# CMI METHOD FOR THE SOLUTION OF LINEAR FORMULATING PROBLEM

**[1]Ms. S. Cynthiya Margaret Indrani , [2]Dr.N.Srinivasan**

[1]Research Scholar& Assistant Professor, [2] Professor and Head

[1] Department of Mathematics,

[1]St. Peter's Institute of Higher Education and Research, Avadi, Chennai-54, India

*ABSTRACT-  In this paper , CMI  method is proposed  to the solution of the linear Formulating problem. This method is very powerful with effortless iterations when compared to the available methods. This method involves less or equivalent number of iterations than  the traditional simplex method . It also saves the worthy time by omitting the calculations of net evaluations.*

*KEY WORDS – Linear Formulating Problem, Optimal Solution, CMI  Method.*
_____

## I.INTRODUCTION

Linear formulating Problem is a Mathematical modelling technique in which a linear function is maximized or minimized when subjected to various restrictions(or constraints) expressed as equation or inequalities. The solution of a linear formulating  problem reduces to finding the optimum value of the linear expression called the objective function.

## II MATHEMATICAL FORMULATION

To Maximize the Objective function Z=CY

Subject to AY=B,   Y≥0

Where Y =q x1 column matrix

A=p x q coefficient matrix

B = p x 1 column matrix

C = 1 X q row matrix.

There are few methods to obtain the solution of the above problem. In that few methods simplex method is the most    general and powerful.

## III PROPOSED CMI  ALGORITHM

To find the optimal solution of any LFP by CMI   method, algorithm is given as follows,

**Step 1 :** Check the objective function of LFP is of maximization or minimization type. If it is to be minimization type then convert it into a maximization type by using the result Min Z = - Max (-Z)

**Step 2 :** Check whether all $b_i$ are non negative. If any $b_i$ is negative then multiply the corresponding constraints by -1.

**Step 3:** Express the given LFP in standard form then obtain an initial feasible solution.

**Step 4 :** <u>**To find the entering column.**</u>

(i)  Only in the initial iteration select the entering column by choosing the greatest coefficient in the objective function $C_j$.

(ii) In successive iterations select the Maximum positive value of the variables ( both basic and non basic).

(iii) If the maximum value attains 1, it should be chosen from body matrix only.

**Step 5 :** <u>**To find the leaving row**</u>

(i)  Select the Minimum value of the variables ( both basic and non basic ) except  0 and 1.

(ii)  If the pivot element attains 0 ,leave that corresponding row and consider the next minimum value

(iii)   If the minimum value occurs in tie break then select the corresponding row of largest element in that entering column.

**Step 6 :** Drop the leaving row and introduce the entering column in the basis and convert the pivot element to unity and all other elements in its column to zero by using usual simplex method and go to next step.

**Step 7 :** Ignore the corresponding row and column. Proceed to step 4 and 5 for remaining elements and repeat the same procedure until an optimum solution is obtained or there is an indication for an unbounded solution.

**Step 8 :** If all rows and columns are ignored then the current solution is an optimum solution.

**IN CASE OF FAILURE:**

i) **While finding the entering column (step-4)**, Once the variable leaves, it should not enter again in the basis in the successive iterations. If it happens, leave that corresponding maximum value and consider the next maximum one from the other variables.

ii) **While finding the leaving row (Step-5),** Once the variable enters , it should not leave from the basis in the successive iterations . If it happens, leave that corresponding minimum value and consider the next minimum one from the other variables

## IV NUMERICAL EXAMPLES

### PROBLEM -1

$$Maximize\ Z = 2y_1 + 4y_2 + y_3$$
Subject to the constraints
$$y_1 + 3y_2 \leq \ 4,$$
$$2y_1 + y_2 \leq \ 3,$$
$$y_2 + 4y_3 \leq \ 3, \text{where } y_1, y_2, y_3 \geq 0$$

**Solution:**

By introducing the slack variables, the set of constraints of the LFP are converted in to the system of equations.
The modified objective function

$$Maximize\ Z = 2y_1 + 4y_2 + y_3 + 0P_1 + 0P_2 + 0P_3$$

**Subject to the constraints**
$$y_1 + 3y_2 + P_1 = \ 4,$$
$$2y_1 + y_2 + P_2 = \ 3,$$
$$y_2 + 4y_3 + P_3 = 3,$$
$$\text{where } y_1, y_2, y_3, P_1, P_2, P_3 \geq 0$$

**Initial iteration.**

| O.F | | | 2 | 4 | 1 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $y_3$ | $P_1$ | $P_2$ | $P_3$ |
| 0 | $P_1$ | 4 | 1 | 3 | 0 | 1 | 0 | 0 |
| 0 | $P_2$ → | 3 | 2 | (1) | 0 | 0 | 1 | 0 |
| 0 | $P_3$ | 3 | 0 | 1 | 4 | 0 | 0 | 1 |

↑

In initial iteration the variable $y_2$ column is having greatest coefficient in objective function. So select that column as an entering column .The minimum value is 2 which is in $P_2$ row.Hence that corresponding row is a leaving row. The Pivotal element is **(1).**

**First iteration:**
By using Step 6, the initial iteration becomes

| O.F | | | 2 | 4 | 1 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $y_3$ | $P_1$ | $P_2$ | $P_3$ |
| 0 | $P_1$ | -5 | -5 | 0 | 0 | 1 | -3 | 0 |
| 4 | $y_2$ | 3 | 2 | 1 | 0 | 0 | 1 | 0 |
| 0 | $P_3$ → | 0 | -2 | 0 | (4) | 0 | -1 | 1 |

↑

In the above iteration the maximum value is 4 which occurs in $y_3$ column, therefore it is an entering column. The minimum value is -5(i.e in $P_1$ row). If we select that one then pivotal element becomes 0, so as per the condition (ii) in step-5 leave that corresponding row and consider the next minimum value. The next minimum value is -2 that occurs in $P_3$ row, hence $P_3$ is the leaving row. The Pivotal element is **(4).**

**Second Iteration:**

By using Step 6, the first iteration becomes

| O.F | | | 2 | 4 | 1 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $y_3$ | $P_1$ | $P_2$ | $P_3$ |
| 0 | $P_1$ → | -5 | (-5) | 0 | 0 | 1 | -3 | 0 |

| 4 | $y_2$ | 3 | 2 | 1 | 0 | 0 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|
| 1 | $y_3$ | 0 | $-\frac{1}{2}$ | 0 | 1 | 0 | $-\frac{1}{4}$ | $\frac{1}{4}$ |
| | | | ↑ | | | | | |

In the above iteration the maximum value is 2 that occurs in $y_1$ and hence it is an entering column. next the minimum value is -5 that occurs in $P_1$ row, hence $P_1$ is a leaving row. The Pivotal element is **(-5).**

**Third iteration:**

By using Step 6, the second iteration becomes

| O.F | | | 2 | 4 | 1 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $y_3$ | $P_1$ | $P_2$ | $P_3$ |
| 2 | $y_1$ | 1 | 1 | 0 | 0 | $\frac{-1}{5}$ | $\frac{3}{5}$ | 0 |
| 4 | $y_2$ | 1 | 0 | 1 | 0 | $\frac{2}{5}$ | $\frac{2}{5}$ | 0 |
| 1 | $y_3$ | $\frac{1}{2}$ | 0 | 0 | 1 | $\frac{-1}{10}$ | $\frac{1}{20}$ | $\frac{1}{4}$ |

Since all the rows and colunms are ignored, hence an optimum basic feasible solution has been reached.

Thereforetheoptimumsolutionis

$$y_1 = 1 \quad , \quad y_2 = 1 \;, \; y_{3 =} \tfrac{1}{2} \quad \text{\& Max Z} = \tfrac{13}{2}$$

**PROBLEM -2:** $Minimize \; Z = -2y_1 - 3y_2$

**Subject to the constraints**

$$2y_1 + 2y_2 \leq \; 7,$$
$$y_1 \leq \; 2,$$
$$y_2 \leq \; 2, \text{ where } y_1, y_2 \geq 0$$

**Solution:** Since the given objective function is of minimization type, convert it as maximization type given as follows.

**Maximize (-Z) = Maximize $Z^* = 2y_1 + 3y_2$**

**Subject to the constraints**

$$2y_1 + 2y_2 \leq \; 7,$$
$$y_1 \leq \; 2,$$
$$y_2 \leq \; 2, \text{ where } y_1, y_2 \geq 0$$

By introducing the slack variables, the set of constraints of the LFP are converted in to the system of equations. The modified objective function

$$\text{Maximize } Z^* = 2y_1 + 3y_2 + 0P_{1+} 0P_{2+} 0P_3$$

**Subject to the constraints**

$$2y_1 + 2y_2 + P_1 = \; 7,$$
$$y_1 + P_2 = \; 2,$$
$$y_2 + P_3 = 2,$$
$$\text{where } y_1, y_2 \;, P_1, P_2, P_3 \geq 0$$

**Initial iteration.**

| O.F | | | 2 | 3 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $P_1$ | $P_2$ | $P_3$ |
| 0 | $P_1$ → | 7 | 2 | (2) | 1 | 0 | 0 |
| 0 | $P_2$ | 2 | 1 | 0 | 0 | 1 | 0 |
| 0 | $P_3$ | 2 | 0 | 1 | 0 | 0 | 1 |
| | | | | ↑ | | | |

In initial iteration the variable $y_2$ is having greatest coefficient in objective function. So select that column as an entering column .The minimum value is 2 which is in $P_1$ row. Hence that corresponding row is a leaving row. The Pivotal element is **(2).**

**First iteration:**

By using Step 6, the initial iteration becomes

| O.F | | | 2 | 3 | 0 | 0 | 0 |
|-----|-----|-----|-----|-----|-----|-----|-----|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $P_1$ | $P_2$ | $P_3$ |
| 3 | $y_2$ | $\dfrac{7}{2}$ | 1 | 1 | $\dfrac{1}{2}$ | 0 | 0 |
| 0 | $P_2$ | 2 | 1 | 0 | 0 | 1 | 0 |
| 0 | $P_3$ $\rightarrow$ | $\dfrac{-3}{2}$ | ( -1) | 0 | $\dfrac{-1}{2}$ | 0 | 1 |

$\uparrow$

In the above iteration the maximum value is 1 which occurs in $y_1$ column(as per condition (iii) in step-4), therefore $y_1$ is an entering column. The minimum value is -1 which occurs in $P_3$ row. So select $P_3$ is a leaving row . The Pivotal element is **(-1)**

**Second iteration:**

By using Step 6, the first iteration becomes

| O.F | | | 2 | 3 | 0 | 0 | 0 |
|-----|-----|-----|-----|-----|-----|-----|-----|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $P_1$ | $P_2$ | $P_3$ |
| 3 | $y_2$ | 2 | 0 | 1 | 0 | 0 | 1 |
| 0 | $P_2$ | $\dfrac{1}{2}$ | 0 | 0 | $\dfrac{-1}{2}$ | 1 | 1 |
| 2 | $y_1$ | $\dfrac{3}{2}$ | 1 | 0 | $\dfrac{1}{2}$ | 0 | -1 |

Since all the rows and colunms are ignored, hence an optimum basic feasible solution has been reached. Therefore the optimumsolutionis

$$y_1 = \frac{3}{2} \quad , \quad y_2 = 2 \quad \& \text{ Max } Z^* = 9$$

But Minimize Z = -Maximize (-Z) = - MaximizeZ$^*$ = -9

**Therefore Min Z = - 9, $y_1 = \frac{3}{2}$ , $y_2 = 2$**

**PROBLEM -3:** $Maximize\ Z = 3y_1 + 2y_2 + 5y_3$

**Subject to the constraints**

$$y_1 + 2y_2 + y_3 \leq 43,$$
$$3y_1 + 2y_3 \leq 46,$$
$$y_1 + 4y_2 \leq 42, \text{ where } y_1, y_2, y_3 \geq 0$$

**Solution:** By introducing the slack variables, the set of constraints of the LFP are converted in to the system of equations.

The modified objective function $Maximize\ Z = 3y_1 + 2y_2 + 5y_3 + 0P_1 + 0P_2 + 0P_3$

**Subject to the constraints**

$$y_1 + 2y_2 + y_3 + P_1 = 43,$$
$$3y_1 + 2y_3 + P_2 = 46,$$
$$y_1 + 4y_2 + P_3 = 42,$$
$$\text{where } y_1, y_2, y_3, P_1, P_2, P_3 \geq 0$$

**Initial iteration.**

| O.F | | | 3 | 2 | 5 | 0 | 0 | 0 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $y_3$ | $P_1$ | $P_2$ | $P_3$ |
| 0 | $P_1$ | 43 | 1 | 2 | 1 | 1 | 0 | 0 |
| 0 | $P_2$ $\rightarrow$ | 46 | 3 | 0 | (2) | 0 | 1 | 0 |
| 0 | $P_3$ | 42 | 1 | 4 | 0 | 0 | 0 | 1 |

$\uparrow$

In initial iteration the variable $y_3$ column is having greatest coefficient in objective function. So select that column as an entering column .The minimum value is 2 which occurs in both $P_1$ $and P_2$ rows .Here tie break occurs, hence by using the condition (iii) in step-5, $P_2$ is a leaving row. The Pivotal element is **(2).**

**First iteration:**

By using Step 6, the initial iteration becomes

| O.F | | | 3 | 2 | 5 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $y_3$ | $P_1$ | $P_2$ | $P_3$ |
| 0 | $P_1$ $\rightarrow$ | 20 | $\dfrac{-1}{2}$ | (2) | 0 | 1 | $\dfrac{-1}{2}$ | 0 |
| 5 | $y_3$ | 23 | $\dfrac{3}{2}$ | 0 | 1 | 0 | $\dfrac{1}{2}$ | 0 |
| 0 | $P_3$ | 42 | 1 | 4 | 0 | 0 | 0 | 1 |

        ↑

In the above iteration  the maximum value is 4 which occurs in $y_2$ column, therefore it is an entering column. The minimum value is -1/2 (i.e in $P_1$ row), hence $P_1$ is the leaving row. The Pivotal element is **(2).**

**Second Iteration:**

By using Step 6, the first iteration becomes

| O.F | | | 3 | 2 | 5 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $y_3$ | $P_1$ | $P_2$ | $P_3$ |
| 2 | $y_2$ | 10 | $\dfrac{-1}{4}$ | 1 | 0 | $\dfrac{1}{2}$ | $\dfrac{-1}{4}$ | 0 |
| 5 | $y_3$ | 23 | $\dfrac{3}{2}$ | 0 | 1 | 0 | $\dfrac{1}{2}$ | 0 |
| 0 | $P_3$ | 2 | 2 | 0 | 0 | $-2$ | 1 | 1 |

Since all the rows and colunms are ignored, hence an optimum basic feasible solution has been reached. Therefore the optimumsolutionis
$$y_1 = 0 \quad , y_2 = 10, \ y_{3 =} 23 \ \& \ \textbf{Max Z} = 135$$

**FAILURE CASE:**

**PROBLEM -4:**     $\textit{Maximize } Z = 4y_1 + 6y_2 + 2y_3$
         **Subject to the constraints**
           $4y_1 - 4y_2 \leq \ 5,$
           $-y_1 + 6y_2 \leq \ 5,$
           $-y_1 + y_2 + y_3 \leq \ 5, \text{where } y_1, y_2 , y_3 \geq 0$

**Solution:** By introducing the slack variables, the set    of constraints of the LFP are converted in to the system of equations.
   The modified objective function
         $\textit{Maximize } Z = 4y_1 + 6y_2 + 2y_3 + 0P_{1+} 0P_{2+} 0P_3$

          **Subject to the constraints**
          $4y_1 - 4y_2 + P_1 = \ 5,$
          $-y_1 + 6y_2 + P_2 = \ \ 5,$
          $-y_1 + y_2 + y_3 + P_3 = 5,$
       where $y_1, y_2 , y_3 , P_1, P_2, P_3 \geq 0$

**Initial iteration.**

| O.F | | | 4 | 6 | 2 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $y_3$ | $P_1$ | $P_2$ | $P_3$ |
| 0 | $P_1$ $\rightarrow$ | 5 | 4 | (-4) | 0 | 1 | 0 | 0 |
| 0 | $P_2$ | 5 | -1 | 6 | 0 | 0 | 1 | 0 |
| 0 | $P_3$ | 5 | -1 | 1 | 1 | 0 | 0 | 1 |

        ↑

In initial iteration the variable $y_2$ column is having greatest coefficient in objective function. So select  that column as an entering column .The minimum value is -4 which is in $P_1$ row.Hence that corresponding row is a leaving row. The Pivotal element is **(-4).**

**First iteration:**

By using Step 6, the initial iteration becomes

| O.F | | | 4 | 6 | 2 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $y_3$ | $P_1$ | $P_2$ | $P_3$ |
| 6 | $y_2$ | $\dfrac{-5}{4}$ | -1 | 1 | 0 | $\dfrac{-1}{4}$ | 0 | 0 |

| 0 | $P_2$ → | $\frac{25}{2}$ | (5) | 0 | 0 | $\frac{3}{2}$ | 1 | 0 |
| 0 | $P_3$ | $\frac{25}{4}$ | 0 | 0 | 1 | $\frac{1}{4}$ | 0 | 1 |

↑

In the above iteration the maximum value is 5 which occurs in $y_1$ column, therefore it is an entering column. The minimum value is -1(i.e in $y_2$ row), but according to condition (ii) in failure case , leave that corresponding row and select the next minimum value. The next minimum value is ¼(i.e in $P_3$ row), If we select that one then the pivotal element becomes 0, so as per the condition (ii) in step-5 leave that corresponding row also and consider the next minimum value. The next minimum value is 3/2 that occurs in $P_2$ row hence $P_2$ is the leaving row. The Pivotal element is **(5)**.

### Second Iteration:
By using Step 6, the first iteration becomes

| O.F | | | 4 | 6 | 2 | 0 | 0 | 0 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $y_3$ | $P_1$ | $P_2$ | $P_3$ |
| 6 | $y_2$ | $\frac{5}{4}$ | 0 | 1 | 0 | $\frac{1}{20}$ | $\frac{1}{5}$ | 0 |
| 4 | $y_1$ | $\frac{5}{2}$ | 1 | 0 | 0 | $\frac{3}{10}$ | $\frac{1}{5}$ | 0 |
| 0 | $P_3$ → | $\frac{25}{4}$ | 0 | 0 | (1) | $\frac{1}{4}$ | 0 | 1 |

↑

In the above iteration the maximum value 1 occurs in $y_3$ column as per the condition (iii) in step-4, $y_3$ is an entering column. For the minimum value, by using the conditions (ii) in failure case skip the $y_2$ and $y_1$ rows and select the next minimum value ie ¼ which is in $P_3$ row, hence $P_3$ is a leaving row. The Pivotal element is **(1)**.

### Third iteration:
By using Step 6, the second iteration becomes

| | | | 4 | 6 | 2 | 0 | 0 | 0 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $C_B$ | BASIS | $y_B$ | $y_1$ | $y_2$ | $y_3$ | $P_1$ | $P_2$ | $P_3$ |
| 6 | $y_2$ | $\frac{5}{4}$ | 0 | 1 | 0 | $\frac{1}{20}$ | $\frac{1}{5}$ | 0 |
| 4 | $y_1$ | $\frac{5}{2}$ | 1 | 0 | 0 | $\frac{3}{10}$ | $\frac{1}{5}$ | 0 |
| 2 | $y_3$ | $\frac{25}{4}$ | 0 | 0 | 1 | $\frac{1}{4}$ | 0 | 1 |

Since all the rows and colunms are ignored, hence an optimum basic feasible solution has been reached. Therefore the optimumsolutionis

$$y_1 = \frac{5}{2} \; , \; y_2 = \frac{5}{4} , \; y_3 = \frac{25}{4} \; \& \; \textbf{Max Z} = 30$$

## V.CONCLUSION
CMI method has been derived to obtain the solution of linear Formulating problem. we observed that the solution obtained in less iteration or at the most equal iterations by our modified technique. It is very easy to apply and reduces our manual work. This method save our worthy time as there is no need to calculate the net evaluations.

## REFERENCES
[1] Kalpana Lokhande; Pranay .Khobragade and .W. Khobragade: Alternative approach to simplex method, International journal of ngineering and innovative Technology, volume 4,Issue 6, pg:123-127
[2] Mrs Lokhande K. G., Khobragade N. W., Khot P.G: Simplex method : An Alternative Approach, International Journal of, International journal of Engineering and innovative Technology, volume 3, Issue 1, pg:426-428
[3] Khobragade N. W., Khot P.G: An Alternative Approach to the Simplex method-I ,Bulletin of pure and applied sciences,volume 23(E) pg:35-40(2004)