Performance Analysis of Kaiser-Hanning Window For Digital Filter Design

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Abstract: Many digital signal processing applications make use of digital filters. That's why, digital filtering is considered as the basic needs of digital signal processing. This paper introduces a new coupled window by adding of two windows, known as Kaiser and Hanning, for the design of finite impulse response (FIR) digital filters. The quality of the proposed window is analyzed in terms of window spectral parameters, main-lobe width (MLW) and side-lobe amplitude, and then compared with other windows, namely Kaiser and Hanning. Results show that the proposed window can provide 1.5 to 4 dB side-lobe amplitude and smaller MLW than other windows for fixed window length (M=51). Equivalent Noise Bandwidth (ENBW) calculated also gives better results i.e. 1.05. Application example for the performance analysis of the proposed window in digital filter design confirms that the window can exhibit a better filtering performance by providing smaller minimum stop-band attenuation (As) for fixed filter length (M=51) compared to Kaiser and Hanning window.

***Index Terms* – FIR, MLW, ENBW, FFT, SNR, As.**

I . INTRODUCTION

Digital filters have a vital role in digital signal processing applications such as digital signal filtering, noise reduction, frequency analysis, multimedia compression, biomedical signal processing and image enhancement etc. The digital filters can reduce or enhance certain aspects of any signal by passing some desired signals through it. It passes the signals according to the specified frequency of pass-band and rejects the other frequencies. The basic filters can be divided into four categories i.e. low-pass, high-pass, band-pass and band-stop. There are two fundamental types of digital filters, on the basis of impulse response, i.e. Infinite Impulse Response (IIR) filters and Finite Impulse Response (FIR) filters [1].
Because FIR digital filters have a linear phase, highly stable, non-recursive structure and arbitrary amplitude frequency characteristic etc., that's why FIR filters are preferred over IIR filters [1]. The design and simulation analysis of the digital filter is quickly and efficiently achieved by using powerful computing capabilities of MATLAB.
FIR filter is described by the difference equation

$$y(n) = h(k)x(n-k) \tag{1}$$

Where, x(n) is input signal and h(k) is impulse response of FIR filter. The transfer function of a causal FIR filter is described by;

$$H(z) = \sum_{k=0}^{M-1} h(k)z^{-k} \tag{2}$$

Designing of digital FIR filter by window method is a simple and effective way [2]. Infinite impulse response of the prescribed filter is truncated by using a Window function in this method. The impulse response coefficient can be obtained in closed form and can be determined very easily and quickly. The most common and widely used window functions are: Rectangular window, Hanning window, Hamming window and Kaiser window.

There are two main applications of windows in digital signal processing: firstly, data analysis based on Fast Fourier transform (FFT), secondly, design of finite impulse response (FIR) filters from infinite impulse response (IIR) filters. For FFT analysis, to reduce the so called 'leakage effect' windows are employed, and for FIR filter design, Gibbs oscillations are attenuated by 'windowing method'. Characteristics considered as desirable for a window in the frequency domain are: narrow main-lobe, small side-lobes and fast side-lobes roll-off [3].

In this paper, a new window is presented that offers about 1.5–4 dB smaller peak side-lobe amplitude than that of the Hanning window, while having the same or even narrower main-lobe width. It also offers about 5–6 dB smaller peak side-lobe amplitude than that of the Kaiser window. Smaller side-lobes represent better SNR in signal processing by reducing the leakage effect, and also give smaller ripples in the frequency response of windowed IIR filters. New window shows better results in terms of ENBW (Equivalent Noise Bandwidth) also. The proposed window is obtained by adding Hanning and Kaiser Window. We compare the performance of the new window with, Hann and Kaiser. Further, in order to examine the efficiency of the proposed window, we design FIR filters by windowing of IIR filters using the mentioned windows.
The rest of the paper is organized as follows: Section II describes the new window and its spectrum. In Section III, part 1 calculates and compares ENBW of proposed window with standard window, in part 2 frequency response of the proposed

window is compared with those of Hann and Kaiser windows. As an application example, FIR low-pass filter design by the windowing method is considered in Section IV. Finally, Section V concludes the paper.

II. PROPOSED WINDOW

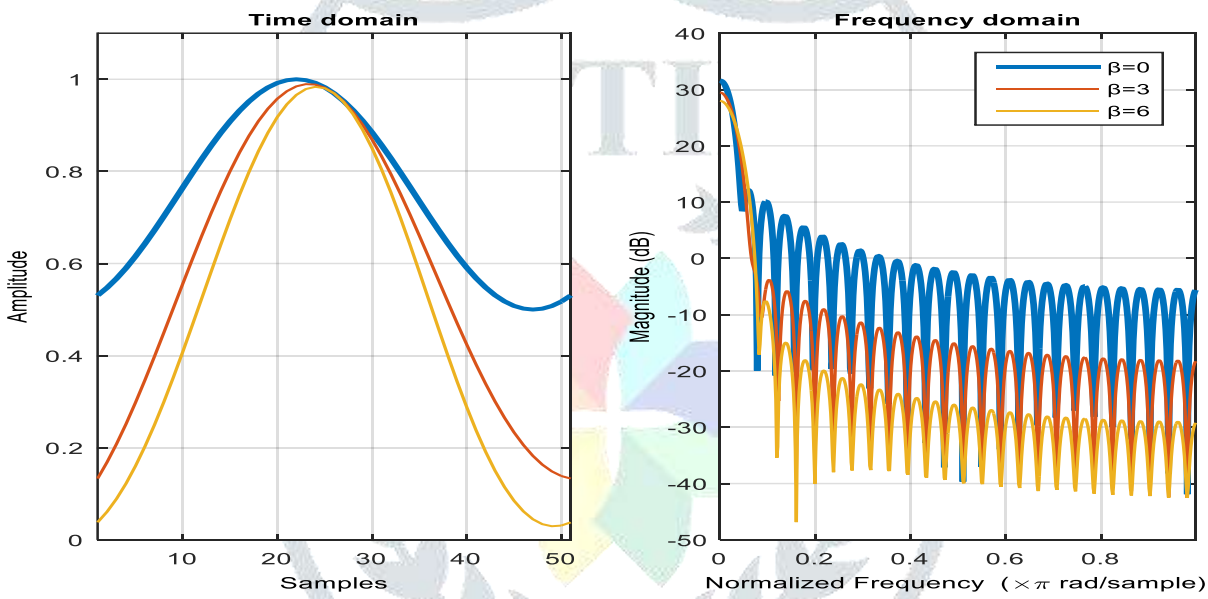
II.A. Description of window

M a window having a length, w(nT), $|n| \leq (M-1)/2$, in the range of zero and the remaining range is a function of time is zero. Due to the page limitation is not given details, for more information [5] to be viewed. In all calculations made in the rest of the paper it is taken as a second sampling period. (That is, T = 1).

II.B. Kaiser-Hanning window

Hanning window and Kaiser window are coupled to form the combination of the following window.

$$w(n)= 0.5\left\{\frac{I_0(\beta\sqrt{1-(2n/M-1)^2})}{I_0(\beta)} + 0.50-0.5\cos 2\pi\left(\frac{n}{M-1}+0.5\right)\right\} \quad (3)$$



(a) (b)

Figure 1 (a) Amplitude characteristic of Kaiser-hanning window for M=51.
(b) Frequency spectrum of Kaiser-hanning window for M=51.

Independent parameters of the Kaiser-Hanning window are M and 'β'. For M = 51 and different 'β', amplitude characteristic values in time is given in Figure 1(a). The performance analysis of the frequency spectrum and the spectral parameters are required to have a narrower main-lobe width. In Figure 1(b), M = 51 and different 'β' values are given for the frequency spectrum of the Kaiser-Hanning window, the window spectral parameter values are reported in Table 1.

Table 1. Kaiser-hanning frequency spectral value for M=51

Window	M	β	RLA(dB)	MLW(dB)
Kaiser-hanning	51	0	-19.4	0.035
Kaiser-hanning	51	3	-33.3	0.046
Kaiser-hanning	51	6	-35.5	0.054

III. PERFORMANCE EVALUATION

III.1 ENBW

The amplitude of the harmonic estimate at a given frequency is biased by the accumulated broadband noise included in the bandwidth of the window. By this logic, the window behaves as a filter, gathering contributions for its estimate over its bandwidth. It is desired to minimize this accumulated noise signal, for the harmonic detection problem, by accomplishing with small-bandwidth windows.

An appropriate measure of this bandwidth is the Equivalent Noise Bandwidth (ENBW) of the window. Figure 2 shows the width of a rectangle filter of the same peak power gain that would gather the same noise-power [2].

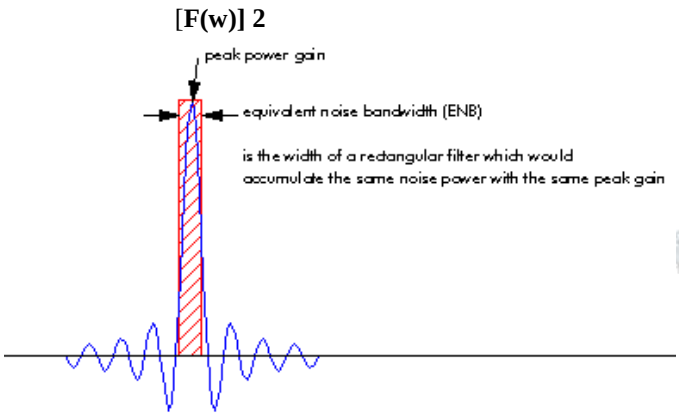


Figure 2 -Equivalent noise bandwidth of window.

$$\text{ENBW}=N\frac{\sum_{n=0}^{M-1}|w(n)|^2}{\left(\sum_{n=0}^{M-1}w(n)\right)^2} \qquad (4)$$

where, w(n) is the window function taken in consideration.

Table2-ENBW comparison of different windows

Window	**ENBW(Bins)**
Rectangular	1
Hamming	1.37
Hanning	1.52
Blackman	1.75
Kaiser-hanning window(β=0)	1.05

III.2 FREQUENCY SPECTRUM

In this section, we compare frequency-domain characteristics of the proposed window with some commonly used windows.

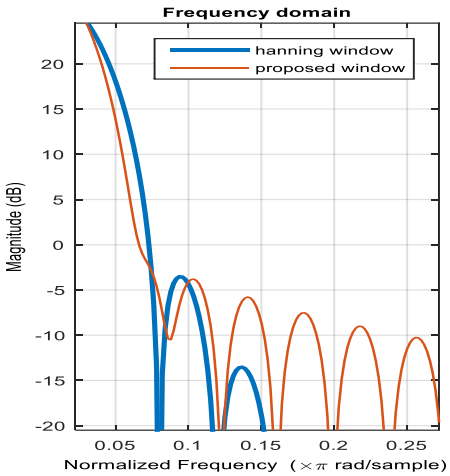
III.2.A. Hanning window

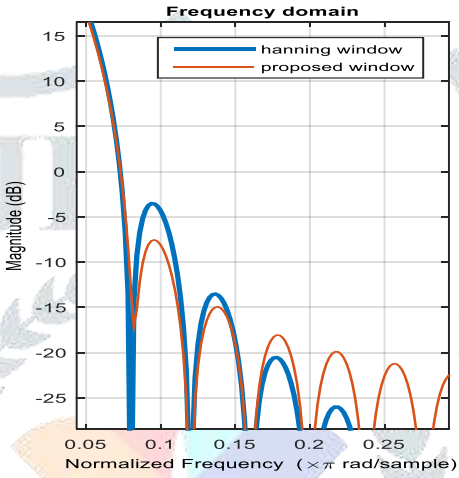
Hanning window has the following shape

$$w_1(n)=0.50-0.5\cos 2\pi\left(\frac{n}{M-1}\right) \qquad (5)$$

Figure 3 compares the frequency spectrum of the proposed and Hanning windows for same order but different values of 'β'. Figure 3(a) shows that, for β=3, at normalized frequency Wr=0.103 rad/s, the proposed window offers both smaller main-lobe width and peak side-lobe level compared to the Hanning window at Wr=0.0957 rad/s. Figure 3(b) consider the window at β=6and Wr=0.0957, the main-lobe widths of the two windows become approximately equal, but the peak side-lobe amplitude of the new window is smaller.

The data presented in Table 3 indicate that the peak side-lobe amplitude of the new window is about 1.5–4 dB smaller than that of Hanning window, while having smaller or equal , main-lobe width for small 'β' values (β<=6).





(a) (b)

Figure 3- Frequency spectrum of the Kaiser-Hanning and Hanning window for different values of 'β' for window order M=51
(a) β=3
(b) β=6

Table 3- Frequency-domain performance comparison of the Kaiser-hanning and Hanning windows

β	Kaiser-Hanning window			Hanning window		
	Wr(rad/s)	Main-lobe width	Side-lobe peak(dB)	Wr(rad/s)	Main-lobe width	Side-lobe peak(dB)
β=3	0.103	2× 0.0468 ×π	- 33	0.0957	2× 0.055×π	-31.5
β=6	0.0957	2× 0.0546×π	-35.52	0.0957	2× 0.055 ×π	-31.52

III.2.B. Kaiser Window

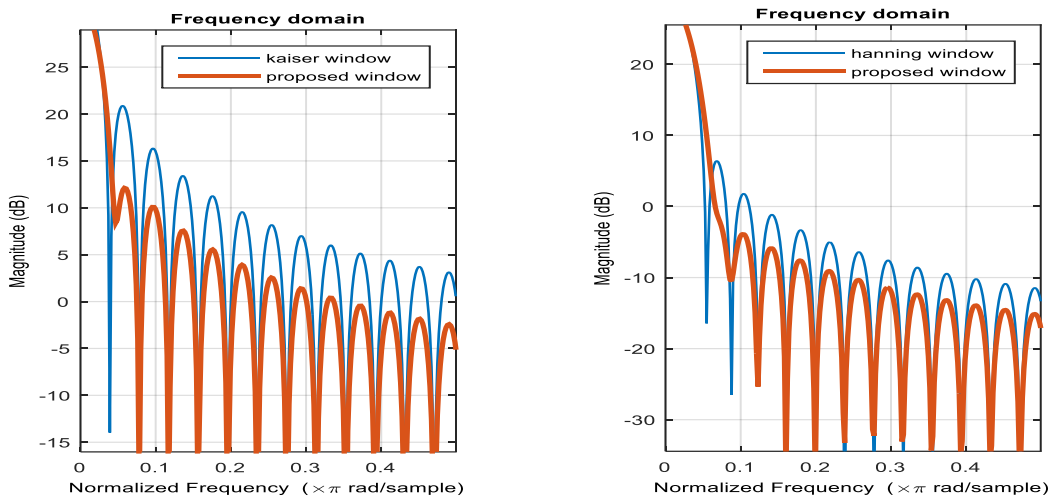
Kaiser window has the following shape [4]

$$w_2(n)= \frac{I_0(\beta\sqrt{1-(2n/M-1)^2})}{I_0(\beta)} \quad , \quad 0< n <M-1 \qquad (6)$$

where β is the tuning parameter of the window to obtain the desired 'main-lobe width – side-lobe peak' trade-off and I_0 is the modified Bessel function of order zero.

Figure 4(a) and (b) demonstrate the performance of the proposed and Kaiser window for M=51 and two different values of β. For β=0, Figure 4(a) shows that the proposed window has smaller side-lobe amplitude, at Wr=0.057, than Kaiser window (5.5 dB), while the proposed window has greater main-lobe width. Figure 4(b) evaluates, at Wr=0.068, for β=3 that the proposed window has smaller side-lobe amplitude than Kaiser window (6 dB), while the proposed window has again greater main-lobe width.

So, the proposed window shows 5.5-6 dB smaller side-lobe amplitude as compared to Kaiser window for the same order and same β value in the expense of main-lobe width for low values of β (β<=3).



(a) (b)

Figure 4- Frequency spectrum of the Kaiser-hanning and Kaiser Window for different values of 'β' for window order M=51
(a) β=0
(b) β=3

Table 4- Frequency-domain performance comparison of the Kaiser-hanning and Kaiser windows

β	Kaiser-hanning window			Kaiser window		
	Wr(rad/s)	Main-lobe width	Side-lobe peak(dB)	Wr(rad/s)	Main-lobe width	Side-lobe peak(dB)
β=0	0.058	2× 0.0468 ×π	-19.06	0.058	2× 0.0313×π	-13.27
β=3	0.068	2× 0.0546×π	-30.47	0.068	2× 0.043 ×π	-24.30

IV. APPLICATION EXAMPLE

It is requisite to find a suitable impulse response for designing a FIR filter that satisfies the prescribed filter specifications. By using a window, w(nT), the impulse response of a realizable non-casual FIR filter can be acquired as [4]

$$h_{nc}(nT) = w(nT)h_{id}(nT) \qquad (7)$$

where $h_{id}(nT)$ is the impulse response of the ideal filter with infinite length. For a given cut-off frequency and sampling period (T), a low pass filter can be found as [4]

$$h_{id}(nT)=\begin{cases} w_{ct}T = \pi & for \quad n = 0 \\ \frac{sinwctnT}{n\pi} & for \quad n \neq 0 \end{cases} \qquad (8)$$

If the non-casual impulse response $h_{nc}(nT)$ is delayed by a period (M-1)/2, a causal impulse response can be obtained as

$$h_c(nT) = h_{nc}[(n - (M - 1)/2)T] \qquad (9)$$

It is familiar that the ripples in pass-band and stop-band regions of the filters designed by the windowing method are approximately equal to each other [3]. Therefore, A*s (*minimum stop-band attenuation) parameter is considered as ripple performance in filter simulation examples [5].
For analyzing the designed filter, its amplitude spectrum must be plotted. The frequency spectrum of a filter can be found from its impulse response as

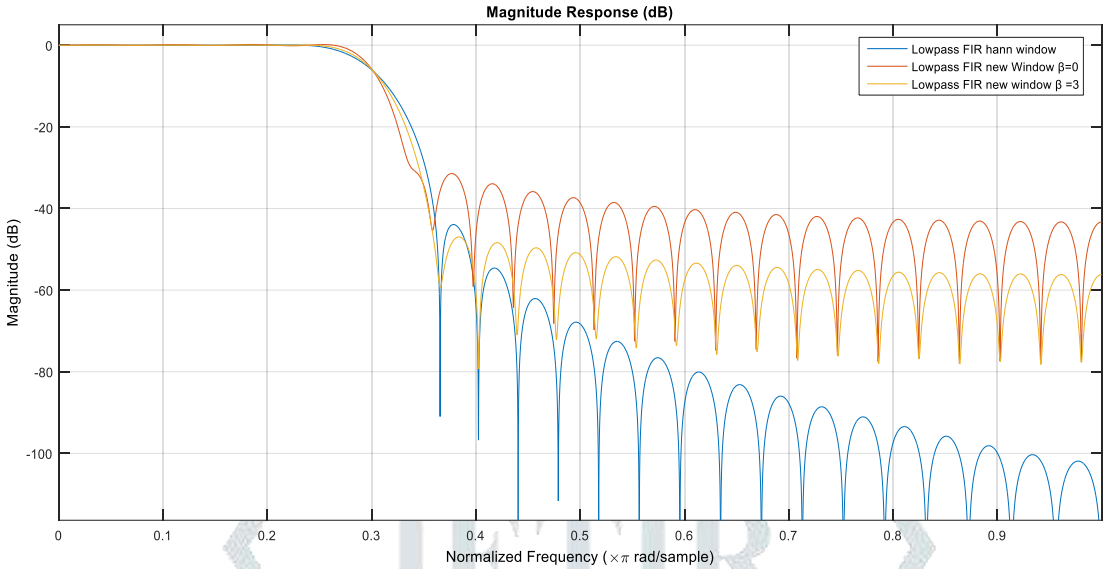
$$H(e^{jwT}) = \sum_{n=0}^{M-1} h(nT)e^{-jwnT} \qquad (10)$$

The amplitude spectrum in logarithmic scale can be calculated [6] from

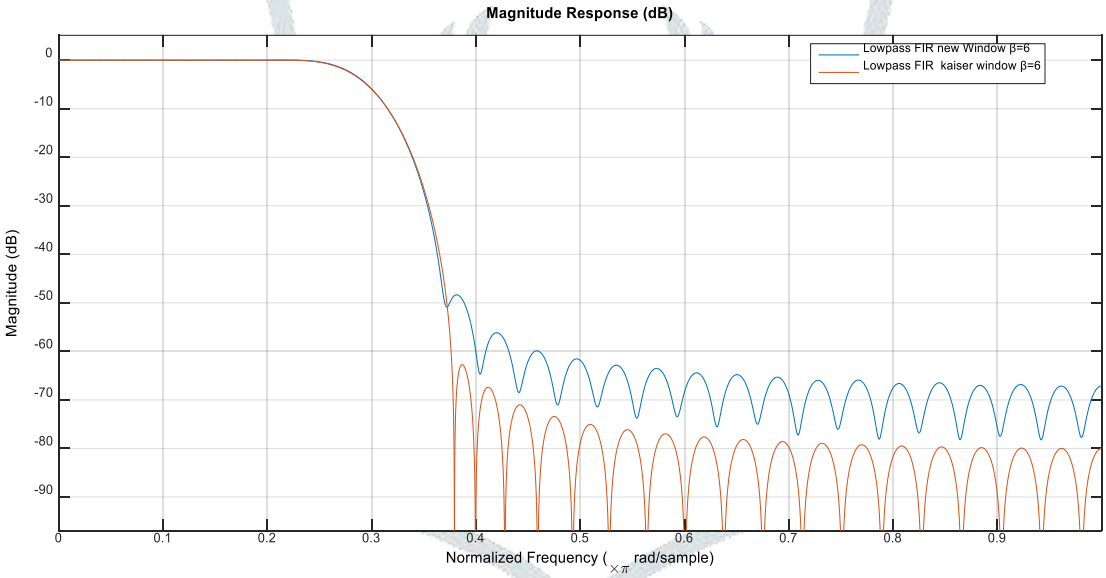
$$H(e^{jwT})_{dB} = 20 \log_{10}(|H(e^{jwT})| / |H(e^{jwT})|_{w=0}) \qquad (11)$$

Using Equations (3), (7), and (8) low-pass filters can be designed by using the proposed window. The amplitude spectrums of the filters designed for various 'β' values at M = 51 by using Eq. (11) are shown in Figure 10. The numerical data for this figure are

summarized in Table 5. It is seen from the figure and table that the proposed window filter has smaller stop-band attenuation (As=-31.45 compared to As=-44.03) than hanning window for small 'β' (β < 3) and shows smaller stop-band attenuation than Kaiser window for β >= 6 (As=-48.37 compared to As=-62.84).



(a)



(b)

Figure 5- Frequency responses of FIR low-pass filters (M = 51) obtained by windowing of an IIR ideal low-pass filter (Wc=0.29π) with the proposed window and

(a) Hamming

(b) Kaiser

Table 5- Characteristics of FIR filters designed by windowing of an ideal low-pass filter with N=50

Window	N(order)	Wr(rad/s)	β	Stop-band attenuation(dB)
Hanning	50	0.376	-	-44.03
Kaiser	50	0.375	6	-62.84
Kaiser-Hanning	50	0.376	0	-31.45
Kaiser-Hanning	50	0.376	3	-47.14
Kaiser-Hanning	50	0.375	6	-48.37

V. CONCLUSION

Evaluation shows that proposed window (Kaiser-Hanning) is better than hanning window by giving 1.5 to 4 dB smaller side-lobe amplitude (-31.52 as compared to -35.52) with smaller or equal main-lobe width for smaller β. For Kaiser window, it shows smaller side-lobe amplitude, 5.5 to 6 dB, with the expense of greater main-lobe width (MLW). ENBW calculated has also way better value, i.e. 1.05, as compared to hanning, hamming, blackman window. The FIR filter designed with the proposed window achieves smaller stop-band attenuation (As=-31.45 as compared to As=-44.03) than that of obtained by using Hanning window for low 'β' values while in case of Kaiser window smaller stop-band attenuation (As=-48.37 as compared to As=-62.84) is achieved for 'β' greater than or equal to 6.

VI. REFERENCES

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