PRODUCT INTEGER CORDIAL LABELING OF SOME WELL KNOWN GRAPHS

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Abstract

In this paper we introduce the concept of product integer cordial labeling. Let G(V, E) be a simple connected graph with p vertices. Let the injective mapping f: V → {1, 2, . . ., p} induce an edge labeling f* on E such that f*(uv) = 1 or 0 according as f(u)f(v) even or odd respectively. Let $e_f(i)$ = number of edges labeled with i, where i = 0 or 1. We call f a product integer cordial if $|e_f(0) - e_f(1)| \leq 1$. A graph G is called product integer cordial if it admits a product integer cordial labeling. We also investigate the product integer cordial labeling of some graphs.

Keywords:- cordial, Integer cordial, product integer cordial labeling.

1. Introduction:

We consider finite, undirected and simple graphs G with vertex set V(G) and Edge set E(G).

The concept of cordial labeling originated from I. Cahit **[1, 2]** in 1987 as a weaker version of graceful and harmonious graphs and was based on {0, 1} binary labeling of vertices. For a deeper insight on cordial labeling one may refer to **[1 - 13]**.The concept of I-cordial labeling was introduced by T. Nicholas and P. Maya **[14]**.

Let f be an injective map from V to $[\frac{-p}{2}, \frac{p}{2}]^*$ or $[\frac{-p}{2}, \frac{p}{2}]$ as p is even or odd respectively, such that $f(u) + f(v) \neq 0$. Let f induce an edge labeling f*: E→ {0, 1} where f*(uv) = 1 if f(u) + f(v) > 0 and f*(uv) = 0 otherwise. We call f an I-cordial labeling of a graph G(V, E) if $|e_f(0) - e_f(1)| \leq 1$, where $e_f(i)$ = number of edges labeled with i, where i = 0 or 1. The graph G is called I-cordial if it admits an I-cordial labeling. **[14]**

In this paper, we introduce the concept of product integer cordial labeling.

Let G = (V, E) be a simple connected graph with p vertices. Let an injective mapping f: V → {1, 2, . . ., p} induce an edge labeling f* on E such that f*(uv) = 1 or 0 according as f(u)f(v) even or odd respectively. Let $e_f(i)$ = number of edges labeled with i, where i = 0 or 1. We call f a product integer cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph G is called product integer cordial if it admits a product integer cordial labeling.

In this paper we introduce the concept of a new variant of cordial labeling, namely, product integer cordial labeling and investigate it for some graphs.

2. Main results:

Definition 2.1:

Let G = (V, E) be a simple connected graph with p vertices. Let the injective mapping f: V → {1, 2, …, p} induce an edge labeling f* on E such that f*(uv) = 1 or 0 according as f(u) f(v) is even or odd respectively. Let $e_f(i)$ = number of edges labeled with i, where i = 0 or 1. We call f a product integer cordial if $|e_f(0) - e_f(1)| \leq 1$. A graph G is called product integer cordial if it admits a product integer cordial labeling.

Theorem 2.2: The cycle C_n is product integer cordial if and only if n is odd.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of C_n.

Case (i) n is even.

Define f: V → {1,2, …,n} as $f(v_i) = \begin{cases} 2i-1 \text{ for } i = 1, 2, \ldots, \frac{n}{2}, \\ 2i-n \text{ for } i = \frac{n}{2}+1, \ldots, n. \end{cases}$

Hence $f^*(v_iv_{i+1}) = 0$; for $i = 1, 2\ldots, (\frac{n}{2} - 1)$

$f^*(v_iv_{i+1}) = 1$; for $i = \frac{n}{2}, \ldots, (n-1)$

$f^*(v_nv_1) = 1$.

Hence $\frac{n}{2} + 1$ edges receive label 1 and $\frac{n}{2} - 1$ edges receive label 0, which imply

$|e_f(1) - e_f(0)| = 2$.

In fact any other labeling f would yield $|e_f(1) - e_f(0)| \geq 2$.

Hence C_n is not product integer cordial when n is even.

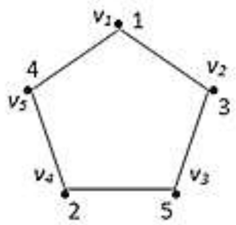
Case (ii) n is odd. $f(v_i) = \begin{cases} 2i-1, \text{ for } i = 1, 2, \ldots, \frac{n+1}{2}, \\ 2i-(n+1), \text{ for } i = \frac{n+3}{2}, \ldots, n. \end{cases}$

Then $f^*(v_iv_{i+1}) = \begin{cases} 0, \text{ for } i = 1, 2, \ldots, \frac{n-1}{2}, \\ 1, \text{ for } i = \frac{n+1}{2}, \ldots, n-1. \end{cases}$

$f^*(v_nv_1) = 1$

Hence $\frac{n-1}{2}$ edges receive label 0 and $\frac{n+1}{2}$ edges receive label 1, which imply $|e_f(1) - e_f(0)| = 1$.

Hence C_n is product integer cordial graph iff n is odd.



Product integer cordial labeling of C_5

Theorem 2.3: The complete bipartite graph $K_{m,n}$, n > m, is product integer cordial iff m = 1.

Case(i) n is even.

Let V = U ∪ W where $U = \{u_1\}$, $W = \{w_1, \ldots, w_n\}$; n is even.

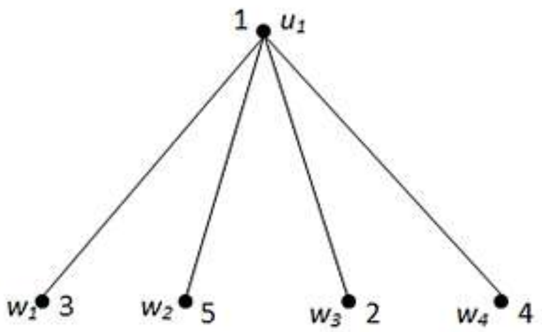
Then p = n + 1, q = n

Let f: V→{1, 2 ,…, n+1} be the function defined as follows:

$f(u_1) = 1$;

$$f(w_i) = \begin{cases} 2i + 1\ ;\ i = 1, 2, \dots, \frac{n}{2}, \\ 2i - n\ ;\ i = \frac{n}{2} + 1, \dots, n. \end{cases}$$

Hence $\frac{n}{2}$ edges receive even labels and $\frac{n}{2}$ edges odd labels.
Hence $|e_f(1) - e_f(0)| = 0$.
Hence $K_{m,n}$ is product integer cordial when m = 1 and n is even.



Product integer cordial labeling of $K_{1,4}$

Case (ii) n is odd.

Define f: V →{1, 2, …, n + 1} as follows:

$$f(w_i) = \begin{cases} 2i + 1 \text{ for } i = 1, 2, \dots, \frac{(n-1)}{2}, \\ 2i - (n - 1) \text{ for } i = \frac{(n+1)}{2}, \dots, n. \end{cases}$$

and $f(u_1) = 1$.

Hence $f^*(u_1 w_i) = 0$, for i = 1, 2, …, $\frac{(n-1)}{2}$

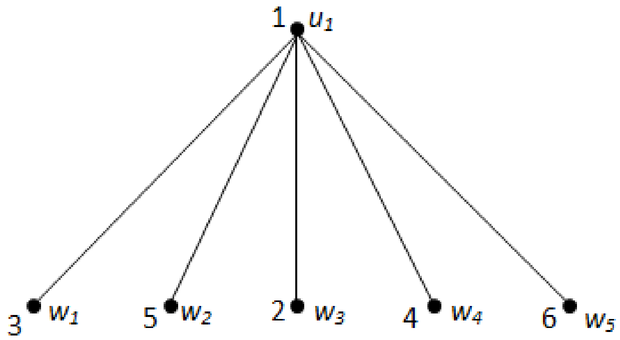
$f^*(u_1 w_i) = 1$, for i = $\frac{(n+1)}{2}$, …, n.

That is, $\frac{n-1}{2}$ edges receive odd labels and $\frac{n+1}{2}$ edges receive even labels.

Hence $|e_f(1) - e_f(0)| \leq 1$.

Hence $K_{m,n}$ is product integer cordial where n is odd and m = 1.

From both cases $K_{m,n}$ is product integer cordial for any n and m = 1.



Product integer cordial labeling of $K_{1,5}$

Case (iii): $m \geq 2$; $n > m$.

Let V= A ∪ B, where |A| = m, |B| = n.

If there exists a product integer cordial labeling f then for the least possible assignment, f must assign odd labels to the m vertices of the partite set A. In this case f would induce $m(\frac{n-m}{2})$ odd labels for the edges.

Therefore $(mn - \frac{m(n-m)}{2})$ edges receive even labels.

That is $e_f(1) = \frac{m(n+m)}{2}$; $e_f(0) = \frac{m(n-m)}{2}$

Therefore the difference between even and odd labels = $|e_f(0) - e_f(1)|$

$$= \left| \frac{m(n+m)}{2} - \frac{m(n-m)}{2} \right|$$

$$= m^2 \geq 1$$

which is a contradiction.

Hence $K_{m,n}$ is product integer cordial graph iff m = 1.

Corollary 2.4: Star graph is product integer cordial for all n.

Theorem 2.5: Path P_n is product integer cordial for any n.

Case (i) n is even.

Let G be a graph with n vertices, n is even, hence q = n – 1.

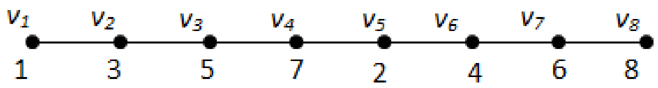
Define f: V (G) → {1, 2, 3, …, n} by

$$f(v_i) = \begin{cases} 2i - 1; \ \text{for } i = 1, 2, \ldots, \frac{n}{2}, \\ 2i - n; \ \text{for } i = \frac{n}{2} + 1, \ldots, n. \end{cases}$$

Then $f^*(v_iv_{i+1}) = 0$; for i = 1, 2, …, $\frac{n}{2} - 1$ and $f^*(v_iv_{i+1}) = 1$; for i = $\frac{n}{2}$, …, n – 1.

Hence $\frac{n}{2} - 1$ edges receive odd labels and $\frac{n}{2}$ edges receive even labels.

This implies $|e_f(1) - e_f(0)| \leq 1$. Hence P_n is product integer cordial when n is even.



Product integer cordial labeling of P_8

Case (ii): n is odd.

Define f: V (G) → {1, 2, …, n} by

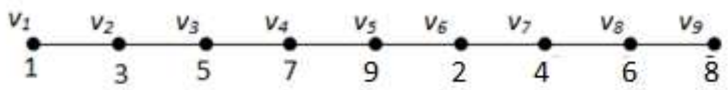
$$f(v_i) = \begin{cases} 2i - 1;\ i = 1, 2, \ldots, \frac{(n+1)}{2}, \\ 2i - (n+1);\ i = \frac{(n+3)}{2}, \ldots, n. \end{cases}$$

Then the induced function f* is obtained as

$$f^*(v_i v_{i+1}) = \begin{cases} 0;\ i = 1, 2, \ldots, \frac{n-1}{2}, \\ 1;\ i = \frac{n+1}{2}, \ldots, n-1. \end{cases}$$

Hence $\frac{n-1}{2}$ edges receive even labels and $\frac{n-1}{2}$ edges receive odd labels.

Hence P_n is product integer cordial when n is odd.



Product integer cordial labeling of P_9

Theorem 2.6: Friendship graph $C_3^{(t)}$ is product integer cordial for all t.

Proof:

Case (i) :t is odd.

The graph $C_3^{(t)}$has 2t +1 vertices and 3t edges. Let w be the apex vertex.

Define f: V→ {1, 2, …, 2t+1} by

$$f(v_i) = \begin{cases} 2i - 1;\ i = 1, 2, \ldots, t, \\ 2i - 2t;\ i = t+1, \ldots, 2t. \end{cases}$$

f(w) = 1.

Then $f^*(wv_i) = 0$; if i = 1, 2, …, t

$f^*(wv_i) = 1$; if i = t+1,… 2t and f(w) = 1.

Also $f^*(v_i v_{i+1}) = \begin{cases} 0;\ i = 1, 3, \ldots, t-2, \\ 1;\ i = t, t+2, \ldots, (2t-1). \end{cases}$

Then $\frac{3t-1}{2}$ edges receive odd labels and $\frac{3t+1}{2}$edges receive even labels.

Hence $|e_f(0) - e_f(1)| \leq 1$.

Hence $C_3^{(t)}$is product integer cordial graph when t is odd.

Case (ii): t is even.

Let f (w) = 1

Define f: V → {1, 2, …, 2t} by

$$f(v_i) = \begin{cases} 2i + 1\,;\ i = 1, 2, \ldots, t, \\ 2i - 2t\,;\ i = t+1, \ldots, 2t. \end{cases}$$

Then the induced function f* as follows:

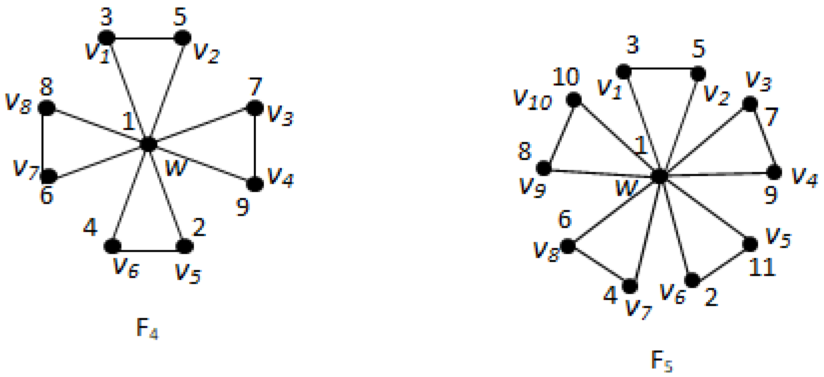
$$f^*(v_iv_{i+1}) = \begin{cases} 0;\ i = 1, 2, \ldots, t, \\ 1;\ i = t+1, \ldots, 2t. \end{cases}$$

and $$f^*(v_iv_{i+1}) = \begin{cases} 0;\ i = 1, 3, \ldots, t-1, \\ 1;\ i = t+1, t+3, \ldots, 2t-1. \end{cases}$$

Thus, $e_f(0) = \frac{3t}{2}$ and $e_f(1) = \frac{3t-2}{2}$.

Hence $|e_f(0) - e_f(1)| = 1$.

Hence $C_3^{(t)}$ is product integer cordial graph for all t.



Product integer cordial labeling of F_4 and F_5

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