COMPLEMENT AND CONNECTIVITY IN A FUZZY GRAPH

P.Priyanka[1] , S.Divya[2] , T.Kalavani[3]
Assistant professor[1] , Ug student[2] , Ug student[3]
Department of mathematics, Adhiyaman arts & science college for women , Uthangarai

ABSTRACT : Connectivity has important role in the area of applications of fuzzy graphs such as fuzzy neural networks and clustering. In this paper criterion for connectivity of a fuzzy graphs and its complement is analysed. The structure of the complement of a fuzzy cycle is also discussed. We show that automorphism groups of a fuzzy graph and its complement are identifical and a relative study of complement and some other operation on fuzzy graphs such as union, join and composition and has been made.

Keyword: Fuzzy relation, fuzzy subset, complement of fuzzy graphs, fuzzy cycle, connectivity in fuzzy graphs, m – strong arcs.

Introduction:

The notion of fuzzy grapy was introduced by Rosenfeld in the year 1975 fuzzy analogues of many Structures in crisp graph theory, like bridges, cut nodes, connectedness, tree and cycles etc were developed after that. A characterized fuzzy tree using its unique maximum spanning tree. A sufficient condition for a node to be a fuzzy cut node is also established. Center problems in fuzzy graphs, blocks in fuzzy graphs and properties of self complementary fuzzy graphs were also studied. We obtained a characterization for blocks in fuzzy graphs, the concepts of strong arcs, fuzzy end nodes and geodesics in fuzzy graphs. We have defined the concepts of strong arcs and strong paths. As far as the applications are concerned of strong arcs and strong paths. As far as the applications are concerned (information networks, electric circuits, etc.), the reduction of flow between pairs of nodes is more relevant and may frequently occur than the total disruption of the flow or the disconnection of the entire network. In this paper we put forward the conditions under which a fuzzy graph and its complement will be connected.

Preliminaries:

Connectivity in fuzzy graph:

The following basic definitions are taken from. A fuzzy graph is a pair G: (σ ,μ), where σ is a fuzzy subset of a set V and μ is a fuzzy relation onσ, i.e.) μ(X, Y) ≤ σ (X) ∧ σ(Y), ∀ X,Y∈V. We assume that V is finite and non empty, μ is reflexive and symmetric. In all the examples σ is chosen suitably.

Also we denote the underlying crisp graph byG^*: (σ^*,μ^*), where σ^* ={u∈V: σ (U) > 0} and μ^* ={(u, v)∈VXV:μ (u, v) > 0}. H= (τ,γ) is called a partial fuzzy sub graph of G if $\tau \leq \sigma$ and $\gamma \leq \mu$. We call H= (τ,γ) a spanning fuzzy sub graph of G =(σ ,μ) if $= \sigma$.

A path P of length n is a sequence of distinct nodes u_0, u_1, u_2 … u_n such that $\mu(u_{i-1}, u_i) > 0$ and degree of membership of a weakest arc is defined as its strength. If $u_0 = u_n$ and n≥3, then P is called a cycle and it is a fuzzy cycle if there is more than one weak arc.

The strength of connectedness between two nodes X, Y is defined as the maximum of strength of all paths between X and Y and is denoted by $CONN_G(x, y)$.

An arc (X, Y) is called a fuzzy bridge in G if the removal of (X, Y) reduces the strength of connectedness between some pair of nodes in G. A connected fuzzy graph is called a fuzzy tree if it contains a spanning sub graph F which is a tree such that, for all arcs (x, Y) not in F, μ (x, y) <$CONN_F(x, y)$.

An arc (u, v) of G is called m – strong if μ (u, v) = ∧ [σ (x) σ(y)]. Suppose G: (σ ,μ) be a fuzzy graph.

The complement of G [4] is denoted as G^c: (σ^c,μ^c),where σ^c = a and μ^c(x, y) = ∧ [σ (x) σ(y)] - μ (x, y) If (u, v) is m – strong, then μ^c(u, v) = 0

I.COMPLEMENT OF A FUZZY GRAPH :

Mordeson has defined the complement of a fuzzy graph G: (σ, μ) as a fuzzy graph G^C:(σ^C, μ^C) where σ^c=σ and μ^c(u ,v)=0 if μ(u ,v)> 0and μ(u ,v)=σ(u) ∧ σ(v) otherwise .It follows from this definition that G^C is a fuzzy graph even if G is not and that $(G^C)^C$=G if and only if G is a strong fuzzy graph. In Fig.1, $(G^C)^C \neq$ G.

Also, automorphism group of G and G^C are not identical; for, consider the fuzzy graph G and G^C in Fig.2. The automorphism group of G consists of two maps h_1 and h_2 where h_1 is the identity map and h_2 is given by the permutation (v_1) (v_2 v_4) (v_3). But the automorphism group of G^C consists of four maps h_1, h_2, h_3 and h_4 where h_1 and h_2 are automorphism of G and h_3 and h_4 are given by h_3 =(v_1 v_3) (v_2) (v_4) and h_4 =(v_1 v_3) (v_2 v_4).

Thes observation motivate us to modify the notion of a fuzzy group.

Definition:

The complement of a fuzzy graph G:(σ, μ) is a fuzzy graph $\bar{G}$: $(\bar{\sigma},\bar{\mu})$ where $\bar{\sigma}\equiv$ σ and $\bar{\mu}$(u, v)=σ(u)∧ σ(v)-μ (u, v) for all u, v in v.

Theorem:

Let G: (σ, μ) be a self complementary fuzzy graph. Then

$$\sum_{u\neq v} \mu\,(u,v) = \frac{1}{2}\sum_{u\neq v}(\sigma(u)\wedge\sigma(v)).$$

Proof:

Let G: (σ, μ) be a self complementary fuzzy graph. Then there exists an isomorphism h:V→ V such that

$\bar{\sigma}$h(u)=σ(u) ∀ u∈ V

and $\bar{\mu}$(h(u),h(v)=μ (u, v)∀ $u,v \in V$.

Now by definition of $\overline{G}$, we have,

$\mu(\overline{h}(u), h(v)) = \sigma(\overline{h}(u) \wedge \sigma(\overline{h}(v)) - \mu(h(u), h(v))$

i.e., $\mu(u, v) = \sigma(u) \wedge \sigma(v) - \mu(h(u), h(v))$

i.e., $\sum_{u \neq v} \mu(u, v) + \sum_{u \neq v} \mu(h(u), h(v)) = \sum_{u \neq v} (\sigma(u) \wedge \sigma(v))$

i.e., $2\sum_{u \neq v} \mu(u, v) = \sum_{u \neq v} (\sigma(u) \wedge \sigma(v))$

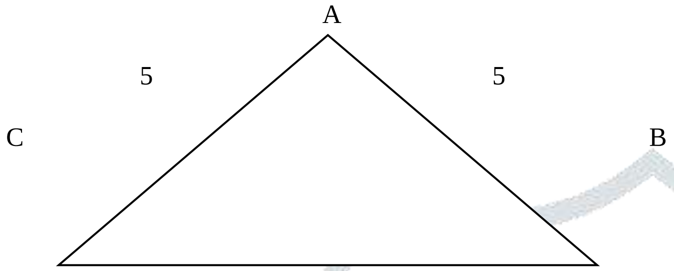
i.e., $\sum_{u \neq v} \mu(u, v) = \frac{1}{2} \sum_{u \neq v} (\sigma(u) \wedge \sigma(v))$.

Hence the theorem.

II.Connectivity inG^c:

It is a fact that the class of fuzzy graphs arc so wide and needs great effort to understand and analyze the structural properties of fuzzy graphs. It was noticed that there are fuzzy graphs which are connected but their complements become disconnected. There may be cases when a fuzzy graph will be a fuzzy tree or fuzzy cycle and this structural property may not be satisfied by its complement. In this paper we propose a criterion by which a fuzzy graph and its complement will be connected simultaneously. We also discuss a result regarding the fuzzy cycle and its complement.

Examples : A fuzzy graph on 3 vertices and its complement.



Proposition:

Let $G = (\sigma, \mu)$ be a connected fuzzy graph with no m – strong arcs then G^c is connected.

PROOF:

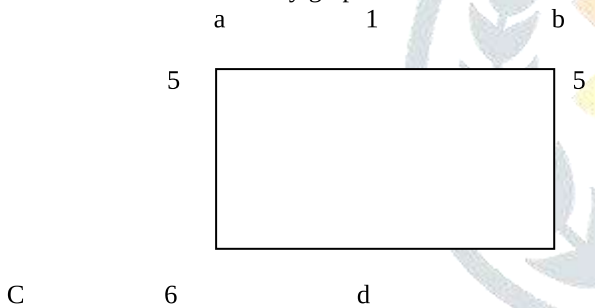
The fuzzy graph G is connected and contain no m – strong arcs. Suppose u, v be two arbitrary nodes of G^c. Then they are also nodes of G. since G is connected there exist a path between u and v in G. Let this path be P.

Then $P = (u_0, u_1)(u_1, u_2) \ldots\ldots (u_{n-1}, u_n)$ where $\mu(u_{i-1}, u_i) > 0 \ \forall$ i.

Since G contain no m – strong arcs, $\mu^c(u_{i-1}, u_i) > 0 \ \forall$ i.

Hence P will be a (u, v) path in G^c also. Therefore G^c is connected.

REMARK: There are fuzzy graph which contain m – strong arcs such that G and G^c are connected.



G

Figure: (a, b) is an m – strong arc in G and still $G^c$is connected.

THEOREM:

Let $G = (\sigma, \mu)$ be a fuzzy graph. G and G^c are connected if and only if G contains at least one connected spanning fuzzy sub graph with no m – strong arcs.

PROOF:

Suppose that G contains a spanning sub graph H that is connected, having no m – strong arcs. Since H contain no m – strong arcs and is connected using proposition – 1. H^c will be a connected spanning fuzzy sub graph of G^c and thus G^c is also connected.

Conversely assume that G and G^c are connected. We have to find a connected spanning sub graph of G that contains no m – strong arcs. Let H be an arbitrary connected spanning sub graph of G. if H contain no m – strong arcs then H is the required sub graph. Suppose H contain one m – strong arc say (u, v). Then arc (u, v) will not be present inG^c. Since G^c is connected there will exist a u-v path in G^c. Let this path be P_1. Let $P_1 = (u_1, u_2)(u_2, u_3) \ldots\ldots (u_{n-1}, u_n)$ where $u_1 = u$ and $u_n = v$

If all the arcs of P_1 are present in G then H – (u, v) together with p_1 will be the required spanning sub graph. If not, there exist at least one arc say (u_1, v_1) in p_1 which is not in G. since G is connected we can replace (u_1, v_1) by another $u_1 - v_1$ path in G. Let this path be p_2. If p_2 contain no m – strong arcs then

H – (u. v) - (u_1, v_1) together with p_1 and p_2 will be the required spanning sub graph. If p_2 contain an m – strong arc than this arc will not be present in G^c. Then replace this arc by a path connecting the corresponding vertices in G^c and proceed as above and since G contain only finite number of arcs finally we will get a spanning sub graph of that contain no m – strong arcs.

If more than one m – strong arc is present in H, then the above procedure can be repeated for all other m – strong arcs of H to get the required spanning sub graph of G.

III.Complement of fuzzy cycles:

Next we examine the case of the complement of fuzzy cycles. By choosing the membership values of arcs and nodes suitably we can construct the complement of fuzzy cycles on 3, 4, 5 vertices as fuzzy cycles.

Examples: $n = 3$, G and G^c are both fuzzy cycles.

Theorem:

Let $G = (\sigma, \mu)$ be a fuzzy graph such that G^* is a cycle with more than 5 vertices. Then $(G^*)^c$ cannot be a cycle.

PROOF:

Given G^* is a cycle having n nodes where $n \geq 6$. Then G^* will have exactly n arcs. Since all the nodes of G are also present in G^c number of nodes of G^c is n. Let the nodes of G and G^c be $v_1, v_2, v_3, \ldots\ldots v_n$.

Then G^c must contain at least the following edges.

$(v_1, v_3),(v_1,v_4)\ldots..(v_1,v_n)$; $(v_2, v_4),(v_2,v_5)\ldots..(v_2,v_n)$; $(v_3, v_5),(v_3,v_6)\ldots..(v_3,v_n)$

Since $n \geq 6$ the total number of edges in G^c will be greater than n. Thus G^c will not be a cycle.

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