

# ANALYSIS OF COMBINATORIAL GAMES

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## Abstract

"Coincident Games" are perfect information games for 2 players. Of course, a SUM of such games is defined as the game in which each player can execute his legal moves in a single total. The study of such sums is a topic called "combinatorial theory of play". Combinatorial game theory is not only an interesting theory in itself, but it is linked to many other areas of mathematics, making it an exciting top. In this work we will introduce some of the existing combinatorial theories. We will also introduce some applications in which this game theory, for example, provides a link to other parts of mathematics. For algebra and coding theory. Combinatorial games are zero sum information games that are perfect for two people, and can theoretically be fully analyzed. However, there are still many problems to solve, R.K. Guy has created a list. A subset of games is called impartial.

**Keywords:** game theory, combinatorial theories, Sprague-Grundy theory

## Introduction

To explore the connection between the games and computer games can be explored from the perspective of IT complexity. Of course, these games will have a combinatorial aspect. In fact, they could only be called "combinatorial games", with the exception of the fact that there is already a field combinatorial game theory and many types of games are well thought out and will not be part of the domain called this field. A combinatorial game is defined as a deterministic and perfect information game for two players, with no random elements. By limiting the notion of the game, the theory of combinatorial games is to create competitively beautiful and unexpected relationships between games and numbers; In this context, a number is simply a kind of special game. But there are many other "games" that have a combinatorial nature, such as the puzzle blocks or the game of Conway or Bridge Life Slides that violate these limitations, but they are interesting to investigate the calculation point of view. The combinatorial game theory needs to be different from all other forms of game theory that appear in the economic environment. The economic theory of games also has many applications in computer science. Popular examples are auctioning and Internet analytics.

## Literature review

In the general field of game theory, a mathematical game is a game with a set of strict rules with simple mathematical parameters. The game has many possible outcomes that provide rewards for the players. The result is uncertain, as many players have to make decisions. An essential difference between mathematical games and most games played in everyday life is that in mathematical games all players should behave rationally. This type of behavior means that all players want to optimize their pay and are aware that other players have the same goal. The combinatorial game theory, a branch of game theory, contains additional parameters that define a game: A combinatorial game consists of two players (called "left" and "right"). There are many positions, but it ends, and the left and right players take turns. Both players need to know what happens, what is called complete information. There is no random element. In normal play, the player who can not make a legal move loses. After all, the rules are set so that the game always ends. Although combinatorial game theory analysis does not require all of these rules to be met, complete information and the absence of a random element are required. There are two main categories of games: non-partisan games, where both players have the same options at a particular position, and partisan games, where both players can have different options. There is a difference between a "good" movement in the theory of combinatorial games and a "good" movement in a game played in real life. A good game is one that wins the player. Although it may seem obvious, a good move in the real world can have several meanings. For example, a move that confuses the opponent or hides the player's strategy may be considered positive in the real world, but not necessarily in the sense of combinatorial game theory.

A game is said to be periodic if its nim-values repeat. The period of a game is the number of values in one cycle of repeated values (Berlekamp et al.). An octal game is a certain type of impartial game that is similar to nim. An octal game is specified by a three-digit octal code  $d_1d_2d_3\dots d_i$ . This code signifies that a player can remove 1, 2, 3...or  $i$  beans from any heap. The player must then leave the heap in  $j, k, l\dots$  non-empty heaps, where  $j, k,$  and  $l$  are the exponents when  $d_i$  is expressed as the sum of powers of two. Every octal game is conjectured to be periodic. Octal games are characterized by their high variation in period length, but no octal games have been found which do not repeat.

## Analysis of two-player combinatorial games

The analysis of combinatorial games with two players revealed another surprising theory based on a mathematical model. Albert, Nowakowski and Wolfe also presented an introductory book on "Lessons in Play"; Another more comprehensive reference is the Winning Ways book by Berlekamp, Conway and Guy. A presentation of a more mathematical book is titled In Numbers and Games by Conway. The basic idea behind the theory is modest: A game for two players can be described by a rooted tree. In the tree, each node has zero or more left branches that correspond to the options that the player can move. At the same

time, the player's right to move may have no or more straight branches corresponding to the move options; In the tree, the hands are linked to the finished games, with the winner being determined by the normal game option or by misery. The combinatorial game theory has interesting parts that represent several methods for implementing and analyzing such games / trees.

### *Two-player Games*

The two-player game dilutes its two player requirement by for solitaire puzzles. It considers it as combinatorial games by considering that solver of the puzzle contests against the null player. Thus it indirectly challenges the designer who has set the challenge. Multiplayer games which have three or more players fall outside the space of combinatorial play as it has some social aspect of alliances that may arise.

The word *game* now should only refer to a two-player combinatorial game. Such games falls in category of perfect test bed for various game program based experiments. These experiments are having own search depth but can be described by simple, well defined rule sets. It is note-worthy that this work doesn't get examined in combinatorial game theory. The said theory is only concerned with the games scrutiny along a view to solving them, or at least finding optimal strategies and emerging players with artificial approach which make them able to challenge human experts. This study context also limits the artificial player as it has little interest to provide self-play simulations. There should also be adequate strength to provide meaningful play outs, which is primary concern for any quality of the game concentrating on it rather than the quality of the player.

### **Zero-Player Games**

Zero-player games; which may be thought as game of simulations: each move is determined from the preceding configuration. Examples that are frequently thought of as games include cellular automata, such as Conway's Game of Life. Generally, the class of zero-player games resembles naturally to ordinary computers or deterministic space-bounded Turing machines—the kinds of computation tools or quantum computers get available. The Constraint Logic formalism does not confine the set of moves available on a constraint graph to an exclusive next move from any given position. To deliberate a deterministic version, further constrain of the legal moves is done. Rather than propose a rule which selects a unique next edge to reverse from each position, determinism is applied independently at each vertex, so that numerous edge reversals may occur on each deterministic "turn".

## Sprague-Grundy Theory

A celebrated result in Combinatorial Game Theory is the characterization of impartial two-player perfect-information games, discovered independently in the 1930's by Sprague and Grundy. Recall that a game is impartial if it does not distinguish between the players Left and Right. The Sprague-Grundy theory states that every finite impartial game is equivalent to an instance of the game of Nim, characterized by a single natural number  $n$ . This theory has since been generalized to all impartial games by generalizing Nim to all ordinals  $n$ ; Nim is a game played with several heaps, each with a certain number of tokens. A Nim game with a single heap of size  $n$  is denoted by  $\_n$  and is called a number. During each move a player can pick any pile and reduce it to any smaller nonnegative integer size. The game ends when all piles have size 0. Thus, a single pile  $\_n$  can be reduced to any of the smaller piles  $\_0, \_1, \dots, \_(n-1)$ . Multiple piles in a game of Nim are independent, and hence any game of Nim is a sum of single-pile games  $\_n$  for various values of  $n$ . In fact, a game of Nim with  $k$  piles of sizes  $n_1, n_2, \dots, n_k$  is equivalent to a one-pile Nim game  $\_n$ , where  $n$  is the binary XOR of  $n_1, n_2, \dots, n_k$ . As a consequence, Nim can be played optimally in polynomial time (polynomial in the encoding size of the pile sizes).

Even more surprising is that every impartial two-player perfect-information game has the same value as a single-pile Nim game,  $\_n$  for some  $n$ . The number  $n$  is called the  $G$ -value, Grundy value, or Sprague-Grundy function of the game. It is easy to define: suppose that game  $x$  has  $k$  options  $y_1, \dots, y_k$  for the first move (independent of which player goes first). By induction, we can compute  $y_1 = \_n_1, \dots, y_k = \_n_k$ . The theorem is that  $x$  equals  $\_n$  where  $n$  is the smallest natural number not in the set  $\{n_1, \dots, n_k\}$ . This number  $n$  is called the minimum excluded value or mex of the set. This description has also assumed that the game is finite, but this is easy to generalize.

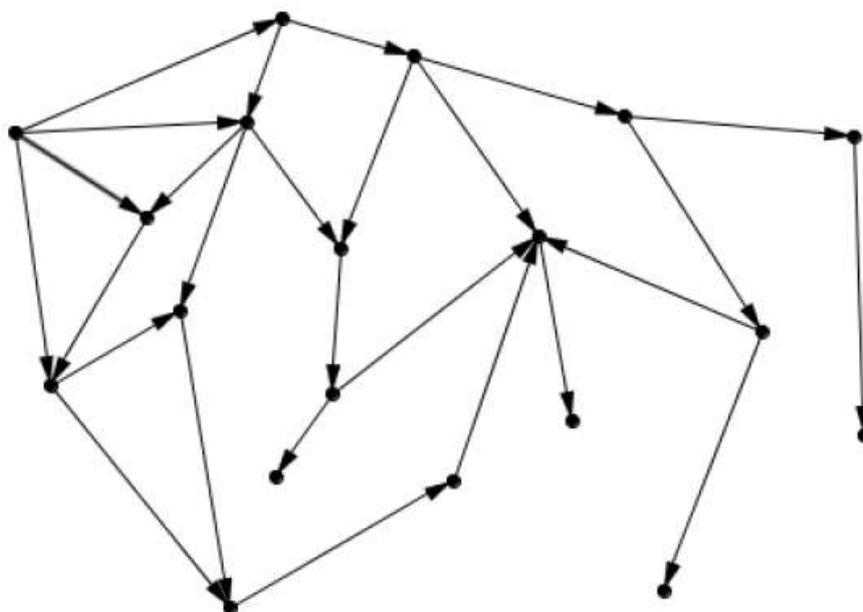


Figure 1 A graph game with no SG-values assigned to positions

The Sprague-Grundy function can increase by at most 1 at each level of the game tree, and hence the resulting number is linear in the maximum number of moves that can be made in the game; the encoding size of the number is only logarithmic in this count. Unfortunately, computing the Sprague-Grundy function for a general game by the obvious method uses time linear in the number of possible states, which can be exponential in the number itself.

## Conclusion

The theory of combinatorial games is a wide field. Over the past forty years, it has evolved into a wide range of games. All of these examples were short games that have finite subtrees and prohibit endless play. The combinatorial theory of short games is essential to the topic and will cover half of the content. Although combinatorial game theory has received little attention over the years, its theory is quite extensive. However, little research has been done on its applications in the field of artificial intelligence. As a result, many areas can still be explored. We've explained some models of combinatorial games and just compare them. Then we introduced the rules of the game and how to add, multiply and compare them.

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