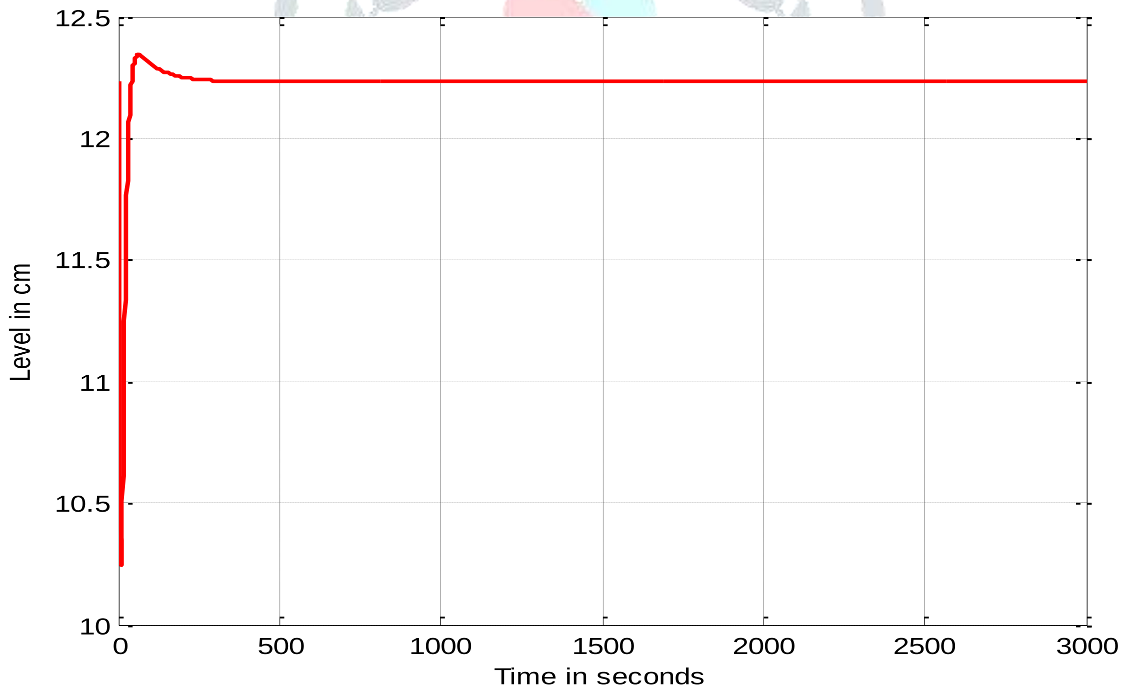


Fig.13 Closed loop interaction response for set point change in tank1 using Decoupling method

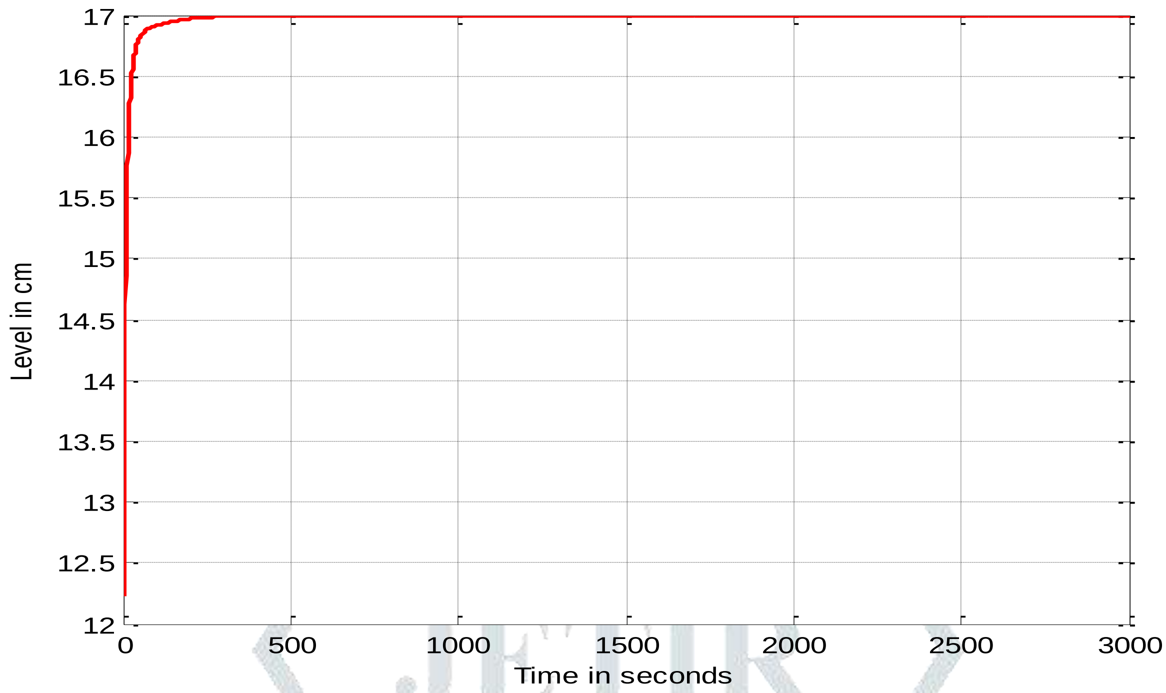


Fig.14 Closed loop response for set point change in tank2 using Decoupling method

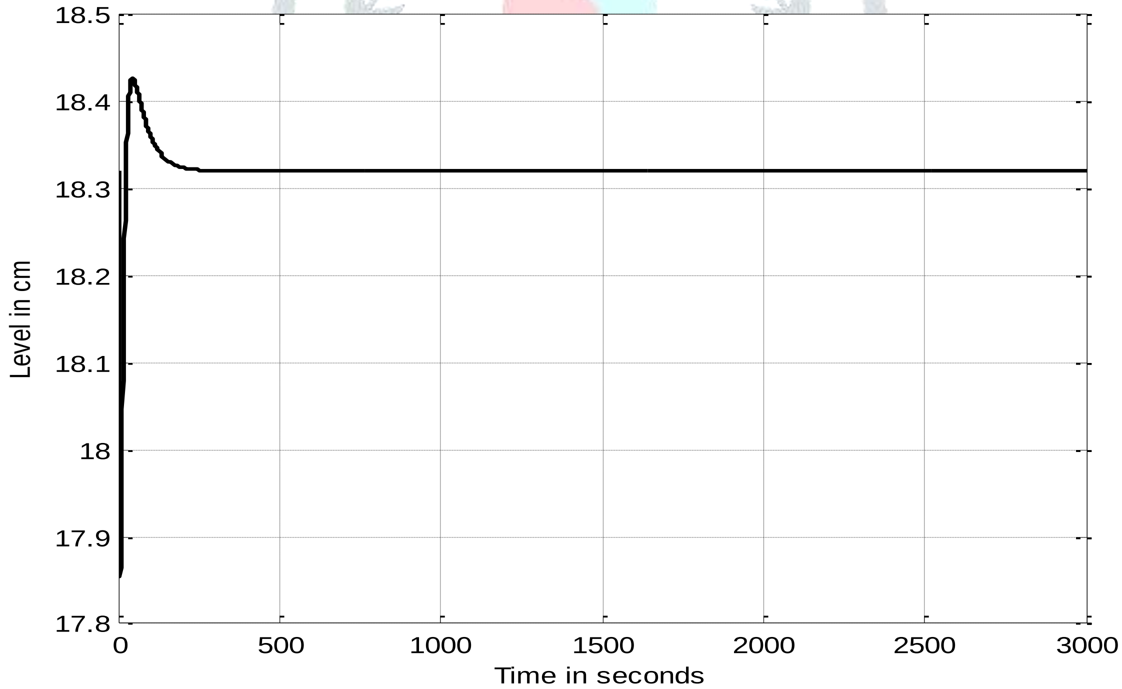


Fig.15 Closed loop interaction response for set point change in tank2 using Decoupling method

Table I1 Performance Measures of Centralized Controllers

Method	For input change in h1		For input change in h2	
	IAE inh1	IAE inh2	IAE inh1	IAE inh2
Davison				
δ=0.25	345.8	293.7	181.5	213
δ=0.50	294.5	258.8	164.8	179.1
δ=0.75	283.2	252.1	150.9	159.2
δ=1.0	286.0	256.8	148.8	153.3
Decoupler	70.86	47.35	13.81	64.68
Tanttu and Lieslehto				
α=0.25	400.7	204	64.04	224.4
α=0.50	801.3	408	128.1	448.8
α=0.75	1202	611.6	192.1	673.2
α=0.10	2011	1694	255.8	897.3

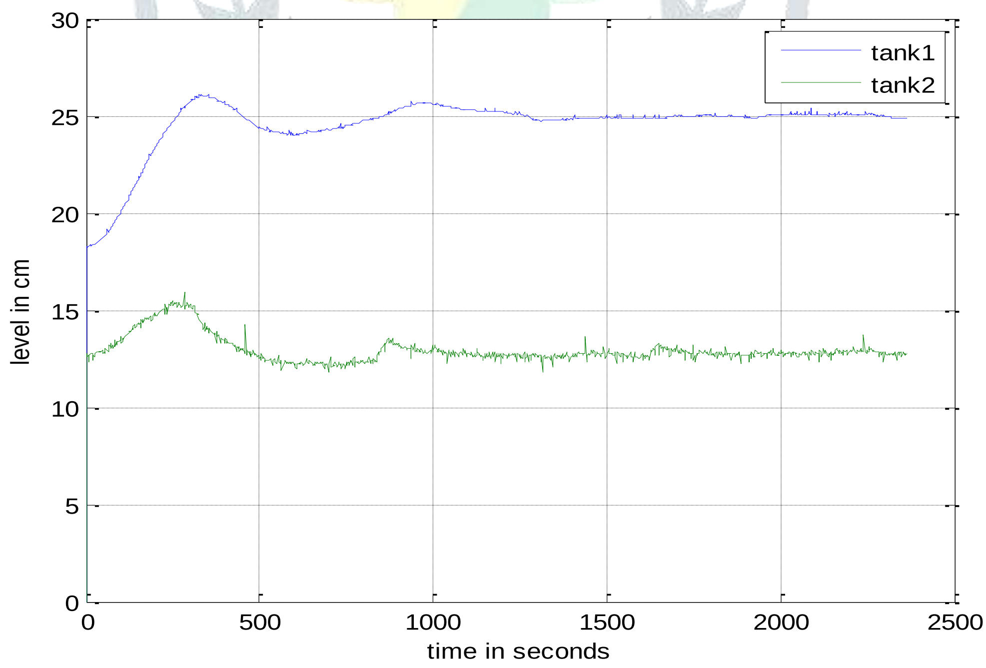


Fig.16 Closed loop real time response of coupled tank for set point change in tank1 using Davison method

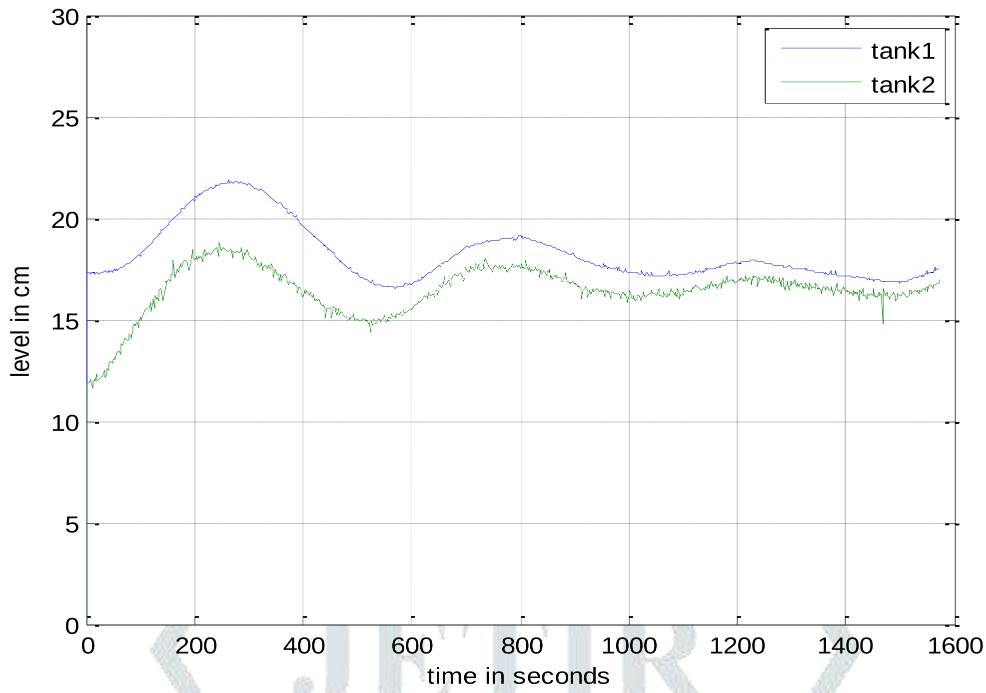


Fig.17 Closed loop real time response of coupled tank for set point change in tank2 using Davison method

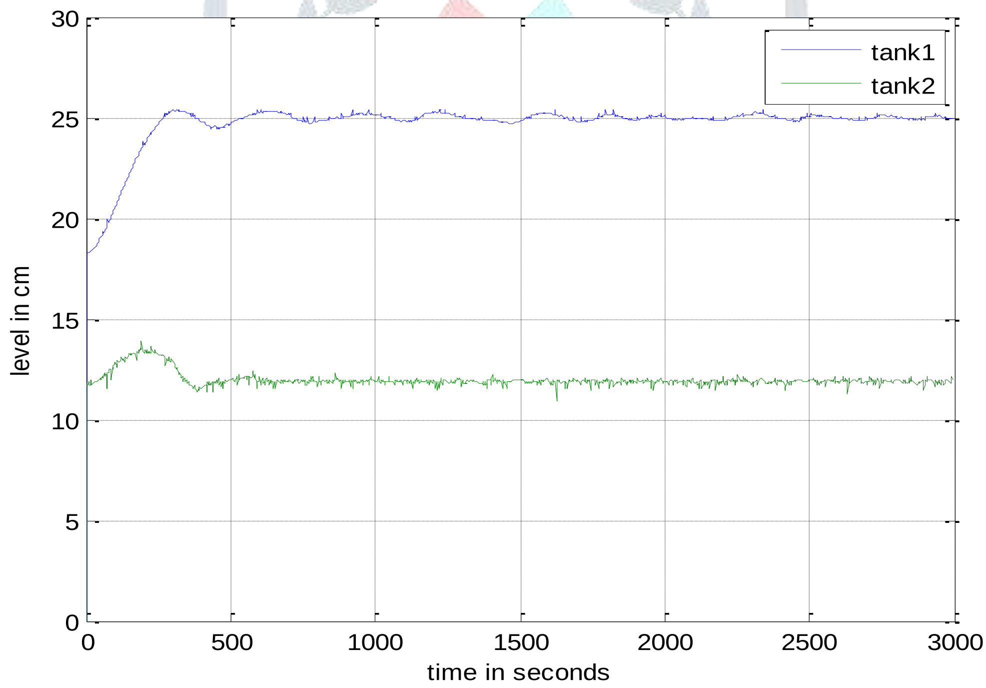


Fig.18 Closed loop real time response of coupled tank for set point change in tank1 from using Tanttu method

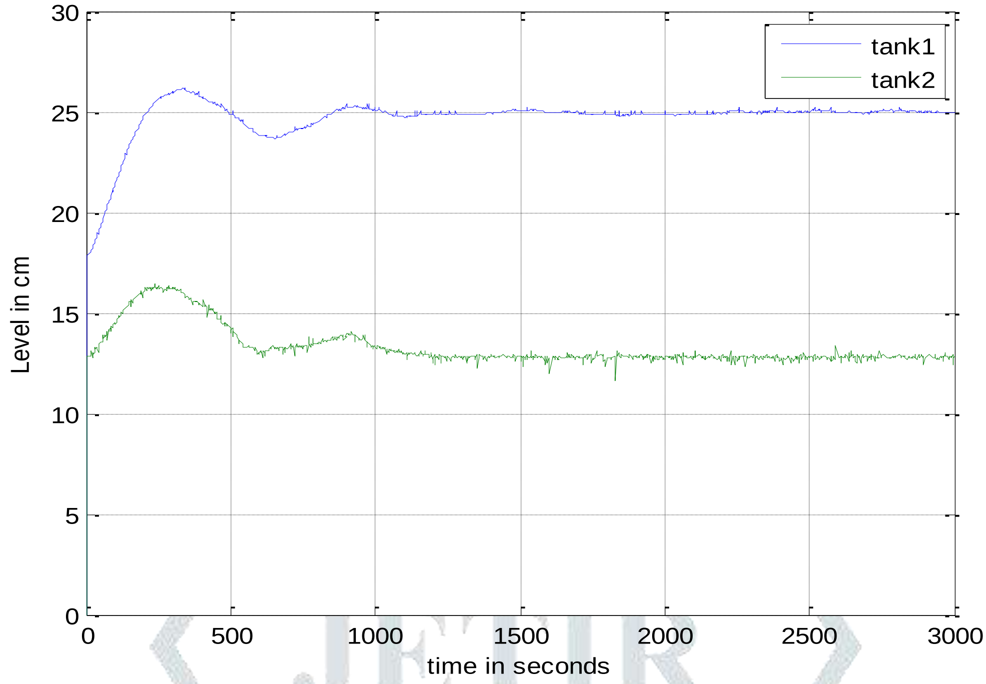


Fig.19 Closed loop real time response of coupled tank for set point change in tank2 using Tanttu method

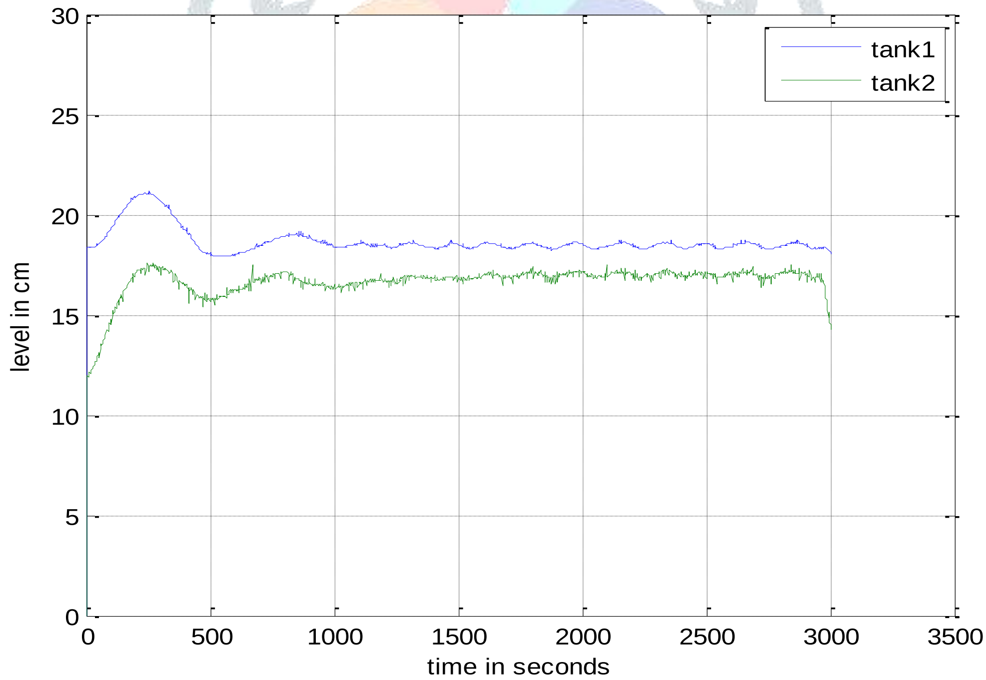


Fig.20 Closed loop real time response of coupled tank for set point change in tank1 susing Decoupler method

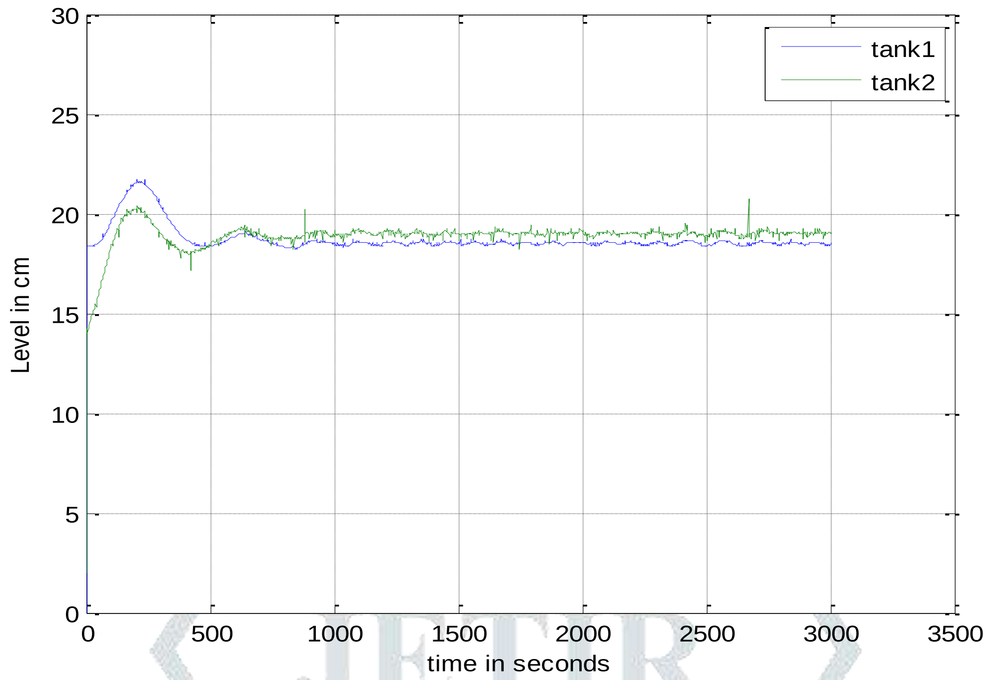


Fig.21 Closed loop real time response of coupled tank for set point change in tank2 using decoupler method

V. CONCLUSIONS

In this work, simple tuning methods, Davisons's method, Tanttu and Lieslehto method and Decoupling method are designed and implemented for coupled tank system. For the centralized controllers designed for coupled tank MIMO system simulations are carried out for servo problems and real time closed loop responses are obtained for the laboratory coupled tank interacting system. IAE values are calculated and tabulated. From the performance measure Davison's method gives better performance than Tanttu and Lieslehto Method. Tanttu and Lieslehto method give sluggish response. IAE values are decreasing for increase in δ values. Tanttu and Lieslehto method gives sluggish response. The controller design of the decoupling method gave the best results. The decoupling method also gave fewer interactions when compared to with other two methods.

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