LINEAR ESTIMATION IN WEIBULL TYPE MODELS

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***Abstract:** Having defined Weibull type distributions as the unification of (i) Weibull (ii) Exponential (iii) Rayleigh (iv) Laplace (v) Reflected Rayleigh and (vi) Reflected (Double) Weibull distributions, derived moments of order statistics of Weibull type distributions to facilitate linear estimation of location and scale parameters for specified values of the shape parameter from doubly censored samples using Lloyd's (1952) method. Also the moments of order statistics and coefficients are evaluated. MATLAB programs have been developed for the evaluation of moments of order statistics and coefficients of BLUEs for specified values of shape values of parameter from doubly censored samples.*

***Index Terms:** Order statistics, Distributions, Reflected (Double) Weibull distribution, Linear estimation*

1. INTRODUCTION

The Weibull Distribution introduced by Weibull (1939), Swedish Physicist, in his paper "The phenomenon of rupture in solid", and formally named after him in1951. He used this model to the data from problems dealing with yield strength of Bofors steel, fiber strength of Indian cotton, fatigue life of an ST-37 steel, statures of adult males born in British Isles, and breadth of beans of Pharsalus vulgaris. Hallinan (1993) has provided an excellent review of the Weibull distribution by presenting historical facts.

The Weibull distribution is undeniably the distribution that has received maximum attention. The genesis of Weibull appears to be,

A random variable X has a Weibull distribution with three parameters, $c(>0)$ shape parameter; $\sigma(>0)$ scale parameter; and μ location parameter such that

$$Y=\left(\frac{X-\mu}{\sigma}\right)^{c} \qquad (1.1)$$

has the standard exponential distribution with probability density function

$$g(y)=Exp(-y);\quad y>0 \qquad (1.2)$$

As a result, the probability density function of the Weibull random variable X is then

$$f(x)=\frac{c}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{c-1} Exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^{c}\right\};\quad x>0 \qquad (1.3)$$

2. WEIBULL TYPE DISTRIBUTIONS

The idea Holla and Bhattacharya (1968) in defining asymmetric reflected distribution is used here to define a new family of distributions called the Weibull type distributions and its probability density function is given by

$$f(x)=\tau\left(\frac{c}{\sigma}\right)\left(\frac{x-\mu}{\sigma}\right)^{c-1} Exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^{c}\right\};\quad \text{if } x\ge 0 \qquad (2.1)$$

$$=(1-\tau)\left(\frac{c}{\sigma}\right)\left(\left|\frac{x-\mu}{\sigma}\right|\right)^{c-1} Exp\left\{-\left(\left|\frac{x-\mu}{\sigma}\right|\right)^{c}\right\};\quad \text{if } x<0 \qquad (2.2)$$

and its cumulative distribution function:

$$F(x)=1-\tau Exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^{c}\right\};\text{ if } x\ge 0 \qquad (2.3)$$

$$=(1-\tau)Exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^{c}\right\};\text{if } x<0 \qquad (2.4)$$

where $\sigma, c>0$ and $0\le\tau\le 1$

The reason for calling the above distribution as Weibull type is that many of the distributions related to Weibull distribution can be obtained from this by suitably choosing the values of c and τ. For example:

I. $\tau=1$ Weibull distribution

II. $\tau=1$ and $c=1$ Exponential distribution

III. $\tau=1$ and $c=2$ Rayleigh distribution

IV. $\tau=0.5$ and $c=1$ Reflected exponential (Laplace) distribution

V. $\tau=0.5$ and $c=2$ Reflected Rayleigh distribution

VI. $\tau=0.5$ Reflected Weibull (Double Weibull) distribution

Weibull type frequency curves for different values of the parameters τ,σ and c, and cdf are shown in Fig.(2.1) through Fig. (2.6)

Figure 2.1: Density function of Weibull type models for various values of parameters c,σ and $\tau=0.25$

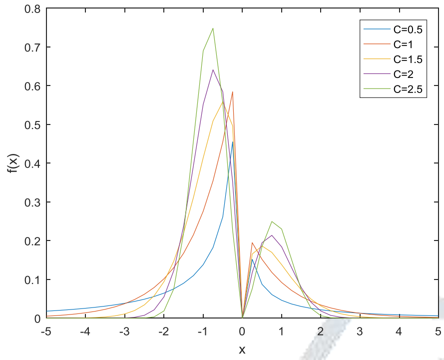


Figure 2.2: Density function of Weibull type models for various values of parameters c,σ and $\tau=0.5$

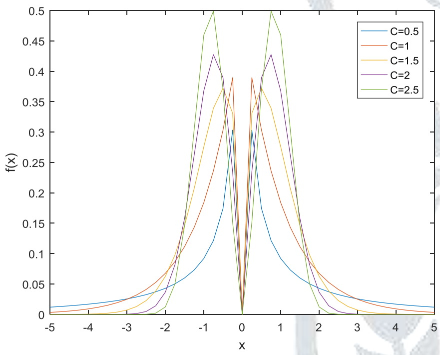


Figure 2.3: Density function of Weibull type models for various values of parameters c,σ and $\tau=0.75$

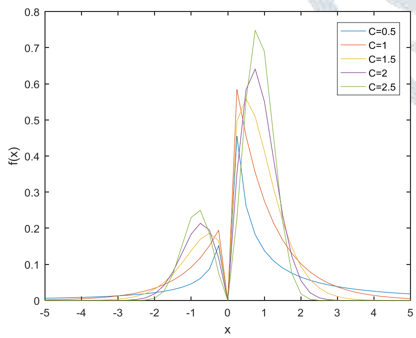


Figure 2.4: Distribution function of Weibull type models for various values of parameters c,σ and $\tau=0.25$

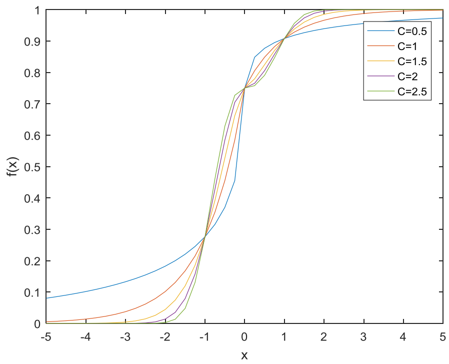


Figure 2.5: Distribution function of Weibull type models for various values of parameters c,σ and $\tau=0.5$

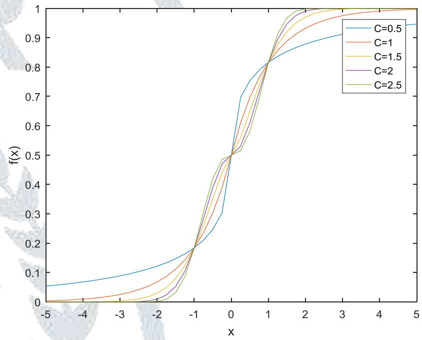
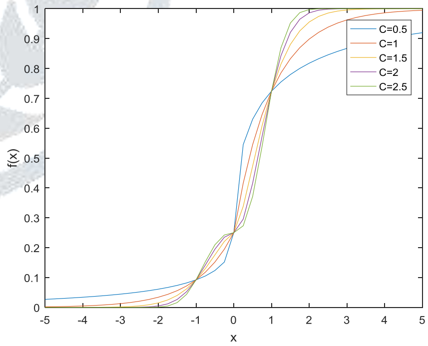


Figure 2.6: Distribution function of Weibull type models for various values of parameters c,σ and $\tau=0.75$



3. MOMENTS OF ORDER STATISTICS

To study the linear estimations, the required moments of order statistics of Weibull type models are derived by us and the details are as under.

$$\text{Let } Y=\left(\frac{x-\mu}{\sigma}\right)$$

Then the standard Weibull type distribution having density

$$g(y)=\tau(c)y^{c-1}Exp\left\{-y^{c}\right\} \text{ ; if } y\geq 0 \qquad (3.1)$$

$$=(1-\tau)(c)|y|^{c-1} Exp\left\{-|y|^{c}\right\};\ \text{if}\ \ y<0 \qquad (3.2)$$

and Cumulative distribution function is

$$G(y)=1-\tau Exp\left\{-y^{c}\right\}\ ;\text{if}\ \ y\geq 0 \qquad (33)$$

$$=(1-\tau)\,Exp\left\{-|y|^{c}\right\};\ \text{if}\ \ y<0 \qquad (3.4)$$

where $c>0$ and $0\leq\tau\leq 1$

Denoting $E\left(y_{i:n}^{(k)}\right)$ as $\alpha_{i:n}^{(k)}$ is given by

$$\alpha_{i:n}^{(k)}=\frac{1}{\beta(i,n-i+1)}\int_{-\infty}^{\infty} y_i^{k}\left[F(y_i)\right]^{i-1}\left[1-F(y_i)\right]^{n-i} g(y_i)\,dy_i$$

$$=\frac{1}{\beta(i,n-i+1)}\left[(-1)^{k}\sum_{m=0}^{i-1}(-1)^{m}\binom{n-i}{m}(1-\tau)^{(i+m)}(i+m)^{-\left(1+\frac{k}{c}\right)}\Gamma\left(1+\frac{k}{c}\right)\right.$$

$$\left.+\sum_{m=0}^{i-1}(-1)^{m}\binom{n-i}{m}\tau^{(n-i+m+1)}(n-i+m+1)^{-\left(1+\frac{k}{c}\right)}\Gamma\left(1+\frac{k}{c}\right)\right] \qquad (3.5)$$

Denoting $E\left(y_{i:n}^{(1)},y_{j:n}^{(1)}\right)$ by $\alpha_{i,j:n}^{(1,1)}$ is given by

$$\alpha_{i,j:n}^{(1,1)}=\frac{n!}{(i-1)!(j-i-1)!(n-j)!}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{y_j} y_i y_j\left[F(y_i)\right]^{i-1}\left[F(y_j)-F(y_i)\right]^{j-i-1}\right.$$

$$\left.\left[1-F(y_j)\right]^{n-j} g(y_i)g(y_j)\,dy_i\,dy_j\right]$$

$$=\frac{n!}{(i-1)!(j-i-1)!(n-j)!}\left[\sum_{a=0}^{n-j}\sum_{b=0}^{j-i-1}(-1)^{(a+b)}\binom{j-i-1}{a}\binom{n-j}{b}(1-\tau)^{(j+b)}\right.$$

$$\frac{\Gamma\left(\frac{1}{c}+1\right)}{(j-i-a+b)^{\left(\frac{1}{c}+1\right)}}\sum_{l=0}^{\infty}\frac{\Gamma\left(\frac{2}{c}+2+l\right)}{\Gamma\left(\frac{1}{c}+2+l\right)(j+b)^{\left(\frac{2}{c}+2+l\right)}}+\sum_{a=0}^{i-1}\sum_{b=0}^{j-i-1}(-1)^{(a+b)}\binom{i-1}{a}$$

$$\binom{j-i-1}{b}(\tau)^{(n-i+a+1)}\frac{\Gamma\left(\frac{1}{c}+1\right)}{(n-j+b+1)^{\left(\frac{1}{c}+1\right)}}\sum_{l=0}^{\infty}\frac{\Gamma\left(\frac{2}{c}+2+l\right)}{\Gamma\left(\frac{1}{c}+2+l\right)(n-j+a+1)^{\left(\frac{2}{c}+2+l\right)}}$$

$$\left.-\sum_{a=0}^{j-i-1}\sum_{b=0}^{a}(-1)^{a}\binom{j-i-1}{a}\binom{a}{b}\tau^{(n-j+a-1)}(1-\tau)^{j+b}\frac{\left\{\Gamma\left(\frac{1}{c}+1\right)\right\}^{2}}{\left\{(j+b)(n-j+a-b)\right\}^{\left(\frac{1}{c}+1\right)}}\right] \qquad (3.6)$$

Special cases:

Substituting $\tau=1$ in (3.5) and (3.6), one can get the results corresponding to Weibull distribution. Similarly, substituting $\left[(\tau=1 \text{ and } c=1)\right]$; $\left[(\tau=1 \text{ and } c=2)\right]$; $\left[(\tau=0.5 \text{ and } c=1)\right]$; $\left[(\tau=0.5 \text{ and } c=2)\right]$ and $\left[(\tau=0.5)\right]$ respectively in (3.5) and (3.6) we get the results corresponding to Exponential; Rayleigh; Laplace; Reflected Rayleigh and Reflected (Double) Weibull distributions

4. LINEAR ESTIMATION [LLOYD (1952)]

Suppose that ***P*** is a family of absolute continuous distribution with cdf's of the form

$$F(x)=G\left(\frac{x-\mu}{\sigma}\right);\ (\sigma>0).$$

In other words, ***P*** is called a location and scale family of distributions as it is depending on location and scale parameters only. We denote these parameters by μ and σ , although they need not be the mean and standard deviation. It follows that $f(x)=F'(x)$ may be written as

$$f(x)=\frac{1}{\sigma}g\left(\frac{x-\mu}{\sigma}\right);\ \ (\sigma>0) \qquad (4.1)$$

And that the standardized variate $Y=\left(\frac{X-\mu}{\sigma}\right)$ has pdf $g(y)$, free of μ and σ. The family of Normal and Uniform distributions provided two important examples.

Since the ordered X and Y variates in "n" random samples are linked by the relationship

$$Y_{i:n}=\left(\frac{X_{i:n}-\mu}{\sigma}\right); \quad i=1,2,\ldots,n \qquad (4.2)$$

The moments of the $Y_{i:n}$ depend only on the form of g, and not on μ and σ. Let

$$E\left(Y_{i:n}^{(k)}\right)=\alpha_{i:n}^{(k)}$$
$$Cov\left(Y_{i:n}^{(s)},Y_{j:n}^{(t)}\right)=\beta_{i,j:n}^{(s,t)}; i,j=1,2,\ldots,n \qquad (4.3)$$

Then

$$\left.\begin{aligned}E\left(X_{i:n}^{(k)}\right)&=\mu+\sigma\alpha_{i:n}^{(k)}\\ Cov\left(X_{i:n}^{(s)},X_{j:n}^{(t)}\right)&=\sigma^2\beta_{i,j:n}^{(s,t)}; i,j=1,2,\ldots,n\end{aligned}\right\} \qquad (4.4)$$

where the $\alpha_{i:n}^{(k)}$ & $\beta_{i,j:n}^{(s,t)}$ can be evaluated once and for all. Thus $E\left(X_{i:n}^{(k)}\right)$ is linear in the parameters μ and σ, with known coefficients, and $Cov\left(X_{i:n}^{(s)},X_{j:n}^{(t)}\right)$ is known apart from σ^2. The Gauss-Markov Least-Squares (LS) theorem may therefore be applied to give unbiased estimators of μ and σ that are BLUES. To see this explicitly, write the first equation of (4.1) as $E(X)=\mu 1+\sigma\alpha$ (4.5)

Or

$$E(X)=A\theta \qquad (4.6)$$

where X,α are, respectively, the column of the $X_{i:n},\alpha_{i:n}$; a column of "n" l's and

$$A=(1,\alpha) \qquad (4.7)$$

$$\theta^T=(\mu,\sigma) \qquad (4.8)$$

Also, let the covariance matrix of the $X_{i:n}$ be $V(X)=\sigma^2 B$ where B is positive definite $(n\times n)$ symmetric matrix of $\beta_{i,j:n}^{(s,t)}$. We have to minimize with respect to θ

$$(x-A\theta)^T\Omega(x-A\theta) \qquad (4.9)$$

where $\Omega=B^{-1}$

Yielding the LS estimator θ^* :

$$\theta^*=\left(A^T\Omega A\right)^{-1}A^T\Omega X \qquad (4.10)$$

The covariance matrix of θ^* is

$$\left(A^T\Omega A\right)^{-1}A^T\Omega\bullet\sigma^2\Omega^{-1}\bullet\Omega A\left(A^T\Omega A\right)^{-1}=\sigma^2\left(A^T\Omega A\right)^{-1} \qquad (4.11)$$

where

$$\left(A^T\Omega A\right)=\begin{pmatrix}1^T\\ \alpha^T\end{pmatrix}\Omega(1,\alpha)$$
$$=\begin{pmatrix}1^T\Omega 1 & 1^T\Omega\alpha\\ \alpha^T\Omega 1 & \alpha^T\Omega\alpha\end{pmatrix}$$

All the elements of matrix being scalar.

It follows from (4.10) that with $\Delta=\det\left(A^T\Omega A\right)$

$$\theta^*=\frac{1}{\Delta}\begin{pmatrix}\alpha^T\Omega\alpha & -\alpha^T\Omega 1\\ -1^T\Omega\alpha & 1^T\Omega 1\end{pmatrix}\begin{pmatrix}1^T\Omega\\ \alpha^T\Omega\end{pmatrix}X$$
$$=\frac{1}{\Delta}\begin{pmatrix}\alpha^T\Omega\alpha 1^T\Omega-\alpha^T\Omega 1\alpha^T\Omega\\ -1^T\Omega\alpha 1^T\Omega+1^T\Omega 1\alpha^T\Omega\end{pmatrix}X$$

$$\left.\begin{aligned}\gamma&=-\alpha^T\Gamma\\ \delta&=1^T\Gamma\end{aligned}\right\} \qquad (4.12)$$

$$\left.\begin{aligned}\hat{\mu} &= \gamma X \\ \hat{\sigma} &= \delta X\end{aligned}\right\} \qquad (4.13)$$

where $\gamma \;\&\; \delta$ coefficients of column matrixes of BLUES for μ and σ respectively and

$$\Gamma = \frac{\Omega\left(1\alpha^T - \alpha 1^T\right)\Omega}{\Delta}, \text{ skew-symmetric matrix.}$$

Also from (4.13)

$$V(\hat{\mu}) = \frac{\alpha^T \Omega \alpha \sigma^2}{\Delta} \qquad (4.14)$$

$$V(\hat{\sigma}) = \frac{1^T \Omega 1 \sigma^2}{\Delta} \qquad (4.15)$$

$$Cov(\hat{\mu}, \hat{\sigma}) = -\frac{1^T \Omega \alpha \sigma^2}{\Delta} \qquad (4.16)$$

Thus the estimations of μ and σ respectively μ^* and σ^* may be expressed as linear function of order statistics, namely,

$$\left.\begin{aligned}\hat{\mu} &= \sum_{i=1}^{n} \gamma_{i:n} X_{i:n} \\ \hat{\sigma} &= \sum_{i=1}^{n} \delta_{i:n} X_{i:n}\end{aligned}\right\} \qquad (4.17)$$

5. REVIEW OF EXISTING LITERATURE

Coefficients of BLUEs of Exponential distribution are expressed in closed form and numeric values of the same are available in Sarhan and Greenberg (1962). Govindarajulu (1966) derived the moments of order statistics and obtained the coefficients of BLUEs of Laplace distribution. Lieblein (1955) derived the moments of order statistics of Weibull distribution. Govindarajulu and Joshi (1968) have tabulated the coefficients of BLUEs of Weibull distribution. Balakrishnan and Kocherla Kota (1985) have derived the moments of order statistics of double Weibull distribution and tabulated the coefficients of BLUEs from complete samples. Dattatreya and Narasimham (1989) have extended the work of Balakrishnan and Kocherla Kota for doubly censored sample. Whereas the equations derived by us can be used for all values of $c > 0$

6. RESULTS

Using (3.5) and (3.6) mean vector of i^{th} order statistic and covariance matrix of i^{th} and j^{th} order statistics of Weibull type models have been computed for the sample size $n = 2(1)15$ for specified values of shape parameter $c = 0.5, 1.0, 1.5, 2.0$ and 3 from doubly censored samples (where r_1 and r_2 respectively denoting number of observations censored on the left and right) and are available with the first author and not presented here to save space. However, they are tabulated here for $c = 1.5$ and the sample size up to $n = 5$ in case of Weibull distribution are presented respectively Table (6.1) and (6.2.)

Table 6.1: Mean vector $\alpha_{i:n}$ with $c = 1.5$ for Ordered Weibull Distribution up to n=5

n	r_1	r_2	i	$\alpha^{(1)}_{i:n}$	n	r_1	r_2	i	$\alpha^{(1)}_{i:n}$	n	r_1	r_2	i	$\alpha^{(1)}_{i:n}$
2	0	0	1	5.686939E-01	4	0	1	2	8.380924E-01	5	3	0	5	5.686939E-01
	0	0	2	5.686939E-01				0	1.436149E+00		0	1	1	3.582547E-01
3	0	0	1	4.339947E-01		0	2	1	5.686939E-01				2	6.612146E-01
			2	8.380924E-01				2	5.686939E-01				3	1.014970E+00
			3	1.436149E+00		1	1	2	5.686939E-01				4	1.576542E+00
	1	0	2	5.686939E-01				3	5.686939E-01		0	2	1	4.339947E-01
			3	5.686939E-01	5	0	0	1	3.087345E-01				2	8.380924E-01
	0	1	1	5.686939E-01				2	5.563353E-01				3	1.436149E+00
			2	5.686939E-01				3	8.185334E-01		0	3	1	5.686939E-01
4	0	0	1	3.582547E-01				4	1.145928E+00				2	5.686939E-01
			2	6.612146E-01				5	1.684195E+00		1	1	2	4.339947E-01
			3	1.014970E+00		1	0	2	3.582547E-01				3	8.380924E-01
			4	1.576542E+00				3	6.612146E-01				4	1.436149E+00
	1	0	2	4.339947E-01				4	1.014970E+00		2	1	3	5.686939E-01
			3	8.380924E-01				5	1.576542E+00				4	5.686939E-01

			4	1.436149E+00		2	0	3	4.339947E-01		1	2	2	5.686939E-01
	2	0	3	5.686939E-01				4	8.380924E-01				3	5.686939E-01
			4	5.686939E-01				5	1.436149E+00					
	0	1	1	4.339947E-01		3	0	4	5.686939E-01					

Table 6.2: Variance Covariance Matrix $\beta^{(1,1)}_{i,j:n}$ with $c=1.5$ Ordered Weibull Distribution up to $n=5$

n	r_1	r_2	i	j	$\beta^{(1,1)}_{i,j:n}$		n	r_1	r_2	i	j	$\beta^{(1,1)}_{i,j:n}$	n	r_1	r_2	i	j
2	0	0	1	1	1.490928E-01	4	0	0	3	3	1.659621E-01	4	1	1	2	3	1.115903E-01
				2	1.115903E-01					4	1.382091E-01				3	3	3.791071E-01
			2	2	3.791071E-01				4	4	3.552506E-01	5	0	0	1	1	4.394100E-02
3	0	0	1	1	8.682970E-02		1	0	2	2	8.682970E-02					2	3.512740E-02
				2	6.775456E-02					3	6.775456E-02					3	2.975924E-02
				3	5.391192E-02					4	5.391192E-02					4	2.558326E-02
			2	2	1.647557E-01				3	3	1.647557E-01					5	2.148023E-02
				3	1.325468E-01					4	1.325468E-01				2	2	7.102863E-02
			3	3	3.670590E-01				4	4	3.670590E-01					3	6.043471E-02
	1	0	2	2	1.490928E-01		2	0	3	3	1.490928E-01					4	5.210268E-02
				3	1.115903E-01					4	1.115903E-01					5	4.384512E-02
			3	3	3.791071E-01				4	4	3.791071E-01				3	3	1.046527E-01
	0	1	1	1	1.490928E-01		0	1	1	1	8.682970E-02					4	9.062060E-02
				2	1.115903E-01					2	6.775456E-02					5	7.653205E-02
			2	2	3.791071E-01					3	5.391192E-02				4	4	1.639601E-01
4	0	0	1	1	5.916751E-02				2	2	1.647557E-01					5	1.393857E-01
				2	4.690569E-02					3	1.325468E-01				5	5	3.451269E-01
				3	3.904259E-02				3	3	3.670590E-01		1	0	2	2	5.916751E-02
				4	3.210483E-02		0	2	1	1	1.490928E-01					3	4.690569E-02
			2	2	1.009778E-01					2	1.115903E-01					4	3.904259E-02
				3	8.457742E-02				2	2	3.791071E-01					5	3.210483E-02
				4	6.986201E-02		1	1	2	2	1.490928E-01				3	3	1.009778E-01

Table 6.2: (Contd.)Variance Covariance Matrix $\beta^{(1,1)}_{i,j:n}$ with $c=1.5$ Ordered Weibull Distribution up to $n=5$

n	r_1	r_2	i	j	$\beta^{(1,1)}_{i,j:n}$		n	r_1	r_2	i	j	$\beta^{(1,1)}_{i,j:n}$	n	r_1	r_2	i	j
5	1	0	3	4	8.457742E-02	5	0	1	1	2	4.690569E-02	5	0	3	1	1	1.490928E-01
				5	6.986201E-02					3	3.904259E-02					2	1.115903E-01
			4	4	1.659621E-01					4	3.210483E-02				2	2	3.791071E-01
				5	1.382091E-01				2	2	1.009778E-01		1	1	2	2	8.682970E-02
			5	5	3.552506E-01					3	8.457742E-02					3	6.775456E-02
	2	0	3	3	8.682970E-02					4	6.986201E-02					4	5.391192E-02
				4	6.775456E-02				3	3	1.659621E-01				3	3	1.647557E-01
				5	5.391192E-02					4	1.382091E-01					4	1.325468E-01
			4	4	1.647557E-01				4	4	3.552506E-01				4	4	3.670590E-01
				5	1.325468E-01		0	2	1	1	8.682970E-02		2	1	3	3	1.490928E-01
			5	5	3.670590E-01					2	6.775456E-02					4	1.115903E-01
	3	0	4	4	1.490928E-01					3	5.391192E-02				4	4	3.791071E-01
				5	1.115903E-01				2	2	1.647557E-01		1	2	2	2	1.490928E-01
			5	5	3.791071E-01					3	1.325468E-01					3	1.115903E-01
	0	1	1	1	5.916751E-02				3	3	3.670590E-01				3	3	3.791071E-01

Using Lloyd's method explained in section 4, the coefficients of BLUEs for specified values $c = 0.5, 1.0, 1.5, 2.0$ and 3 for $n = 2(1)15$ in case of doubly censored case are evaluated and are available with the first author.. However, the same are tabluated for $n = 5$ and $c = 1.5$ in case of Weibull distribution and are presented in Table (6.3)

Table 6.3. Coefficients of the BLUE's of the parameters of Weibull Distribution from doubly censored samples up to n=5.

n	r_1	r_2	i	$\gamma_{i:n}$	$\delta_{i:n}$	$V(\hat{\mu})$	$V(\hat{\sigma})$	$Cov(\hat{\mu},\hat{\sigma})$
2	0	0	1	1.851207E+00	-1.496776E+00	4.339403E-01	6.833460E-01	-4.447475E-01
	0	0	2	-8.512072E-01	1.496776E+00			
3	0	0	1	1.511623E+00	-1.264153E+00	1.786213E-01	3.259511E-01	-1.768970E-01
			2	-1.316446E-01	4.462387E-01			
			3	-3.799789E-01	8.179140E-01			
	1	0	2	1.851207E+00	-1.496776E+00	4.339403E-01	6.833460E-01	-4.447475E-01
			3	-8.512072E-01	1.496776E+00			
	0	1	1	1.851207E+00	-1.496776E+00	4.339403E-01	6.833460E-01	-4.447475E-01
			2	-8.512072E-01	1.496776E+00			
4	0	0	1	1.347633E+00	-1.149404E+00	1.039011E-01	2.106039E-01	-1.008727E-01
			2	-1.765360E-03	2.112075E-01			
			3	-1.133358E-01	3.685735E-01			
			4	-2.325318E-01	5.696233E-01			
	1	0	2	1.511623E+00	-1.264153E+00	1.786213E-01	3.259511E-01	-1.768970E-01
			3	-1.316446E-01	4.462387E-01			
			4	-3.799789E-01	8.179140E-01			
	2	0	3	1.851207E+00	-1.496776E+00	4.339403E-01	6.833460E-01	-4.447475E-01
			4	-8.512072E-01	1.496776E+00			
	0	1	1	1.511623E+00	-1.264153E+00	1.786213E-01	3.259511E-01	-1.768970E-01
			2	-1.316446E-01	4.462387E-01			
			0	-3.799789E-01	8.179140E-01			
	0	2	1	1.851207E+00	-1.496776E+00	4.339403E-01	6.833460E-01	-4.447475E-01

Table.6.3: (Contd.) Coefficients of the BLUE's of the parameters of Weibull Distribution from doubly censored samples up to n=5.

n	r_1	r_2	i	$\gamma_{i:n}$	$\delta_{i:n}$	$V(\hat{\mu})$	$V(\hat{\sigma})$	$Cov(\hat{\mu},\hat{\sigma})$
4	0	2	2	-8.512072E-01	1.496776E+00			
	1	1	2	1.851207E+00	-1.496776E+00	4.339403E-01	6.833460E-01	-4.447475E-01
			3	-8.512072E-01	1.496776E+00			
5	0	0	1	1.248089E+00	-1.077608E+00	7.018379E-02	1.543652E-01	-6.724932E-02
			2	4.310673E-02	1.122860E-01			
			3	-3.666281E-02	2.200210E-01			
			4	-9.174470E-02	3.067266E-01			
			5	-1.627884E-01	4.385745E-01			
	1	0	2	1.347633E+00	-1.149404E+00	1.039011E-01	2.106039E-01	-1.008727E-01
			3	-1.765360E-03	2.112075E-01			
			4	-1.133358E-01	3.685735E-01			
			5	-2.325318E-01	5.696233E-01			
	2	0	3	1.511623E+00	-1.264153E+00	1.786213E-01	3.259511E-01	-1.768970E-01
			4	-1.316446E-01	4.462387E-01			
			5	-3.799789E-01	8.179140E-01			
	3	0	4	1.851207E+00	-1.496776E+00	4.339403E-01	6.833460E-01	-4.447475E-01
			5	-8.512072E-01	1.496776E+00			
	0	1	1	1.347633E+00	-1.149404E+00	1.039011E-01	2.106039E-01	-1.008727E-01
			2	-1.765360E-03	2.112075E-01			

			3	-1.133358E-01	3.685735E-01			
			4	-2.325318E-01	5.696233E-01			
	0	2	1	1.511623E+00	-1.264153E+00	1.786213E-01	3.259511E-01	-1.768970E-01
			2	-1.316446E-01	4.462387E-01			
			3	-3.799789E-01	8.179140E-01			
	0	3	1	1.851207E+00	-1.496776E+00	4.339403E-01	6.833460E-01	-4.447475E-01
			2	-8.512072E-01	1.496776E+00			
	1	1	2	1.511623E+00	-1.264153E+00	1.786213E-01	3.259511E-01	-1.768970E-01
			3	-1.316446E-01	4.462387E-01			
			4	-3.799789E-01	8.179140E-01			
	2	1	3	1.851207E+00	-1.496776E+00	4.339403E-01	6.833460E-01	-4.447475E-01
			4	-8.512072E-01	1.496776E+00			
	1	2	2	1.851207E+00	-1.496776E+00	4.339403E-01	6.833460E-01	-4.447475E-01

Future Work: It is planned to extend the work on Gamma, Amoroso and Burr type models.

References

[1] Balakrishnan, N. and Kocherlakota, S. 1985. On the double Weibull distribution: Order statistics and estimation. Sankhya B, (47), pp. 161-178.

[2] Dattatreya Rao, A.V. and Narasimham, V.L. 1989. Linear estimation in double Weibull distribution. Sankhya, The Indian Journal of Statistics, Vol.51, Series B, (1), pp 24-64.

[3] Govindarajulu, Z. 1966. Best linear estimates under symmetric censoring of the parameters of a double Exponential population. J. Amer. Statist. Assoc., Vol.61, 248-258.

[4] Govindarajulu. Z and Joshi, M. 1968. Best linear unbiased estimation of location-scale distribution of Weibull distribution using order observations. Rep. Stat. Appl. Res., JUSE 15, pp 57-77.

[5] Hallinan Jr, A. J. 1993. A review of the Weibull distribution. Journal of quality technology, Vol. 25, Issue 2, pp 85-93.

[6] Harter, H.L. and Satya D. Dubey. 1967. Theory and tables for tests of hypotheses concerning the mean and the variance of a Weibull population. Office of aerospace research, United states Air Force, U.S.

[7] Holla, M.S. and Bhattacharya, S.K. 1968. On a compound Gaussian distribution. Ann. Inst, Statist. Math., 20, pp. 331-336.

[8] Lieblein, J. 1955. On moments of order statistics from the Weibull distribution. Annals of mathematical statistics, 26, 330-33.

[9] Lloyd. E. H. 1952. Least squares estimation of location and scale parameters using order statistics. Biometrika, 39, pp. 88-95.

[10] Sarhan, A.E. and Greenberg, B.G. (Eds.) 1962. Contributions to Order Statistics. John Wiley: New York.

[11] Weibull, W. 1939. The phenomenon of rupture in solid. Ingeniors, vetenskaps akademien (The Royal Swedish Institute for Engineering Research), Handlingar, NR 153.