Optimization for Economic Manufacturing Quantity in Production Process

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Abstract : In the conventional Economic Manufacturing Quantity (EMQ), it is assumed that the product for the production process is perfect, Eventhough, in the real life manufacturing process scenario, there is a possibility of producing faulty items. Thus it is inevitable to include the cost of quality of the product in the EMQ model. In this paper a modified EMQ model with quality loss and inventory cost is attempted. Based on the expected inventory cost for the modified EMQ model, the optimum run length and process mean are obtained under the assumption that the quality characteristic follows Gamma distribution.

***Key words*:** Economic Manufacturing Quantity, Process Mean, Taguchi's Symmetric Quadratic Quality Loss Function.

1. Introduction

Conventional Economic Manufacturing Quantity (EMQ) model is assumed implicitly that items are produced with perfect quality. However, product quality is not always perfect and is usually a function of the production process. In the recent literatures, some attempts have been made related with the economic models including production, maintenance and quality, e.g., Lin [1], Rahim and Ohta [2,3], Hariga and Al-Fawzan [4], Rahim and A1-Hajailan [5], Chen [6], and Chen and Lai [7]. Chen and Chung [8] presented the quality selection problem to the imperfect production system for obtaining the optimum production run length and target value. Rahim and Tuffaha [9] further proposed the modified Chen and Chung's [8] model with quality loss and sampling inspection. Chen [10] considered the process mean of in-control is not equal to the target value for modified Rahim and Tuffaha's [9] model. Chen [12] proposed the modified economic manufacturing quantity model with quality loss and inventory cost. The optimum production run length and process mean are determined under the assumption that the quality characteristic is normally distributed.

In this paper, the modified EMQ model with inventory cost and quality loss is analysed under the assumption that the quality characteristic follows gamma distribution. The symmetric quadratic quality loss function is applied for determining the product quality. The optimum production run length and process mean with the minimum expected total cost are obtained by solving modified EMQ model. The variations of expected inventory cost are studied for the changes in the different parameters involved in the model by using numerical illustration.

2. Economic Manufacturing Quantity Model

Silver and Peterson [11] have defined the expected inventory cost of EMQ model includes the set-up cost and the holding cost as follows:

$$ETC = \frac{Sd}{pT} + \frac{B(p-d)T}{2} \qquad (1)$$

where ETC is the expected total cost per unit time; d is the demand rate; S is the set-up cost for each production run ; p is the production rate; B is the inventory holding cost and T is the production run length in each production cycle (time).

The first derivative of ETC for T equal to zero, and solves for economic manufacturing quantity, Q (=pT). The optimum economic manufacturing quantity and production run length are respectively given by

$$Q^*_E = \sqrt{\frac{2dSp}{B(p-d)}}$$

and

$$T_E^* = \sqrt{\frac{2dS}{p(p-d)B}}$$

3.Modified EMQ Model

According to Rahim and Tuffaha [9] , there are some assumptions made in the modified EMQ model:

(i) The quality characteristic is distributed according to Gamma distribution with known parameter α and unknown parameter β which is to be determined.

(ii) When the production cycle starts, the process is in control state. Once the shift has occurred, the process will remain in an out-of-control state until it is discovered by inspection and followed by some restoration work. Otherwise, the out-of-control state will continue until the end of the production run.

(iii) The symmetric quadratic quality loss function is adopted for evaluating the product quality.

(iv) The elapse time until the occurrence of the assignable cause assumed to be exponentially distributed with mean $1/\lambda$.

(v) The process mean of in-control process, otherwise, manufacturing process, is (β/α) and not equal to the target value. The process mean of out-of-control process is β.

The expected quality loss per item for each production cycle T has been stated by Rahim and Tuffaha [9] and that is given by

$$c = g_1 + \frac{(1-e^{-\lambda T})(g_0 - g_1)}{\lambda T} \qquad (2)$$

where g_o and g_1 are expected quality loss per item of in-control and that of out-of- control processes respectively.

They are defined as follows:

$$g_0 = \int_o^\infty k(x-m)^2 f(x)\, dx \qquad (3)$$

and

$$g_1 = \int_o^\infty k(y-m)^2 f(y)\, dy \qquad (4)$$

Here f(x) and f(y) are the probability density functions of Gamma distribution with parameter (α,β) and β respectively. m is the target value. k is the quality loss co-efficient. On applying the probability density function in the equations (3) and (4) which yield.

$$g_o = \frac{k}{\alpha^2}\{(\beta-\alpha m)^2 + \beta\} \qquad (5)$$

and

$$g_1 = k\,\{(\beta-m)^2 + \beta\} \qquad (6)$$

Hence, the expected total cost per unit time for modified EMQ model with inventory cost and quality loss is as follows:

$$ETC_1 = \frac{sd}{PT} + \frac{B(P-d)T}{2} + dc \qquad (7)$$

Now, apply the expressions (5) and (6) in the equation (2) and the resultant expression for c is substituted in (7)

$$ETC_1 = \frac{sd}{PT} + \frac{B(P-d)T}{2} + dk\,\{(\beta-m)^2+\beta\} + dk\left[\frac{1}{\alpha^2}\{(\beta-\alpha m)^2+\beta\} - \{(\beta-m)^2+\beta\}\right]\left(\frac{1-e^{-\lambda T}}{\lambda T}\right) \quad (8)$$

To minimize the expected total cost per unit, partially differentiate (8) with respect to β and T and equating the equations to zero.

$$\frac{\partial ETC_1}{\partial \beta} = dk\ \{2(\beta-m)+1\} + dk\left[\frac{1}{\alpha^2}\{2(\beta-\alpha m)+1\} - \{2(\beta-m)+1\}\right]\left(\frac{1-e^{-\lambda T}}{\lambda T}\right) = 0 \quad (9)$$

and

$$\frac{\partial ETC_1}{\partial T} = -\frac{sd}{PT^2} + \frac{B(p-d)}{2} + dk\left[\frac{1}{\alpha^2}\{(\beta-\alpha m)^2+\beta\} - \{(\beta-m)^2+\beta\}\right]\left(-\frac{(1-e^{-\lambda T})}{\lambda T^2} + \frac{e^{-\lambda T}}{T}\right) = 0 \quad (10)$$

On solving the equations (9), we get

$$\beta^* = \frac{\alpha^2\lambda T(2m-1) - (1-e^{-\lambda T})[-2\alpha m+1+\alpha^2(2m-1)]}{2\{\alpha^2\lambda T + (1-e^{-\lambda T})[1-\alpha^2]\}} \quad (11)$$

Substitute the expression (11) in the equation (10) and we get,

$$-\frac{sd}{PT^2} + \frac{B(p-d)}{2} + dk\left[\frac{1}{\alpha^2}\left\{\left(\frac{\alpha^2\lambda T(2m-1) - (1-e^{-\lambda T})[-2\alpha m+1+\alpha^2(2m-1)]}{2\{\alpha^2\lambda T + (1-e^{-\lambda T})[1-\alpha^2]\}} - \alpha m\right)^2\right.\right.$$

$$\left.+\left(\frac{\alpha^2\lambda T(2m-1) - (1-e^{-\lambda T})[-2\alpha m+1+\alpha^2(2m-1)]}{2\{\alpha^2\lambda T + (1-e^{-\lambda T})[1-\alpha^2]\}}\right)\right\}$$

$$-\left\{\left(\frac{\alpha^2\lambda T(2m-1) - (1-e^{-\lambda T})[-2\alpha m+1+\alpha^2(2m-1)]}{2\{\alpha^2\lambda T + (1-e^{-\lambda T})[1-\alpha^2]\}} - m\right)^2\right.$$

$$\left.\left.+\left(\frac{\alpha^2\lambda T(2m-1) - (1-e^{-\lambda T})[-2\alpha m+1+\alpha^2(2m-1)]}{2\{\alpha^2\lambda T + (1-e^{-\lambda T})[1-\alpha^2]\}}\right)\right\}\right]\left(-\frac{(1-e^{-\lambda T})}{\lambda T^2} + \frac{e^{-\lambda T}}{T}\right) = 0 \quad (12)$$

In this stage, there is very cumbersome to estimate the optimum values of T* and β*. Therefore, utilize Numerical analysis methodology and get the required optimum values.

4.Numerical Illustration

Suppose that the production rate is p=40 items per unit time. The demand rate is d=30 items per unit time. The holding cost is B=0.1 per item per unit time and the set-up cost is S=50 per production run. The quality characteristic is gamma distributed with unknown parameter β and known parameter α=2. The quality loss coefficient k=5, the target value m=10, and the parameter of exponential distribution λ=0.05.

Based on the above assumptions, the required β*,T* and ETC_1* are estimated and presented in the tables 1 to 8. Similarly, the curves for expected total cost for unit time are exhibited from figures 1 to 8.

Table 1: The effect of α on the optimal solution for modified EMQ model.

α	β^*	T^*	$ETC_1{}^*$
2	12.3091	21.7366	2679.2
3	13.9899	13.7826	4509.6
4	14.9961	10.2707	6008.8
5	15.6679	8.2154	7175.3
6	16.1493	6.8534	8090.5

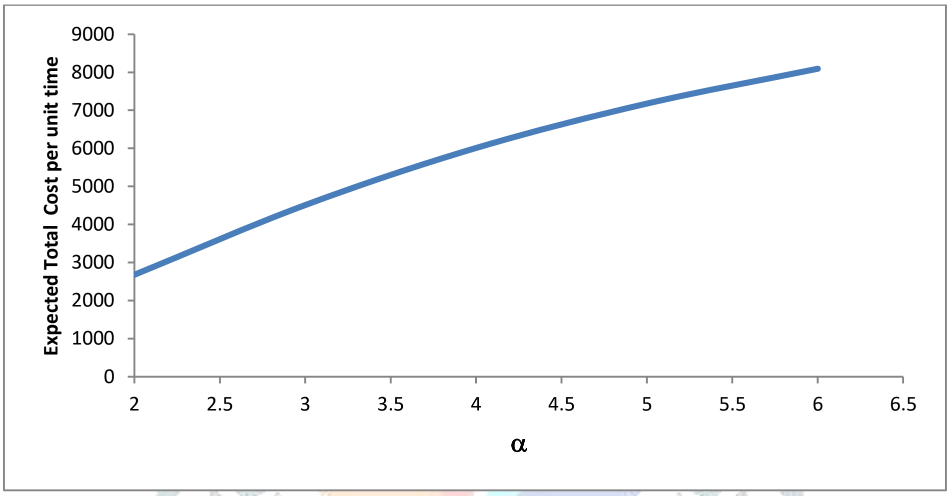

Fig.1

Table 2: The effect of λ on the optimal solution for modified EMQ model.

λ	β^*	T^*	$ETC_1{}^*$
0.03	12.2877	36.5656	2685.8
0.04	12.3009	27.2678	2681.6
0.05	12.3091	21.7366	2679.2
0.06	12.3149	18.0687	2677.7
0.07	12.3193	15.4582	2676.8

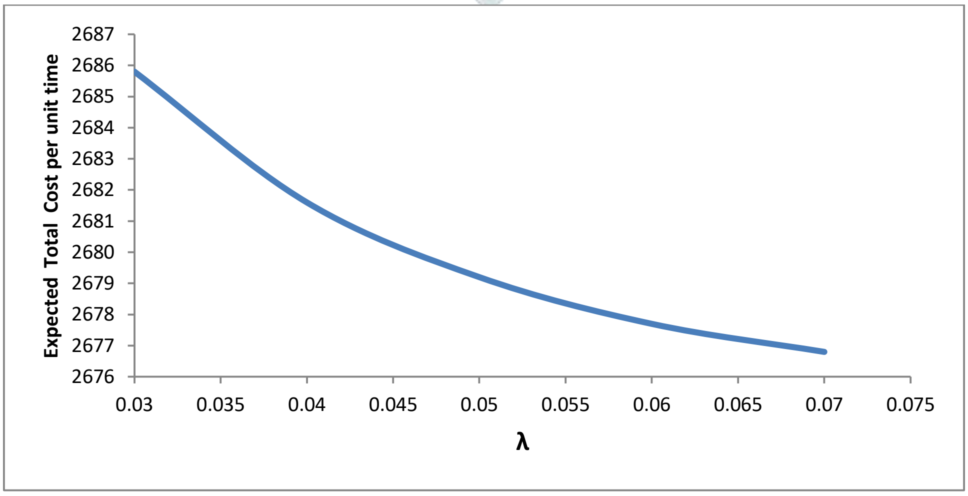

Fig.2

Table3: The effect of m on the optimal solution for modified EMQ model.

m	β^*	T^*	$ETC_1{}^*$
8	9.6337	23.1316	1879.8
9	10.9720	22.3342	2262.8
10	12.3091	21.7366	2679.2
11	13.6453	21.2715	3129.0
12	14.9809	20.8988	3612.2

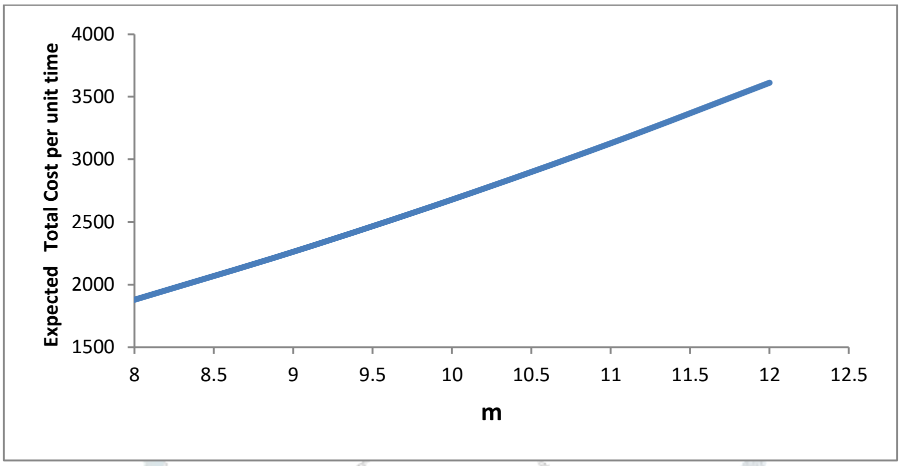

Fig.3

Table 4: The effect of p on the optimal solution for modified EMQ model.

p	β^*	T^*	$ETC_1{}^*$
32	12.3334	21.5099	2671.0
36	12.3212	21.6230	2675.1
40	12.3091	21.7366	2679.2
44	12.2970	21.8511	2683.4
48	12.2848	21.9668	2687.7

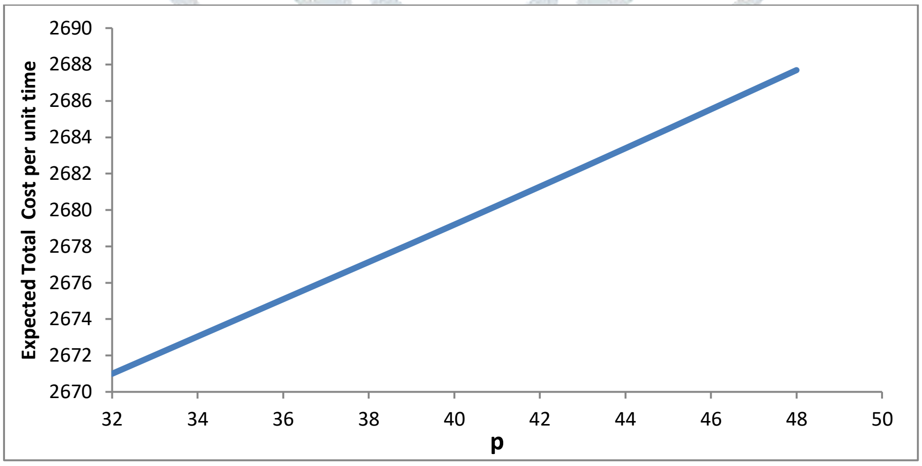

Fig.4

Table 5: The effect of d on the optimal solution for modified EMQ model.

d	β^*	T^*	$ETC_1{}^*$
24	12.2796	22.0163	2152.1

27	12.2961	21.8595	2415.7
30	12.3091	21.7366	2679.2
33	12.3197	21.6377	2942.8
36	12.3284	21.5562	3206.4

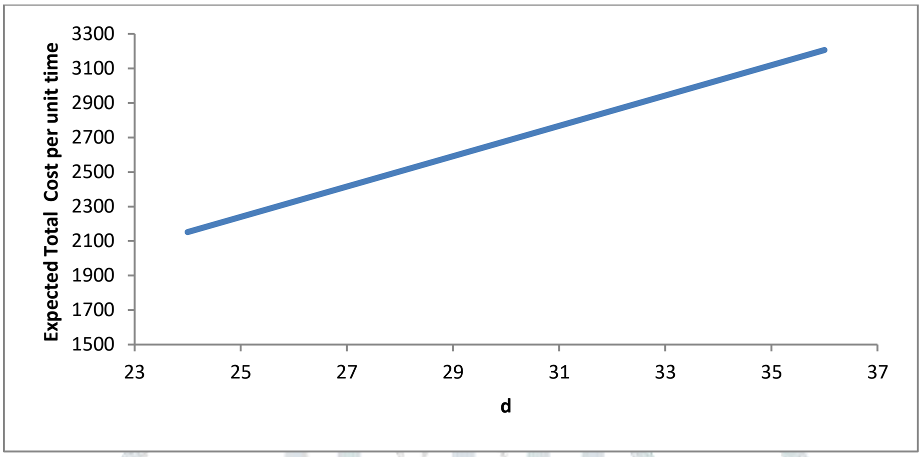

Fig.5

Table 6: The effect of B on the optimal solution for modified EMQ model.

B	β^*	T^*	ETC_1^*
0.1	12.3091	21.7366	2679.2
0.2	12.2796	22.0163	2690.2
0.3	12.2494	22.3077	2701.2
0.4	12.2185	22.6117	2712.5
0.5	12.1869	22.9297	2723.8

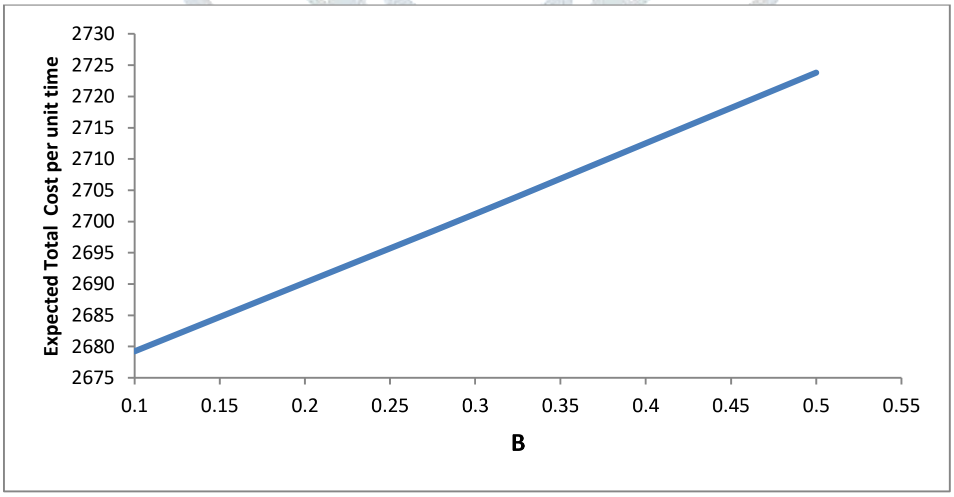

Fig.6

Table 7: The effect of k on the optimal solution for modified EMQ model.

k	β^*	T^*	ETC_1^*
3	12.2926	21.8925	1612.6
4	12.3030	21.7945	2145.9

5	12.3091	21.7366	2679.2
6	12.3132	21.6983	3212.5
7	12.3161	21.6711	3745.9

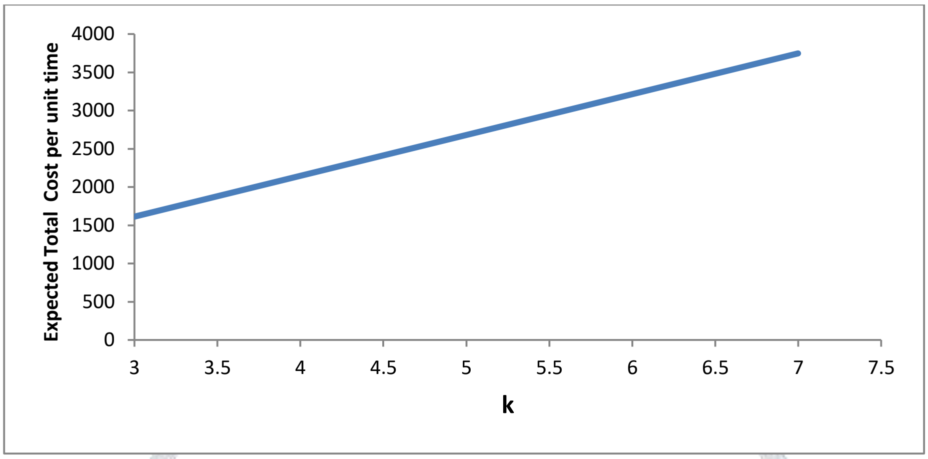

Fig.7

Table 8: The effect of S on the optimal solution for modified EMQ model.

S	β^*	T^*	ETC_1^*
40	12.3082	21.7454	2678.9
45	12.3086	21.7409	2679.0
50	12.3091	21.7366	2679.2
55	12.3096	21.7323	2679.4
60	12.3100	21.7279	2679.6

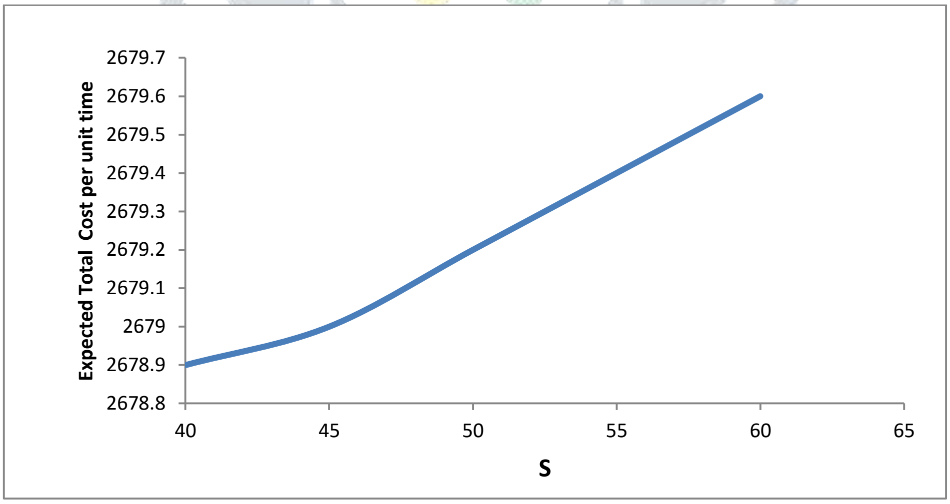

Fig.8

Conclusion

The behavior of ETC_1^* for changing the parameters β^* and T^* are presented through the tables 1 to 8 and figures 1 to 8.

ETC_1^* increases when α, m, d, k, S and β^* increases but T^* decreases.

ETC_1^* increases when p, B and T^* increases but β^* decreases.

ETC_1^* decreases when λ and β^* increases but T^* decreases.

A modified EMQ model have been presented based on the minimum expected total cost per unit time. The solution procedure of proposed method is easy, clear and efficient.

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