THE INDEPENDENT DISTANCE - 2 DOMINATION IN GRAPHS

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Abstract : ***A distance - 2 dominating set $D \subseteq V$ of a graph G is an independent distance - 2 dominating set if the induced sub graph < D > has no edges. The independent distance - 2 domination number $i_{\leq 2}(G)$ is the minimum cardinality of a minimal independent distance - 2 dominating set. In this paper, we defined the notion of independent distance - 2 domination in a graph. We obtained many bounds on independent distance - 2 domination number. Exact values of this new parameter were obtained for some standard graphs and some special graphs. Nordhaus - Gaddum type results were also obtained for this new parameter.***

IndexTerms - Dominating set, independent dominating set, distance - 2 dominating set, independent distance - 2 dominating set, independent distance - 2 domination number, cocktail graph, n- Andrásfai graph, Harary graph, crown graph, windmill graph, fan graph, and musical graph.

I. INTRODUCTION

All graphs considered here are simple, connected, finite and undirected. Let n and m denote the order and size of the graph G. We used the terminology of [9]. Let $\Delta(G)(\delta(G))$ denotes the maximum (minimum) degree and $\lfloor x \rfloor(\lceil x \rceil)$ the greatest (least) integer less (greater) than or equal to x. The independence number β_0(G) is the maximum cardinality among the independent set of vertices of G. A vertex cover of G is a set of vertices that covers all the edges and the minimum cardinality of a vertex cover is $\alpha_0(G)$. The girth g(G) of a graph G is the length of the shortest cycle in G. The radius of G is rad(G) = min{ecc(v): v∈ V} and diam(G) = max{ecc(v):v∈ V},where ecc(v) is eccentricity of a vertex which is defined as max{dis (u,v): v∈ V }in [9].

A non-empty subset $D \subseteq V$ of vertices in a graph G = (V, E) is called a dominating set if every vertex in V-D is adjacent to at least one vertex of D. The domination number of G is the minimum cardinality of a minimal dominating set and it is denoted by γ (G). A recent survey of γ (G) can be found in [10].The upper domination number $\Gamma(G)$ equals the maximum cardinality of a minimal dominating set of G [9].

A dominating set $D \subseteq V$ of a graph G is an independent dominating set if the induced sub-graph < D > has no edges. The independent domination number $i(G)$ is the minimum cardinality of a minimal independent dominating set of G.

The minimal independent dominating set in a graph G is an independent dominating set that contains no independent dominating set as a proper subset.

An independent set D is called a maximal independent set if and only if for every vertex u ε V-D, the set $D \cup \{u\}$ is not independent.

A non-empty set D of vertices in a graph G = (V, E) is a distance - 2 dominating set if every vertex in V-D is within distance - 2 from at least one vertex in D. The distance - 2 domination number $\gamma_{\leq 2}(G)$ of G equals the minimum cardinality of a minimal distance - 2 dominating set in G [9].

The purpose of this paper is to introduce the concept of independent distance - 2 domination in graphs.

Definition 1.1

A distance - 2 dominating set $D \subseteq V$ of a graph G is an independent distance - 2 dominating set if the induced sub graph < D > has no edges. The independent distance - 2 domination number $i_{\leq 2}(G)$ is the minimum cardinality of a minimal independent distance - 2 dominating set.

The minimal independent distance - 2 dominating set in a graph G is an independent distance - 2 dominating set that contains no independent distance - 2 dominating set as a proper subset.

The distance - 2 open neighborhood of a vertex $v \in V$ is the set, $N_{\leq 2}(v)$ of vertices within distance of two from (v).

Example: 1.2

Consider the graph G given below

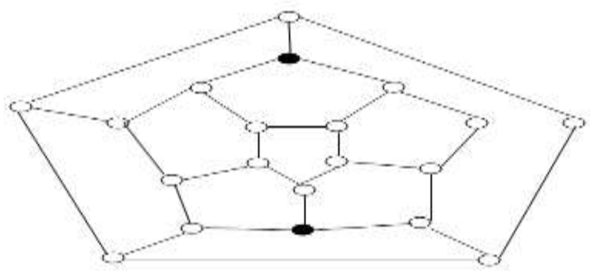

Figure.1

Here $i_{\le 2}(G) = 2$

2. Standard graphs and exact values of $i_{\le 2}(G)$

The independent distance - 2 domination number $i_{\le 2}(G)$ of some standard graphs is given below.

2.1. **Observation**

1. For any Path P_n, for n ≥ 2

$$i_{\le 2}(P_n) = \left\lceil \frac{n}{5} \right\rceil$$

2. For any Cycle C_n, for n ≥ 3

$$i_{\le 2}(C_n) = \left\lceil \frac{n+1}{5} \right\rceil$$

3. For any Wheel graph W_n, for n ≥ 3

$$i_{\le 2}(W_n) = 1$$

4. For any Friendship graph F_n, for n ≥ 2

$$i_{\le 2}(F_n) = 1$$

5. For any Complete graph K_n, for n ≥ 3

$$i_{\le 2}(K_n) = 1$$

6. For any Star graph $K_{1,m}$, for m ≥1

$$i_{\le 2}(K_{1,m}) = 1$$

7. For any Complete bipartite graph $K_{n,m}$, for m ≥ n,

$$i_{\le 2}(K_{n,m}) = 1$$

8. For any Book graph B_n, for n ≥3

$$i_{\le 2}(B_n) = 1$$

9. For any Helm graph H_n, for n ≥3

$$i_{\le 2}(H_n) = 1$$

10. For a Grid graph $P_{i,j}$, for i =2,3. j ≥2

$$i_{\le 2}(P_{i,j}) = \left\lceil \frac{j}{3} \right\rceil$$

11. For any n-Barbell graph, for n ≥3

$$i_{\le 2}(n - barbell) = 1$$

12. For any Ladder Rung graph L_n, for n ≥1

$$i_{\le 2}(L_n) = n$$

Proposition 2.2
For any graph G, $\gamma_{\le 2}(G) = i_{\le 2}(G) = \gamma(G) = 1$ if and only if G = K_n or W_n.

Proposition 2.3
For any graph G, $\gamma_{\le 2}(G) = i_{\le 2}(G)$ if and only if G is any one of the common graph of Path, Cycle, Friendship graph, Book graph, Helm graph, Star graph, Complete and Complete bipartite graph.

Proposition 2.4
For any graph G, $\gamma_{\le 2}(G) = i_{\le 2}(G) = 1$ if and only if G is a Friendship graph or a Complete bipartite graph or a Book graph or a Helm graph or a Star graph.

Proposition 2.5
For any connected graph G with n ≥2, then $\left\lceil \frac{n}{\Delta(G)+1} \right\rceil = i_{\le 2}(G)$ if and only if the graph G is a F_n, or a K_n, or a W_n or a $K_{1,m}$.

3. Bounds on the independent distance - 2 domination number of a graph G

Theorem 3.1
For any graph G, $i_{\le 2}(G) \le i(G)$.

Proof
Every independent dominating set of G is an independent distance - 2 dominating set of G, Thus we have, $i_{\le 2}(G) \le i(G)$.

Theorem 3.2
For any graph G, $\gamma_{\leq 2}(G) \leq i_{\leq 2}(G)$.
Proof
Every independent distance - 2 dominating set of G is a distance - 2 dominating set of G. Thus we have, $\gamma_{\leq 2}(G) \leq i_{\leq 2}(G)$.

Theorem 3.3
For any graph G, $\gamma_{\leq 2}(G) \leq i_{\leq 2}(G) \leq i(G)$.
Proof
By the Theorem 3.2 and Theorem 3.3, we have, $\gamma_{\leq 2}(G) \leq i_{\leq 2}(G) \leq i(G)$.

Proposition 3.4
For any graph G, the cardinality of dominating set of a graph G is more than the cardinality of an independent distance - 2 dominating set i.e., $i_{\leq 2}(G) \leq \gamma(G)$.

Theorem 3.5
For any graph G, $\gamma_{\leq 2}(G) \leq i_{\leq 2}(G) \leq \beta_0(G)$.
Proof
By the reference [8, 9], we have, for any graph G, $i(G) \leq \beta_0(G) \leq \Gamma(G)$
By the Theorem 3.3, we have, $\gamma_{\leq 2}(G) \leq i_{\leq 2}(G) \leq \beta_0(G)$.

Theorem 3.6
For any graph G, $\gamma_{\leq 2}(G) \leq i_{\leq 2}(G) \leq \gamma(G) \leq i(G) \leq \beta_0(G) \leq \Gamma(G)$.
Proof
By the reference [8], we have, for any graph G, $\gamma(G) \leq i(G) \leq \beta_0(G) \leq \Gamma(G)$
Also, by the Theorem 3.3 and Proposition 3.4,
we have, $\gamma_{\leq 2}(G) \leq i_{\leq 2}(G) \leq \gamma(G) \leq i(G) \leq \beta_0(G) \leq \Gamma(G)$.

Theorem 3.7
For a graph G with n vertices and maximum degree $\Delta(G)$,$i_{\leq 2}(G) \leq n - \Delta(G)$.
Proof
From the reference [8], we have, for a graph G with n vertices and maximum degree $\Delta(G)$, $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \leq i(G) \leq n - \Delta(G)$. Using Theorem 3.1, we have, $i_{\leq 2}(G) \leq n - \Delta(G)$.

Theorem 3.8
If G is a graph without isolated vertices on n, then $i_{\leq 2}(G) \leq n + 2 - \gamma(G) - \left\lceil \frac{n}{\gamma(G)} \right\rceil$.
Proof
By the reference [8], If G is an isolate – free graph on n, then $i(G) \leq n + 2 - \gamma(G) - \left\lceil \frac{n}{\gamma(G)} \right\rceil$.
Using Theorem 3.1, we have, $i_{\leq 2}(G) \leq n + 2 - \gamma(G) - \left\lceil \frac{n}{\gamma(G)} \right\rceil$.

Theorem 3.9
If a graph G contains an isolated vertex, then a minimal independent dominating set of G is an independent distance - 2 dominating set of G.
Proof
Let v be an isolated vertex of the graph G. Then v is in every independent dominating set D of G. Hence D is an independent distance - 2 dominating set of G.
Corollary 3.10
If G contains an isolated vertex, then $i_{\leq 2}(G) = \gamma_{\leq 2}(G)$.

Allan and Lasker proved that $i(G) = \gamma(G)$ for all claw- free graphs.

Theorem 3.11
If G is $K_{1,3}$ – free graph, then $i_{\leq 2}(G) = \gamma_{\leq 2}(G)$.

Theorem 3.12
For any tree T_n with m cut vertices, $i_{\leq 2}(T_n) \leq m + 1$.

Theorem 3.13
For any tree T_n, $i_{\leq 2}(T_n) = \gamma_{\leq 2}(T_n)$.

Theorem 3.14
For $k \geq 3$, if G is $k_{1.k}$ free then $i_{\leq 2}(G) \leq (k-2)\gamma(G) - (k-3)$.
Proof
By the reference [4], we have, For $k \geq 3$, if G is $k_{1.k}$ free then,

$i(G) \le (k-2)\gamma(G) - (k-3)$.
And also, By the Theorem 3.1, We have, $i_{\le 2}(G) \le i(G)$.
Hence $i_{\le 2}(G) \le (k-2)\gamma(G) - (k-3)$.

Theorem 3.15
If T_n is a tree with n vertices and l leaves, then $i_{\le 2}(T_n) \le \frac{n+l}{3}$.
Proof
By the reference [8], we have, If T_n is a tree with n vertices and l leaves, then $i(G) \le \frac{n+l}{3}$
And also, By the Theorem 3.1, We have, $i_{\le 2}(G) \le i(G)$.
Hence $i_{\le 2}(G) \le \frac{n+l}{3}$.

Theorem 3.16
If a graph G has $\delta(G) \ge 2$ and $g(G) \ge 5$, then $i_{\le 2}(G) \le \left\lceil n - \frac{\left\lfloor \frac{g(G)}{3} \right\rfloor}{2} \right\rceil$, where $\delta(G)$ is the minimum degree of G and $g(G)$ is the girth of G.
Proof
By the reference [9], we have, If a graph G has $\delta(G) \ge 2$ and $g(G) \ge 5$, then $\gamma(G) \le \left\lceil n - \frac{\lfloor g(G)/3 \rfloor}{2} \right\rceil$. And also, By the Proposition 3.4, We have, $i_{\le 2}(G) \le \gamma(G)$.
Hence $i_{\le 2} \le \left\lceil n - \frac{\lfloor g(G)/3 \rfloor}{2} \right\rceil$.

Theorem 3.17
For any graph G with no isolated vertices, $1 \le i_{\le 2}(G) \le n$.
Proof
The lower and upper bounds follows from observation 2.1

Norduaus – Gaddum Type results

Theorem 3.18
Let G and $\overline{G}$ be two graphs with no isolated vertices, then
(i). $2 \le i_{\le 2}(G) + i_{\le 2}(\bar{G}) \le 2n$
(ii). $1 \le i_{\le 2}$ (G). $i_{\le 2}(\bar{G}) \le n^2$

Theorem 3.19
If a graph G has diam(G) = 2, then $i_{\le 2}(G) \le \delta(G)$.
Proof
By the reference [9], If a graph G has diam(G) = 2, then $\gamma(G) \le \delta(G)$. By the Proposition 3.4 $i_{\le 2}(G) \le \gamma(G)$.
Combining these two we get the result.

Theorem 3.20
If G is an isolate – free graph on n vertices, then $i_{\le 2}(G) \le n + 2 - 2\sqrt{n}$.
Proof
By the reference [8], we have, if G is an isolate – free graph on n vertices, then $i(G) \le n + 2 - 2\sqrt{n}$.
By the Proposition 3.4 $i_{\le 2}(G) \le \gamma(G)$. Combining these two we get the result.
Theorem 3.21
If a graph G of order n has the minimum degree at least δ, then $i_{\le 2}(G) \le n + 2\delta - 2\sqrt{n\delta}$.
Proof
By the reference [8], if the graph G of order n has the minimum degree at leastδ, then $i_{\le 2}(G) \le n + 2\delta - 2\sqrt{n\delta}$.
By the Proposition 3.4 $i_{\le 2}(G) \le \gamma(G)$.Combining these two we get the result.

Theorem 3.22
Let G be any connected graph and let D be the minimum independent distance - 2 dominating set of G, then $\Delta(\langle D \rangle) < \Delta(G)$.

Proposition 3.23
For any graph G with order n, then $i_{\le 2}(G) + i(G) \le n$.

Proposition 3.24
For any graph G with order n, then $i_{\le 2}(G) + \gamma(G) \le n$.

Proposition 3.25
For any graph G with order n, then $i_{\le 2}(G) + \gamma_{\le 2}(G) \le n$.

4. Some special Graphs with Independent distance - 2 domination number

Definition 4.1
The Cocktail party graph is the graph consisting of two rows of paired vertices in which all the vertices except the paired ones are connected with an edge and is denoted by CP_k, where k = 2n for all $n \geq 2$. It is also called hyper Octahedral graph or Roberts graph.

Theorem 4.2
If any graph G = CP_k is the cocktail party graph of k vertices where k = 2n for all ≥ 2 , then $i_{\leq 2}(G) = 1$.
Proof
Let G = CP_k is the cocktail party graph of k vertices where k = 2n for all $n \geq 2$, by the definition of cocktail party graph is the graph consisting of two rows of paired vertices in which all the vertices except the paired ones are connected with an edge. As per the definition of distance - 2 dominating set, every vertex u in V(G) is a distance - 2 dominating set of G. And also it is an independent distance - 2 dominating set of G. We have $i_{\leq 2}(G) = 1$.

Example 4.3
Let G = CP_k is the cocktail party graph of order k = 10.

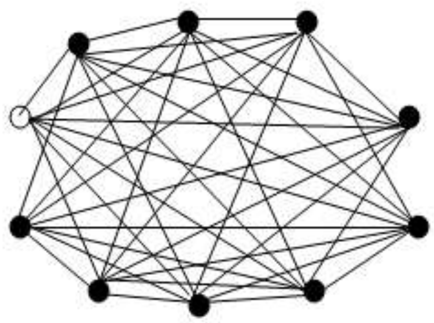

Figure. 2
Here $i_{\leq 2}(CP_k) = 1$.

Observation 4.4
For the cocktail party graph of k vertices where k = 2n for all ≥ 2 , we have
1. $\gamma_{\leq 2}(CP_k) = i_{\leq 2}(CP_k) = 1$.
2. $i_{\leq 2}(CP_k) \leq \gamma(CP_k)$.
3. $i(CP_k) = \gamma(CP_k)$

Definition 4.5
The n- $Andrásfai$ graph is a circulant graph on (3n-1) vertices whose indices are given by the integers 1,2,3…(3n-1) that are congruent to 1(mod 3). The $Andrásfai$ graph have diameter 2 for $n \geq 1$ and is denoted by A_k, where k = 3n-1.

Example 4.6
Consider the $Andrásfai$ graph A_{11} with n = 4

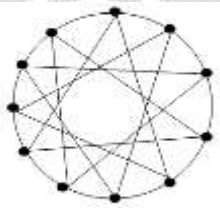

Figure. 3
Here $i_{\leq 2}(A_{11}) = 1$.

Theorem 4.7
Let A_k be the $Andrásfai$ graph with k vertices where k = 3n-1, then $i_{\leq 2}(A_k) = 1$.
Proof
By the definition of the n- $Andrásfai$ graph is a circulant graph on (3n-1) vertices whose indices are given by the integers 1,2,3…(3n-1) that are congruent to 1(mod 3). The $Andrásfai$ graph have diameter 2 for $n \geq 1$. Each and every single vertex of the $Andrásfai$ graph satisfies the condition of the independent distance - 2 domination. Hence we have $i_{\leq 2}(G) = 1$.

Observation 4.8
For the n- $Andrásfai$ graph of k vertices where k = 3n-1 for all $n \geq 1$, we have, $\gamma_{\leq 2}(A_k) = i_{\leq 2}(A_k) = 1$.

Definition 4.9
The Harary graph $H_{k,n}$ is a particular example of a k connected graph with n graph vertices having the smallest possible number of edges.

Theorem 4.10
For any Harary graph $H_{k,n}$,then $i_{\leq 2}\left(H_{k,n}\right) = 1$ for $k \geq 3$.
Proof
Let $u \in V(H_{k,n})$. Every vertices in $V(H_{k,n}) - u$ is within the distance two to u vertex. By the definition, a set D of vertices in a graph G = (V, E) is a distance - 2 dominating set if every vertex in V-D is within distance 2 of at least one vertex in D. Hence we have $\gamma_{\leq 2}\left(H_{k,n}\right) = 1$. Thus we get $i_{\leq 2}(G) = 1$ for $k \geq 3$.

Example 4.11
Consider Goldner – Harary Graph

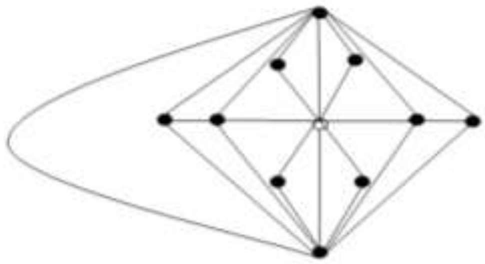

Figure. 4
Independent distance – 2 domination number of the Goldner Harary graph is 1.

Definition 4.12
The crown graph S_n^0 for an integer $n \geq 3$ is the graph with vertex set $\{x_0, x_1, x_2, \ldots x_{n-1}, y_0, y_1, y_2, \ldots y_{n-1}\}$ and edge set $\{(x_i, y_j): 0 \leq i, j \leq n - 1, i \neq j\}$. S_n^0 is therefore equivalent to the complete bipartite graph $K_{n,n}$ with horizontal edges removed.

Theorem 4.13
For any Crown graph the cardinality of an independent distance - 2 dominating set is two.
Proof
Let $D = \{(u, v), where\ u\ and\ v\ are\ not\ adjacent\ in\ V\}$, then every vertex in V-D is within distance two to the set D. Thus D satisfies the condition of the definition of an independent distance - 2 domination. Hence D is an independent distance - 2 dominating set of the Crown graph and $i_{\leq 2}(S_n^0) = 2$.

Observation 4.14
For the Crown graph, $i_{\leq 2}(S_n^0) = \gamma_{\leq 2}(S_n^0) = \gamma(S_n^0) = i(S_n^0) = 2$.

Definition 4.15
The Windmill graph $W_n^{(m)}$ is the graph obtained by taking m copies of the complete graph K_n with a vertex in common.

Theorem 4.16
Suppose the Graph G is the Windmill graph then the independent distance - 2 domination number is one.
Proof
By the definition of the Windmill graph, every copy of complete graph is connected with a common vertex. Then the common vertex is taken into the independent distance – 2 dominating vertex of the Windmill graph. Hence $i_{\leq 2}(W_n^m) = 1$.

Observation 4.17
For the Windmill graph, $i_{\leq 2}(W_n^m) = \gamma_{\leq 2}(W_n^m) = \gamma(W_n^m) = i(W_n^m) = 1$

Definition 4.18
A fan graph $F_{m,n}$ is defined as the graph by joining $\overline{K_m}$ and P_n graphs where $\overline{K_m}$ is the empty graph on m vertices and P_n is the path graph on n vertices.

Theorem 4.19
For any Fan graph $F_{m,n}$, then $i_{\leq 2}(F_{m,n}) = 1$.
Proof
Let D be any one vertex in $\overline{K_m}$ from a Fan graph. Every vertex in V-D of a Fan graph are within distance - 2 to the set D. Then the set D is the independent distance – 2 dominating set of the Fan graph. Hence $i_{\leq 2}(F_{m,n}) = 1$.

Definition 4.20
The Musical graph $n \geq 3$ of order n consists of two parallel copies of cycle graph C_n, in which all the paired vertices and the neighboured vertices are connected and is denoted b M_{2n}, for all $n \geq 3$.

Theorem 4.21
For any Musical graph M_{2n}, for all $n \geq 3$, then $i_{\leq 2}(M_{2n}) = \left\lceil \frac{n+1}{5} \right\rceil$.

Observation 4.22
For any Musical graph M_{2n}, for all $n \geq 3$, then $i_{\leq 2}(M_{2n}) = i_{\leq 2}(C_n)$.

Definition 4.23

The Banana tree is a graph obtained by connecting one leaf of each of n copies of an k- star graph with a single root vertex that is distinct from all the stars, it is denoted by $B_{n,k.}$

Example 4.24

Consider the Banana tree $B_{2,5}$

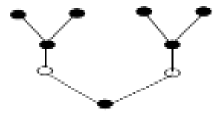

Figure. 5

Here $i_{\leq 2}(B_{2,4}) = 2.$

Theorem 4.25

For any Banana tree $B_{n,k}$, then $i_{\leq 2}(B_{n,k}) = n.$

Proof

Let D be the set of all n- vertices from a Banana tree $B_{n,k.}$ Then every vertex in V- D is within distance - 2 to the set D. Hence $i_{\leq 2}(B_{n,k}) = n.$

Observation 4.26

For any Banana tree $B_{n,k}$, we have

1. $\gamma_{\leq 2}(B_{n,k}) = i_{\leq 2}(B_{n,k}) = n.$
2. $i_{\leq 2}(B_{n,k}) \leq \gamma(B_{n,k}).$
3. $\gamma(B_{n,k}) = n + 1.$

Definition 4.27

The Gear graph also sometimes known as a bipartite wheel graph is a wheel graph with a graph vertex added between each pair of adjacent graph vertices of the outer cycle. The Gear graph $G_{n,}$ has 2n+1 nodes and 3n edges.

Theorem 4.28

Let $G_{n,}$ be any Gear graph, then $i_{\leq 2}(G_n) = 1.$

Observation 4.29

For any Gear graph $G_{n,}$ we have

1. $\gamma_{\leq 2}(G_n) = i_{\leq 2}(G_n) = 1.$
2. $i_{\leq 2}(G_n) \leq \gamma(G_n).$

Definition 4.30

Double star is the graph obtained by joining the centre of two stars $K_{1,n}$ and $K_{1.m}$ with an edge**.**

Theorem 4.31

For any double star graph the independent distance - 2 domination number is one.

5. Applications of independent distance - 2 dominating sets

We extended the concept of dominating sets to independent distance - 2 dominating sets, there are more useful models to many real-world problems. Indeed, much of the motivation for the study of independent distance – 2 domination arises from problems involving locating optimally a hospital, police station, fire station, or any other emergency service facility.

5.1 Radio stations

Suppose that we have a collection of small villages in a remote part of the world. We would like to locate radio stations in some of these villages so that messages can be broadcasted to all the villages in the region. But since the installations of radio stations are costly, we want to locate as few as possible which can cover all other villages. Let each village be represented by a vertex. An edge between two villages is labeled with the distance, say in kilometers. The distance between the two villages is shown in fig.6. Let us assume that a radio station has a broadcast range of hundred kilometers. In this case, we seek an independent distance - 2 dominating set among all the vertices within the distance of 100 kilometers. Clearly this fig.7 gives the independent distance - 2 dominating set.

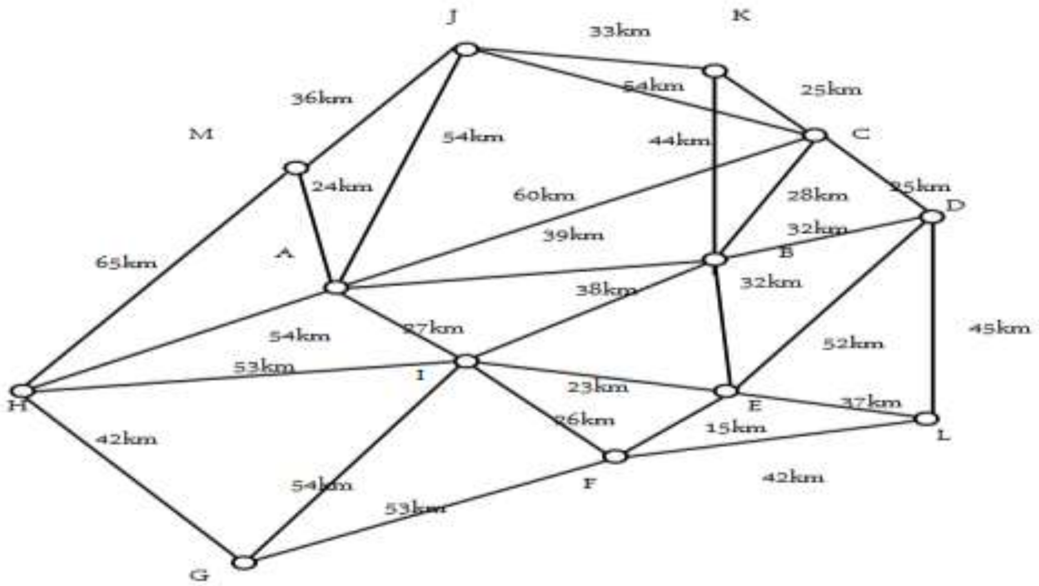

Figure. 6

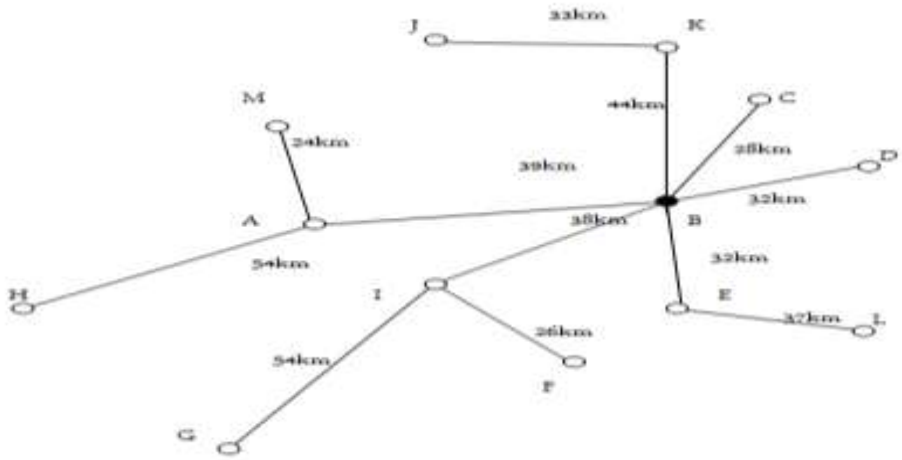

Figure. 7

5.2 Computer Communication Networks

The independent distance - 2 dominating set plays an important role in computer and communication networks to route the information between the nodes. We consider a computer network modeled by a Hyper cube. The vertices of the Hyper cube represents computer and edges represent direct communication link between two computers. So, in this model we have 16 computers or processors and each processor can pass information to the processor to which it is directly connected. Our problem is to collect information from all processors and we would like to do it relatively often and relatively fast. So, we identify a small set of processors called collecting processors and ask each processor to send its information to one of the small sets of collecting processors. We assume that at most a two – unit delay between the time a processor sends its information and the time it arrives at a nearest collector is allowed. In this case, we have to find an independent distance - 2 dominating set of all processors. The set of vertices {marked in dark} forms an independent distance - 2 dominating set in the hypercube network in fig.8.

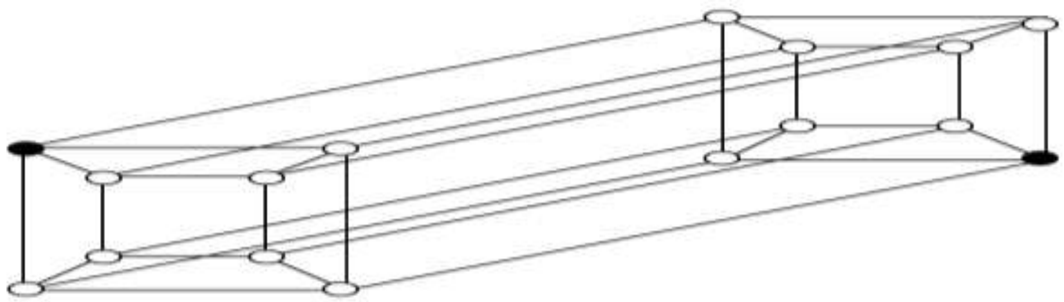

Figure. 8

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