Changing and Unchanging Strong Restrained Domination number of a Graphs

M. Selvaloganayaki[1] and P. Namasivayam[2]

[1]Research Scholar, P.G. & Research Department of Mathematics, The M.D.T. Hindu College, Tirunelveli – 627 010, Tamilnadu, India.

(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627 012, Tamilnadu, India)

[2]Associate Professor, P.G. & Research Department of Mathematics, The M.D.T. Hindu College, Tirunelveli – 627 010, Tamilnadu.

(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627 012, Tamilnadu, India)

Abstract: ***Let G = (V, E) be a simple graph. A Subset S of V is said to be strong restrained dominating set or restrained strong dominating set of G if for every u ∈ V – S, there exists elements v ∈ S and w ∈ V – S such that v and w strongly dominates u. The minimum cardinality of a strong restrained dominating set of G is called the strong restrained domination number of G and is denoted by $\gamma_{srd}(G)$. In this paper, changing and unchanging strong restrained domination number of a graphs are determined.***

Keywords: Domination, strong domination, restrained domination, strong restrained domination.

AMS Subject Classification Number(2010): 05C69.

1. INTRODUCTION

Throughout this paper, finite, undirected, simple graph is considered. Let G = (V,E) be a simple graph. The degree of any vertex u in G is the number of edges incident with u and is denoted by deg u. The minimum and maximum degree of a vertex is denoted by $\delta(G)$ and $\Delta(G)$ respectively. A vertex of degree one is called a pendant (end) vertex and a vertex which is adjacent to an end vertex is called a support vertex. A set $S \subseteq V$ is a dominating set of G if every vertex not in S is adjacent to a vertex in S. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set [1]. The concept of strong domination in graphs was introduced by Sampathkumar and Puspalatha[5] and the restrained domination was introduced by Domke [2] et al. A set $S \subseteq V(G)$ is said to be a strong dominating set of G if every vertex $v \in V – S$ is strongly dominated by some vertex u in S. A set $S \subseteq V(G)$ is a restrained dominating set of G, if every vertex not in S is adjacent to a vertex in S and to a vertex in V – S. The restrained domination number of a graph G, denoted by $\gamma_r(G)$, is the minimum cardinality of a restrained dominating set in G. The strong restrained domination was introduced by Selvaloganayaki and Namasivayam [6]. For all graph theoretic terminologies and notations, Harary [3] is referred to. In this paper, changing and unchanging strong restrained domination number of a graphs are characterized.

Definition 1.1: Let G = (V, E) be a simple graph. A subset S of V is said to be a strong restrained dominating set of G if for every u ∈ V – S, there exists v ∈ S and w ∈ V – S such that v and w strongly dominate u. The minimum cardinality of a strong restrained dominating set of G is called the strong restrained domination number of G and is denoted by $\gamma_{srd}(G)$.
The existence of a strong restrained dominating set of G is guaranteed, since V(G) is a strong restrained dominating set of G.

Example 1.2: Consider the following graph G,

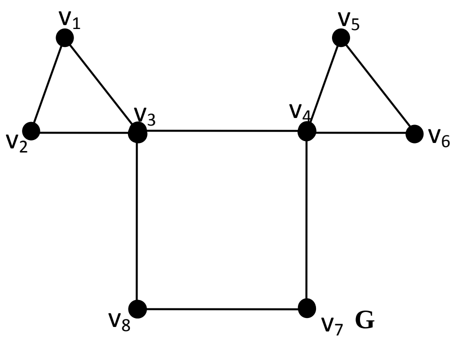

Figure 1

S = {v_3, v_4} is a strong restrained dominating set of G. Since every vertex in V – S has one strong neighbour in S and one strong neighbour in V – S.

Result 1.3: For the path P_m, $\gamma_{srd}(P_m) = \begin{cases} n+2 \text{ if } m = 3n \\ n+3 \text{ if } m = 3n+1 \\ n+4 \text{ if } m = 3n+2 \end{cases}$ where $n \geq 1$.

Result 1.4: $\gamma_{srd}(C_n) = \gamma_r(C_n) = n - 2\left\lfloor \frac{n}{3} \right\rfloor$, $n \geq 3$.
Result 1.5: $\gamma_{srd}(K_n) = 1$, $n \geq 3$.
Result 1.6: $\gamma_{srd}(W_n) = 1$, $n \geq 4$.
Result 1.7: For $n \geq 1$, $\gamma_{srd}(K_{1,n}) = n + 1$.
Result 1.8: For $r, s \geq 1$, $\gamma_{srd}(D_{r,s}) = r + s + 2$.
Result 1.9: Let $G = K_{m,n}$, where $m,n \in N$. Then $\gamma_{srd}(G) = \begin{cases} 2 & if \quad m = n \\ m+n & otherwise \end{cases}$
Result 1.10: Let G be a connected graph.
(i). If G has a unique full degree vertex u then any strong restrained dominating set of G contains u.
(ii). If G has two full degree vertices v and w, then any strong restrained dominating set of G contains v and w.
Result 1.11: If G is a graph with at least 3 full degree vertices, then $\gamma_{srd}(G) = 1$.

2. Main Result: In this chapter, the changing and unchanging values of γ_{srd} when a vertex is removed and an edge is removed from a graph is studied.

Definition 2.1 [4]: Following the notation used in the case of domination, we partition the vertex set V(G) into subsets V_0, V_+, V_- as follows:
$V_{se}^0(G) = \{v \in V(G): \gamma_{se}(G) = \gamma_{se}(G - v)\}$
$V_{se}^+(G) = \{v \in V(G): \gamma_{se}(G) < \gamma_{se}(G - v)\}$
$V_{se}^-(G) = \{v \in V(G): \gamma_{se}(G) > \gamma_{se}(G - v)\}$.

Theorem 2.2: Let $G = P_{3n}$, $n \geq 1$. Let v_i be a vertex of P_{3n}. Then $V_{srd}^+(G) = V(G)$.
Proof: Case i: Let v_i be an end vertex of P_{3n}. Thus $P_{3n} - v_i = P_{3n-1}$. $\gamma_{srd}(P_{3n-1}) = n + 3$ and $\gamma_{srd}(P_{3n}) = n + 2$. Therefore $\gamma_{srd}(P_{3n} - v_i) > \gamma_{srd}(P_{3n})$. Hence $v_i \in V_{srd}^+(G)$.
Case ii: Suppose $v_i = v_2$ or $v_i = v_{3n-1}$. Thus $P_{3n} - v_i = P_1 \cup P_{3n-2}$. $\gamma_{srd}(P_{3n-2}) = n + 2$. Therefore $\gamma_{srd}(P_{3n} - v_i) > \gamma_{srd}(P_{3n})$. Hence $v_i \in V_{srd}^+(G)$.
Case iii: Suppose $v_i = v_3$ or $v_i = v_{3n-2}$. Thus $P_{3n} - v_i = P_2 \cup P_{3n-3}$. $\gamma_{srd}(P_{3n-3}) = n + 1$. Therefore $\gamma_{srd}(P_{3n} - v_i) > \gamma_{srd}(P_{3n})$. Hence $v_i \in V_{srd}^+(G)$.
Case iv: Suppose $v_i = v_{3k}$, $2 \leq k \leq n - 1$. Thus $P_{3n} - v_i = P_{3k-1} \cup P_{3n-3k}$. $\gamma_{srd}(P_{3k-1}) = k + 3$ and $\gamma_{srd}(P_{3n-3k}) = n - k + 2$. Hence $\gamma_{srd}(P_{3n} - v_i) = n + 5$. Therefore $\gamma_{srd}(P_{3n} - v_i) > \gamma_{srd}(P_{3n})$. Hence $v_i \in V_{srd}^+(G)$.
Case v: Suppose $v_i = v_{3k+1}$, $1 \leq k \leq n - 2$. Thus $P_{3n} - v_i = P_{3k} \cup P_{3n-3k-1}$. $\gamma_{srd}(P_{3k}) = k + 2$ and $\gamma_{srd}(P_{3n-3k-1}) = n - k + 3$. Hence $\gamma_{srd}(P_{3n} - v_i) = n + 5$. Therefore $\gamma_{srd}(P_{3n} - v_i) > \gamma_{srd}(P_{3n})$. Hence $v_i \in V_{srd}^+(G)$.
Case vi: Suppose $v_i = v_{3k+2}$, $1 \leq k \leq n - 2$. Thus $P_{3n} - v_i = P_{3k+1} \cup P_{3n-3k-2}$. $\gamma_{srd}(P_{3k+1}) = k + 3$ and $\gamma_{srd}(P_{3n-3k-2}) = n - k + 2$. Hence $\gamma_{srd}(P_{3n} - v_i) = n + 5$. Therefore $\gamma_{srd}(P_{3n} - v_i) > \gamma_{srd}(P_{3n})$. Hence $v_i \in V_{srd}^+(G)$. In all the cases, $V_{srd}^+(G) = V(G)$. Hence the theorem.

Theorem 2.3: $V_{srd}^0(P_m) = \emptyset$, where $m = 3n + 1, 3n + 2$, $n \geq 1$.
Proof: Case i: Let $G = P_{3n+1}$. Suppose vi $\in V_{srd}^0(G)$, where $1 \leq i \leq 3n + 1$. Then $\gamma_{srd}(G - v_i) = \gamma_{srd}(G)$.
Subcase ia: Let v_i be an end vertex of G. Thus $G - v_i = P_{3n}$. $\gamma_{srd}(P_{3n}) = n + 2$ and $\gamma_{srd}(P_{3n+1}) = n + 3$. Therefore $\gamma_{srd}(G - v_i) < \gamma_{srd}(G)$, a contradiction. Therefore v_i cannot be an end vertex of G.
Subcase ib: Suppose $v_i = v_2$ or $v_i = v_{3n}$. Thus $G - v_i = P_1 \cup P_{3n-1}$. $\gamma_{srd}(P_{3n-1}) = n + 3$ and $\gamma_{srd}(G - v_i) = n + 4$. Therefore $\gamma_{srd}(G - v_i) > \gamma_{srd}(G)$, a contradiction. Therefore $v_i \neq v_2$ and $v_i \neq v_{3n}$.
Subcase ic: Suppose $v_i = v_3$ or $v_i = v_{3n-1}$. Thus $G - v_i = P_2 \cup P_{3n-2}$. $\gamma_{srd}(P_{3n-2}) = n + 2$ and $\gamma_{srd}(G - v_i) = n + 4$. Therefore $\gamma_{srd}(G - v_i) > \gamma_{srd}(G)$, a contradiction. Therefore $v_i \neq v_3$ and $v_i \neq v_{3n-1}$.
Subcase id: Suppose $v_i = v_{3k}$, $2 \leq k \leq n - 1$. Thus $G - v_i = P_{3k-1} \cup P_{3n-3k+1}$. $\gamma_{srd}(P_{3k-1}) = k + 3$ and $\gamma_{srd}(P_{3n-3k+1}) = n - k + 3$. Hence $\gamma_{srd}(G - v_i) = n + 6$. Therefore $\gamma_{srd}(G - v_i) > \gamma_{srd}(G)$, a contradiction. Therefore $v_i \neq v_{3k}$, $2 \leq k \leq n - 1$.
Subcase ie: Suppose $v_i = v_{3k+1}$, $1 \leq k \leq n - 1$. Thus $G - v_i = P_{3k} \cup P_{3n-3k}$. $\gamma_{srd}(P_{3k}) = k + 2$ and $\gamma_{srd}(P_{3n-3k}) = n - k + 2$. Hence $\gamma_{srd}(G - v_i) = n + 4$. Therefore $\gamma_{srd}(G - v_i) > \gamma_{srd}(G)$, a contradiction. Therefore $v_i \neq v_{3k+1}$, $1 \leq k \leq n - 1$.
Subcase if: Suppose $v_i = v_{3k+2}$, $1 \leq k \leq n - 2$. Thus $G - v_i = P_{3k+1} \cup P_{3n-3k-1}$. $\gamma_{srd}(P_{3k+1}) = k + 3$ and $\gamma_{srd}(P_{3n-3k-1}) = n - k + 3$. Hence $\gamma_{srd}(G - v_i) = n + 6$. Therefore $\gamma_{srd}(G - v_i) > \gamma_{srd}(G)$, a contradiction. Therefore $v_i \neq v_{3k+2}$, $1 \leq k \leq n - 2$. Thus there is no v_i belong to $V_{srd}^0(G)$. Therefore $V_{srd}^0(P_{3n+1}) = \emptyset$.
Case ii: Let $G = P_{3n+2}$. Suppose $v_i \in V_{srd}^0(G)$, where $1 \leq i \leq 3n + 2$. Then $\gamma_{srd}(G - v_i) = \gamma_{srd}(G)$.
Subcase iia: Let v_i be an end vertex of G. Thus $G - v_i = P_{3n+1}$. $\gamma_{srd}(P_{3n+1}) = n + 3$ and $\gamma_{srd}(P_{3n+2}) = n + 4$. Therefore $\gamma_{srd}(G - v_i) < \gamma_{srd}(G)$, a contradiction. Therefore v_i cannot be an end vertex of G.
Subcase iib: Suppose $v_i = v_2$ or $v_i = v_{3n+1}$. Thus $G - v_i = P_1 \cup P_{3n}$. $\gamma_{srd}(P_{3n}) = n + 2$ and $\gamma_{srd}(G - v_i) = n + 3$. Therefore $\gamma_{srd}(G - v_i) < \gamma_{srd}(G)$, a contradiction. Therefore $v_i \neq v_2$ and $v_i \neq v_{3n+1}$.
Subcase iic: Suppose $v_i = v_3$ or $v_i = v_{3n}$. Thus $G - v_i = P_2 \cup P_{3n-1}$. $\gamma_{srd}(P_{3n-1}) = n + 3$ and $\gamma_{srd}(G - v_i) = n + 5$. Therefore $\gamma_{srd}(G - v_i) > \gamma_{srd}(G)$, a contradiction. Therefore $v_i \neq v_3$ and $v_i \neq v_{3n}$.
Subcase iid: Suppose $v_i = v_{3k}$, $2 \leq k \leq n - 1$. Thus $G - v_i = P_{3k-1} \cup P_{3n-3k+2}$. $\gamma_{srd}(P_{3k-1}) = k + 3$ and $\gamma_{srd}(P_{3n-3k+2}) = n - k + 4$. Hence $\gamma_{srd}(G - v_i) = n + 7$. Therefore $\gamma_{srd}(G - v_i) > \gamma_{srd}(G)$, a contradiction. Therefore $v_i \neq v_{3k}$, $2 \leq k \leq n - 1$.
Subcase iie: Suppose $v_i = v_{3k+1}$, $1 \leq k \leq n - 1$. Thus $G - v_i = P_{3k} \cup P_{3n-3k+1}$. $\gamma_{srd}(P_{3k}) = k + 2$ and $\gamma_{srd}(P_{3n-3k+1}) = n - k + 3$. Hence $\gamma_{srd}(G - v_i) = n + 5$. Therefore $\gamma_{srd}(G - v_i) > \gamma_{srd}(G)$, a contradiction. Therefore $v_i \neq v_{3k+1}$, $1 \leq k \leq n - 1$.

Subcase iif: Suppose $v_i = v_{3k+2}$, $1 \le k \le n - 1$. Thus $G - v_i = P_{3k+1} \cup P_{3n-3k}$. $\gamma_{srd}(P_{3k+1}) = k + 3$ and $\gamma_{srd}(P_{3n-3k}) = n - k + 2$. Hence $\gamma_{srd}(G - v_i) = n + 5$. Therefore $\gamma_{srd}(G - v_i) > \gamma_{srd}(G)$, a contradiction. Therefore $v_i \neq v_{3k+2}$, $1 \le k \le n - 1$. Thus there is no v_i belong to $V^0_{srd}(G)$. Therefore $V^0_{srd}(P_{3n+2}) = \emptyset$. Hence the theorem.

Theorem 2.4: Let $G = C_m$, $m \ge 4$. Then $V^+_{srd}(G) = V(G)$.
Proof: Case i: Let $G = C_{3n}$, $n \ge 2$. Let $v \in V(G)$. Then $\gamma_{srd}(G) = n$, $G - v$ is a path P_{3n-1} and $\gamma_{srd}(P_{3n-1}) = n + 3$. Therefore $\gamma_{srd}(G - v) > \gamma_{srd}(G)$.
Case ii: Let $G = C_{3n+1}$, $n \ge 1$. Let $v \in V(G)$. Then $\gamma_{srd}(G) = n + 1$, $G - v$ is a path P_{3n} and $\gamma_{srd}(P_{3n}) = n + 2$. Therefore $\gamma_{srd}(G - v) > \gamma_{srd}(G)$.
Case iii: Let $G = C_{3n+2}$, $n \ge 1$. Let $v \in V(G)$. Then $\gamma_{srd}(G) = n + 2$, $G - v$ is a path P_{3n+1} and $\gamma_{srd}(P_{3n+1}) = n + 3$. Therefore $\gamma_{srd}(G - v) > \gamma_{srd}(G)$. Therefore $V^+_{srd}(G) = V(G)$. Hence the theorem.

Remark 2.5: Let $G = C_3$. Let $v \in C_3$. Then $\gamma_{srd}(G) = 1$, $G - v$ is a path P_2 and $\gamma_{srd}(P_2) = 2$. Therefore $\gamma_{srd}(G - v) > \gamma_{srd}(G)$. Therefore $V^+_{srd}(G) = V(G)$.

Theorem 2.6: Let $G = K_{1,n}$, $V^-_{srd}(G) = V(G)$, $n \ge 2$.
Proof: Let $V(G) = \{v, v_1, v_2, \ldots, v_n\}$ and $E(G) = \{vv_i / 1 \le i \le n\}$, $\gamma_{srd}(K_{1,n}) = n + 1$.
Case i: $G - v$ is nK_1. $\gamma_{srd}(nK_1) = n$. Therefore $\gamma_{srd}(G - v) < \gamma_{srd}(G)$. Hence $v \in V^-_{srd}(G)$.
Case ii: $G - v_i$, $1 \le i \le n$ is a star $K_{1,n-1}$ and $\gamma_{srd}(K_{1,n-1}) = n$. Therefore $\gamma_{srd}(G - v) < \gamma_{srd}(G)$. Hence $v_i \in V^-_{srd}(G)$. Therefore $V^-_{srd}(G) = V(G)$. Hence the theorem.

Theorem 2.7: $V^-_{srd}(W_n) = \emptyset$, $n \ge 4$
Proof: Let $G = W_n$, $n \ge 4$. Let $V(G) = \{v, v_i / 1 \le i \le n\}$, $E(G) = \{vv_i, v_iv_{i+1} / 1 \le i \le n - 1\} \cup \{v_nv_1\}$ and $\gamma_{srd}(W_n) = 1$. Suppose $v, v_i \in V^-_{srd}(W_n)$, $1 \le i \le n$. Then $\gamma_{srd}(G - v) < \gamma_{srd}(G)$ and $\gamma_{srd}(G - v_i) < \gamma_{srd}(G)$.
Case i: $G - v$ is a cycle C_n and $\gamma_{srd}(C_n) = n - 2\left\lfloor\frac{n}{3}\right\rfloor$. Therefore $\gamma_{srd}(G - v) > \gamma_{srd}(G)$. Hence $v \in V^+_{srd}(G)$, a contradiction.
Case ii: $G - v_i = P_n + K_1$ and $\gamma_{srd}(P_n + K_1) = 1$. Therefore $\gamma_{srd}(G - v_i) = \gamma_{srd}(G)$. Hence $v \in V^0_{srd}(G)$, a contradiction. From cases (i) and (ii), there is no v, v_i belong to $V^-_{srd}(G)$. Therefore
$V^-_{srd}(W_n) = \emptyset$. Hence the theorem.

Theorem 2.8: Let $G = K_{m,n}$, $m, n \ge 2$. Then $V(G) = \begin{cases} V^+_{srd}(G) \text{ if } m = n \\ V^-_{srd}(G) \text{ if } m < n \end{cases}$
Proof: Let $G = K_{m,n}$, $m, n \ge 2$. Let $V(G) = \{u_i, v_j / 1 \le i \le m, 1 \le j \le n\}$.
Case i: Suppose $m = n$. Let $v \in V(G)$. Then $\gamma_{srd}(G) = 2$, $G - v = K_{m,n-1}$ (or) $K_{n,m-1}$. Hence $\gamma_{srd}(G - v) = m + n - 1 > \gamma_{srd}(G)$. Therefore $v \in V^+_{srd}(G)$. Hence $V^+_{srd}(G) = V(G)$.
Case ii: Suppose $m < n$.
Subcase iia: Suppose $n - m = 1$, $\gamma_{srd}(G) = m + n$.
Subsubcase iiai: $G - u_i$ is a complete bipartite graph $K_{m-1,n}$, then $\gamma_{srd}(G - u_i) = m + n - 1 < \gamma_{srd}(G)$.
Subsubcase iiaii: $G - v_i$ is also a complete bipartite graph $K_{m,n-1}$, $m = n - 1$, then $\gamma_{srd}(G - v_i) = 2 < \gamma_{srd}(G)$.
Subcase iib: Suppose $n - m \neq 1$, $\gamma_{srd}(G) = m + n$.
Subsubcase iibi: $G - u_i$ is a complete bipartite graph $K_{m-1,n}$, then $\gamma_{srd}(G - u_i) = m + n - 1 < \gamma_{srd}(G)$.
Subsubcase iibii: $G - v_i$ is also a complete bipartite graph $K_{m,n-1}$, $m = n - 1$, then $\gamma_{srd}(G - v_i) = m + n - 1 < \gamma_{srd}(G)$. Hence $u_i, v_i \in V^-_{srd}(G)$. Therefore $V^-_{srd}(G) = V(G)$. Hence the theorem.

Theorem 2.9: Let $G = D_{r,s}$, $r, s \ge 1$. Then $V^-_{srd}(G) = V(G)$.
Proof: Let $v \in V(G)$, $\gamma_{srd}(G) = r + s + 2$. Thus $G - v = K_{1,r} \cup sK_1$ (or) $rK_1 \cup K_{1,s}$ (or) $D_{r,s-1}$ (or) $D_{r-1,s}$, $\gamma_{srd}(G - v) = r + s + 1 < \gamma_{srd}(G)$. Hence $v \in V^-_{srd}(G)$. Therefore $V^-_{srd}(G) = V(G)$. Hence the theorem.

Definition 2.10 [4]: Following the notation used in the case of domination, we partition the edge set $E(G)$ into subsets E_0, E_+, E_- as follows:

$$E^o_{se}(G) = \{e \in G;\ \gamma_{se}(G) = \gamma_{se}(G - e)\}$$
$$E^+_{se}(G) = \{e \in G;\ \gamma_{se}(G) < \gamma_{se}(G - e)\}$$
$$E^-_{se}(G) = \{e \in G;\ \gamma_{se}(G) > \gamma_{se}(G - e)\}.$$

Theorem 2.11: Let $G = P_{3n}$, $n \ge 2$. Let e_i be a edge of P_{3n}. Then $E^+_{srd}(G) = E(G)$.
Proof: Let $G = P_{3n}$, $n \ge 2$. Let $V(G) = \{v_i / 1 \le i \le 3n\}$ and $E(G) = \{v_iv_{i+1} / 1 \le i \le 3n - 1\}$.
Case i: Suppose $e_i = e_1$ or $e_i = e_{3n-1}$. Thus $P_{3n} - e_i = P_1 \cup P_{3n-1}$. $\gamma_{srd}(P_{3n-1}) = n + 3$, $\gamma_{srd}(P_{3n} - e_i) = n + 4$. Therefore $\gamma_{srd}(P_{3n} - e_i) > \gamma_{srd}(P_{3n})$. Hence $e_i \in E^+_{srd}(G)$.
Case ii: Suppose $e_i = e_2$ or $e_i = e_{3n-2}$. Thus $P_{3n} - e_i = P_2 \cup P_{3n-2}$. $\gamma_{srd}(P_{3n-2}) = n + 2$, $\gamma_{srd}(P_{3n} - e_i) = n + 4$. Therefore $\gamma_{srd}(P_{3n} - e_i) > \gamma_{srd}(P_{3n})$. Hence $e_i \in E^+_{srd}(G)$.
Case iii: Suppose $e_i = e_{3k}$, $1 \le k \le n - 1$. Thus $P_{3n} - e_i = P_{3k} \cup P_{3n-3k}$. $\gamma_{srd}(P_{3k}) = k + 2$ and $\gamma_{srd}(P_{3n-3k}) = n - k + 2$. Hence $\gamma_{srd}(P_{3n} - e_i) = n + 4$. Therefore $\gamma_{srd}(P_{3n} - e_i) > \gamma_{srd}(P_{3n})$. Hence $e_i \in E^+_{srd}(G)$.
Case iv: Suppose $e_i = e_{3k+1}$, $1 \le k \le n - 2$. Thus $P_{3n} - e_i = P_{3k+1} \cup P_{3n-3k-1}$. $\gamma_{srd}(P_{3k+1}) = k + 3$ and $\gamma_{srd}(P_{3n-3k-1}) = n - k + 3$. Hence $\gamma_{srd}(P_{3n} - e_i) = n + 6$. Therefore $\gamma_{srd}(P_{3n} - e_i) > \gamma_{srd}(P_{3n})$. Hence $e_i \in E^+_{srd}(G)$.

Case v: Suppose $e_i = e_{3k+2}$, $1 \le k \le n - 2$. Thus $P_{3n} - e_i = P_{3k+2} \cup P_{3n-3k-2}$. $\gamma_{srd}(P_{3k+2}) = k + 4$ and $\gamma_{srd}(P_{3n-3k-2}) = n - k + 2$. Hence $\gamma_{srd}(P_{3n} - e_i) = n + 6$. Therefore $\gamma_{srd}(P_{3n} - e_i) > \gamma_{srd}(P_{3n})$. Hence $e_i \in E^+_{srd}(G)$. In all the cases, $E^+_{srd}(G) = E(G)$. Hence the theorem.

Theorem 2.12: $E^-_{srd}(P_m) = \emptyset$, where $m = 3n + 1$, $n \ge 2$, $3n + 2$, $n \ge 1$.

Proof: Case i: Let $G = P_{3n+1}$, $n \ge 2$. Suppose $e_i \in E^-_{srd}(G)$, where $1 \le i \le 3n + 1$. Then $\gamma_{srd}(G - e_i) < \gamma_{srd}(G)$.

Subcase ia: Suppose $e_i = e_1$ or $e_i = e_{3n}$. Thus $G - e_i = P_1 \cup P_{3n}$. $\gamma_{srd}(P_{3n}) = n + 2$ and $\gamma_{srd}(G - e_i) = n + 3$. Therefore $\gamma_{srd}(G - e_i) = \gamma_{srd}(G)$, a contradiction. Therefore $e_i \ne e_1$ and $e_i \ne e_{3n}$.

Subcase ib: Suppose $e_i = e_2$ or $e_i = e_{3n-1}$. Thus $G - e_i = P_2 \cup P_{3n-1}$. $\gamma_{srd}(P_{3n-1}) = n + 3$ and $\gamma_{srd}(G - e_i) = n + 5$. Therefore $\gamma_{srd}(G - e_i) > \gamma_{srd}(G)$, a contradiction. Therefore $e_i \ne e_2$ and $e_i \ne e_{3n-1}$.

Subcase ic: Suppose $e_i = e_{3k}$, $1 \le k \le n - 1$. Thus $G - e_i = P_{3k} \cup P_{3n-3k+1}$. $\gamma_{srd}(P_{3k}) = k + 2$ and $\gamma_{srd}(P_{3n-3k+1}) = n - k + 3$. Hence $\gamma_{srd}(G - e_i) = n + 5$. Therefore $\gamma_{srd}(G - e_i) > \gamma_{srd}(G)$, a contradiction. Therefore $e_i \ne e_{3k}$, $1 \le k \le n - 1$.

Subcase id: Suppose $e_i = e_{3k+1}$, $1 \le k \le n - 1$. Thus $G - e_i = P_{3k+1} \cup P_{3n-3k}$. $\gamma_{srd}(P_{3k+1}) = k + 3$ and $\gamma_{srd}(P_{3n-3k}) = n - k + 2$. Hence $\gamma_{srd}(G - e_i) = n + 5$. Therefore $\gamma_{srd}(G - e_i) > \gamma_{srd}(G)$, a contradiction. Therefore $e_i \ne e_{3k+1}$, $1 \le k \le n - 1$.

Subcase ie: Suppose $e_i = e_{3k+2}$, $1 \le k \le n - 2$. Thus $G - e_i = P_{3k+2} \cup P_{3n-3k-1}$. $\gamma_{srd}(P_{3k+2}) = k + 4$ and $\gamma_{srd}(P_{3n-3k-1}) = n - k + 3$. Hence $\gamma_{srd}(G - e_i) = n + 7$. Therefore $\gamma_{srd}(G - e_i) > \gamma_{srd}(G)$, a contradiction. Therefore $e_i \ne e_{3k+2}$, $1 \le k \le n - 2$. Thus there is no e_i belong to $E^-_{srd}(G)$. Therefore $E^-_{srd}(P_{3n+1}) = \emptyset$.

Case ii: Let $G = P_{3n+2}$, $n \ge 1$. Suppose $e_i \in E^-_{srd}(G)$, where $1 \le i \le 3n + 2$. Then $\gamma_{srd}(G - e_i) < \gamma_{srd}(G)$.

Subcase iia: Suppose $e_i = e_1$ or $e_i = e_{3n+1}$. Thus $G - e_i = P_1 \cup P_{3n+1}$. $\gamma_{srd}(P_{3n+1}) = n + 3$ and $\gamma_{srd}(G - e_i) = n + 4$. Therefore $\gamma_{srd}(G - e_i) = \gamma_{srd}(G)$, a contradiction. Therefore $e_i \ne e_1$ and $e_i \ne e_{3n+1}$.

Subcase iib: Suppose $e_i = e_2$ or $e_i = e_{3n}$. Thus $G - e_i = P_2 \cup P_{3n}$. $\gamma_{srd}(P_{3n}) = n + 2$ and $\gamma_{srd}(G - e_i) = n + 4$. Therefore $\gamma_{srd}(G - e_i) = \gamma_{srd}(G)$, a contradiction. Therefore $e_i \ne e_2$ and $e_i \ne e_{3n}$.

Subcase iic: Suppose $e_i = e_{3k}$, $1 \le k \le n - 1$. Thus $G - e_i = P_{3k} \cup P_{3n-3k+2}$. $\gamma_{srd}(P_{3k}) = k + 2$ and $\gamma_{srd}(P_{3n-3k+2}) = n - k + 4$. Hence $\gamma_{srd}(G - e_i) = n + 6$. Therefore $\gamma_{srd}(G - e_i) > \gamma_{srd}(G)$, a contradiction. Therefore $e_i \ne e_{3k}$, $1 \le k \le n - 1$.

Subcase iid: Suppose $e_i = e_{3k+1}$, $1 \le k \le n - 1$. Thus $G - e_i = P_{3k+1} \cup P_{3n-3k+1}$. $\gamma_{srd}(P_{3k+1}) = k + 3$ and $\gamma_{srd}(P_{3n-3k+1}) = n - k + 3$. Hence $\gamma_{srd}(G - e_i) = n + 6$. Therefore $\gamma_{srd}(G - e_i) > \gamma_{srd}(G)$, a contradiction. Therefore $e_i \ne e_{3k+1}$, $1 \le k \le n - 1$.

Subcase iie: Suppose $e_i = e_{3k+2}$, $1 \le k \le n - 1$. Thus $G - e_i = P_{3k+2} \cup P_{3n-3k}$. $\gamma_{srd}(P_{3k+2}) = k + 4$ and $\gamma_{srd}(P_{3n-3k}) = n - k + 2$. Hence $\gamma_{srd}(G - e_i) = n + 6$. Therefore $\gamma_{srd}(G - e_i) > \gamma_{srd}(G)$, a contradiction. Therefore $e_i \ne e_{3k+2}$, $1 \le k \le n - 1$. Thus there is no e_i belong to $E^-_{srd}(G)$. Therefore $E^-_{srd}(P_{3n+2}) = \emptyset$. Hence the theorem.

Result 2.13: Let $G = P_3$. $G - e = P_1 \cup P_2$ (or) $P_2 \cup P_1$. $\gamma_{srd}(G - e) = 3 = \gamma_{srd}(G)$. Hence $e \in E^0_{srd}(G)$. Therefore $E^0_{srd}(G) = E(G)$.

Result 2.14: Let $G = P_4$. $G - e = P_1 \cup P_3$ (or) $P_2 \cup P_2$ (or) $P_3 \cup P_1$. $\gamma_{srd}(G - e) = 4 = \gamma_{srd}(G)$. Hence $e \in E^0_{srd}(G)$. Therefore $E^0_{srd}(G) = E(G)$.

Theorem 2.15: Let $G = C_m$, $m \ge 3$. Then $E^+_{srd}(G) = E(G)$.

Proof: Case i: Let $m = 3n$, $n \ge 1$. Let $e \in E(G)$. Then $\gamma_{srd}(G) = n$, $G - e$ is a path P_{3n} and $\gamma_{srd}(P_{3n}) = n + 2$. Therefore $\gamma_{srd}(G - e) > \gamma_{srd}(G)$. Hence $e \in E^+_{srd}(G)$.

Case ii: Let $m = 3n + 1$, $n \ge 1$. Let $e \in E(G)$. Then $\gamma_{srd}(G) = n + 1$, $G - e$ is a path P_{3n+1} and $\gamma_{srd}(P_{3n+1}) = n + 3$. Therefore $\gamma_{srd}(G - e) > \gamma_{srd}(G)$. Hence $e \in E^+_{srd}(G)$.

Case iii: Let $m = 3n + 2$, $n \ge 1$. Let $e \in E(G)$. Then $\gamma_{srd}(G) = n + 2$, $G - e$ is a path P_{3n+2} and $\gamma_{srd}(P_{3n+2}) = n + 4$. Therefore $\gamma_{srd}(G - e) > \gamma_{srd}(G)$. Hence $e \in E^+_{srd}(G)$. Therefore $E^+_{srd}(G) = E(G)$. Hence the theorem.

Theorem 2.16: Let $G = K_{1,n}$, $E^0_{srd}(G) = E(G)$, $n \ge 2$.

Proof: Let $e \in E(G)$, $\gamma_{srd}(K_{1,n}) = n + 1$. Thus $G - e$ is $K_{1,n-1} \cup K_1$. $\gamma_{srd}(K_{1,n-1} \cup K_1) = n + 1$. Therefore $\gamma_{srd}(G - e) = \gamma_{srd}(G)$. Hence $e \in E^0_{srd}(G)$. Therefore $E^0_{srd}(G) = E(G)$. Hence the theorem.

Theorem 2.17: Let $G = D_{r,s}$, $r, s \ge 1$. Then $E^0_{srd}(G) = E(G)$.

Proof: Let $e \in E(G)$, $\gamma_{srd}(G) = r + s + 2$. Thus $G - e = D_{r-1,s} \cup K_1$ (or) $K_1 \cup D_{r,s-1}$ (or) $K_{1,r} \cup K_{1,s}$, $\gamma_{srd}(G - e) = r + s + 2 = \gamma_{srd}(G)$. Hence $e \in E^0_{srd}(G)$. Therefore $E^0_{srd}(G) = E(G)$. Hence the theorem.

Theorem 2.18: Let $G = K_n$, $n \ge 5$. Then Then $E^0_{srd}(G) = E(G)$.

Proof: Let $e \in E(G)$, $\gamma_{srd}(G) = 1$. $G - e$ has at least 3 full degree vertices, by result 1.11, $\gamma_{srd}(G - e) = 1$. Therefore $\gamma_{srd}(G - e) = \gamma_{srd}(G)$. Hence $e \in E^0_{srd}(G)$. Therefore $E^0_{srd}(G) = E(G)$. Hence the theorem.

Result 2.19: Let $G = K_4$. Let $e \in E(G)$, $\gamma_{srd}(G) = 1$. $G - e$ has two full degree vertices, by theorem 1.10, any strong restrained dominating set of G contains two full degree vertices and there is no vertex to strongly dominate the remaining two vertices, they also belong to strong restrained dominating set of G. Hence $\gamma_{srd}(G - e) = 4$. Therefore $\gamma_{srd}(G - e) > \gamma_{srd}(G)$. Hence $e \in E^+_{srd}(G)$. Therefore $E^+_{srd}(G) = E(G)$.

Theorem 2.20: Let $G = W_n$, $n \ge 5$. Then $E^-_{srd}(G) = \emptyset$.

Proof: Let $V(G) = \{v, v_i / 1 \le i \le n\}$, $E(G) = \{e_i = v_i v_{i+1} / 1 \le i \le n - 2\} \cup \{e_{n-1} = v_{n-1} v_1\} \cup \{e_{i+n-1} = v v_i / 1 \le i \le n - 1\}$ and $\gamma_{srd}(W_n) = 1$. Suppose $e_i, e_{i+n-1}, e_{n-1} \in E^-_{srd}(G)$. Then $\gamma_{srd}(G - e_i) < \gamma_{srd}(G)$, $\gamma_{srd}(G - e_{i+n-1}) < \gamma_{srd}(G)$ and $\gamma_{srd}(G - e_{n-1}) < \gamma_{srd}(G)$.

Case i: $G - e_i$ (or) $G - e_{n-1} = P_n + K_1$ and $\gamma_{srd}(P_n + K_1) = 1$. Therefore $\gamma_{srd}(G - e_i) = \gamma_{srd}(G) = \gamma_{srd}(G - e_{n-1})$. Hence $e_i, e_{n-1} \in E^0_{srd}(G)$, a contradiction.

Case ii: Let S be the strong restrained dominating set of $G - e_{i+n-1}$. $G - e_{i+n-1}$ contain only one maximum degree vertex v, v belongs to S and since there is no vertex to strongly dominate v_i in V – S, v_i belongs to S. Hence $\gamma_{srd}(G - e_{i+n-1}) = 2$. Therefore $\gamma_{srd}(G - e_{i+n-1}) >$

γ_{srd}(G). Hence $e_{i+n-1} \in E^{+}_{srd}$(G), a contradiction. From cases (i) and (ii), there is no edges belong to E^{-}_{srd}(G). Therefore E^{-}_{srd}(G) = ∅. Hence the theorem.

Result 2.21: Let G = W_4. Let e ∈ E(G), γ_{srd}(G) = 1. G – e has two full degree vertices, by theorem 1.10, any strong restrained dominating set of G contains two full degree vertices and there is no vertex to strongly dominate the remaining two vertices, they also belong to strong restrained dominating set of G. Hence γ_{srd}(G – e) = 4. Therefore γ_{srd}(G – e) > γ_{srd}(G). Hence e ∈ E^{+}_{srd}(G). Therefore E^{+}_{srd}(G) = E(G).

Theorem 2.22: Let G = $K_{m,n}$, m, n ≥ 1. Let V(G) = (V_1, V_2), $|V_1|$ = m and $|V_2|$ = n. Then E(G) =
$$\begin{cases} E^{0}_{srd}(G) \text{ if } m = n,\ m,n \neq 2 \text{ and } |m-n| \geq 2 \\ E^{-}_{srd}(G) \text{ if } |m-n| = 1 \end{cases}$$
Proof: Case i: Suppose m = n, m, n ≠ 2. Let e = u_iv_j, 1 ≤ i, j ≤ n. {u_k, v_t}, 1 ≤ k, t ≤ n, k ≠ i, t ≠ j is a strong restrained dominating set of G – e. Clearly γ_{srd}(G – e) = 2 = γ_{srd}(G). This is true for any e ∈ E(G). Hence E^{0}_{srd}(G) = E(G).
Case ii: Suppose | m – n | ≥ 2. γ_{srd}(G) = m + n. Let e = u_iv_j, 1 ≤ i ≤ m, 1 ≤ j ≤ n. Then V(G) is the unique strong restrained dominating set of G – e. Hence E^{0}_{srd}(G) = E(G).
Case iii: Suppose |m – n| = 1. γ_{srd}(G) = m + n. Let V_1 = {v_1, v_2, …., v_m} and V_2 = {u_1, u_2, ….., u_n}. Let e = v_iu_j, 1 ≤ i ≤ m, 1 ≤ j ≤ n. Since the maximum degree vertices are not adjacent with one another, they belong to any strong restrained dominating set of G. Then S = {v_k / 1 ≤ k ≤ m, k ≠ i} ∪ {u_t, u_j / 1 ≤ t ≤ n, t ≠ j} is a strong restrained dominating set of G – e. Therefore |S| = m + 1. Hence γ_{srd}(G – e) ≤ m + 1. Also, no set with less than m vertices forms a strong restrained dominating set of G – e. Therefore γ_{srd}(G – e) ≥ m + 1. Hence γ_{srd}(G – e) = m + 1. Therefore γ_{srd}(G – e) < γ_{srd}(G). Hence e ∈ $E^{-}_{srd}(G)$. Therefore E^{-}_{srd}(G) = E(G).

Remark 2.23: Suppose m = n = 2, γ_{srd}($K_{2,2}$) = 2. Since $K_{2,2}$ – e = P_4, γ_{srd}($K_{2,2}$ – e) = 4. Hence γ_{srd}($K_{2,2}$ – e) > γ_{srd}($K_{2,2}$). Therefore e ∈ E^{+}_{srd}($K_{2,2}$). Hence E^{+}_{srd}($K_{2,2}$) = E($K_{2,2}$).

3. CONCLUSION

In this paper, the authors studied changing and unchanging strong restrained domination number of a graphs. Similar studies can be made on this type.

REFERENCES

[1] Acharya. B. D, Waliker. H. B and Sampathkumar. E, ***Domination Theory in Graphs***, Mehta Research Institute, Alagabad, 1981.
[2] Domke.G.S, et al., ***Restrained domination in graphs***, Discrete Mathematics, 203, pp.61-69, (1999).
[3] Harary. F, ***Graph Theory***, Adison Wesley, Reading Mass (1969).
[4] Meena, N, Subramanian. A and Swaminathan. V, ***Changing and Unchanging GSE for Vertex Removal in Graphs***, Proceedings of the International Conference on Applied Mathematics and Theoretical Computer Science - 2013, ISBN 978-93-82338-35-2 2013 © Bonfring
[5] Sampathkumar. E and Pushpa Latha. L, ***Strong weak domination and domination balance in a graph***, Discrete Math., 161: 235 – 242, 1996.
[6] Selvaloganayaki. M and Namasivayam. P, ***Strong restrained domination number for some standard graphs,*** Advances in Domination Theory II, Vishwa International Publication (2013).