

DOUBLE ACCEPTANCE SAMPLING PLANS FOR TIME TRUNCATED LIFETESTS

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Abstract: In this article, double acceptance sampling plans for truncated life tests are developed when the lifetimes of test items follows Transmuted Weibull Distribution and Transmuted Size Biased Exponential Distribution. The minimum sample sizes are determined when the consumer's risk and the test termination time are specified. The operating characteristic values are calculated for different consumer's confidence levels fixing the producer's risk. Probability of acceptance and producer's risk are also discussed with the help of tables and examples. A comparative study of double acceptance sampling plans for transmuted weibull distribution with transmuted size biased exponential distribution is also performed.

Keywords: Transmuted Weibull Distribution, Transmuted Size Biased Exponential Distribution, Double Acceptance Sampling Plan (DASP), Consumer's risk, Operating Characteristics (OC), Producer's risk, Truncated Life Test.

I. INTRODUCTION

Quality is now not only an option or aim of companies, but a necessity for business in a global market. Thus, the quality has become a differentiation tool between competitive enterprises. Two important tools for ensuring quality are the statistical quality control and the acceptance sampling.

The acceptance sampling is an inspecting procedure applied in statistical quality control. Acceptance sampling is a part of operations management and services quality maintenance. It is important for industrial, but also for business purposes helping the decision – making process for the purpose of quality management. Producers are very careful about the quality of their products so that they do not face any difficulty in the acceptance when the consumer comes to buy them. Acceptance sampling is most likely to be useful in situations when testing is destructive, or when the cost of 100% inspection is extremely high, or when 100% inspection is not technologically feasible or would require so much calendar time that the production schedule would be seriously impacted. Sampling plans are hypothesis tests regarding the product that has been submitted for an appraisal and subsequent acceptance or rejection. The decision is based on the pre-specified criteria and the amount of defects or defective units found in the sample. Accepting or rejecting a lot is analogous to not rejecting or rejecting the null hypothesis in a hypothesis test.

A single acceptance sampling plan (SASP) is a specified plan that establishes the minimum sample size to be used for testing. In most Acceptance Sampling Plans for a truncated life test, the major issue is to determine the sample size from a lot under consideration. It is implicitly assumed in the usual sampling plan that only a single item is put in a tester. On the basis of information obtained from this first sample, we accept or reject the lot. If a good lot is rejected on the basis of this information, its probability is called the type-I error (producer's risk) and it is denoted by α . The probability of accepting the bad lot is known as the type-II error (consumer's risk) and it is denoted as β . So, an acceptance sampling plan consists of the number of units on test (n) and the acceptance number (c) such that if there are at most c failures out of n , the lot is accepted.

More recently, Aslam (2007) proposed double acceptance sampling plans (DASP_s) based on truncated life tests when the lifetime of an item follows the Rayleigh distribution and Srinivasa Rao, Ghitany, and Kantam (2009) developed DASP_s based on truncated life tests following a Marshall-Olkin Extended Lomax distribution.

SASP_s based on truncated life tests for a variety of distributions were discussed by Epstein (1954), Sobel and Tischendorf (1959), Goode and Kao (1961), Gupta and Groll (1961), Gupta (1962), Kantam and Rosaiah (1998), Kantam, Rosaiah, and Srinivasa Rao (2011), Baklizi (2003), Baklizi and El Masri (2004), Rosaiah and Kantam (2005), Rosaiah, Kantam and Santosh Kumar (2007), Tsai and Wu (2006), Balakrishnan, Leiva, and Lopez (2007), Aslam and Kantam (2003) and Srinivasa Rao et al. (2009).

In acceptance sampling based on truncated life tests, we assume the following: (1) the units are destructible or are degraded after the life test, and (2) there are several distributions which are used to model, the product life reasonably well. Thus, considering similar risk and operating conditions and the assumptions (1) and (2), the consumer will benefit with smaller number of units required to test. For this reason, we could use a distribution that gives the smallest sample size.

In this paper we propose a plan to find the probability of acceptance for the double acceptance sampling assuming the experiment is truncated at pre-assigned time and lifetime follows Transmuted Weibull Distribution and Transmuted Size Biased Exponential Distribution. The probability density function (pdf) and the cumulative distribution function (cdf) of the Transmuted Weibull Distribution are given by

$$f(t; \sigma) = \frac{\eta}{\sigma} \left(\frac{t}{\sigma}\right)^{\eta-1} e^{-\left(\frac{t}{\sigma}\right)^\eta} \left[1 - \lambda + 2\lambda e^{-\left(\frac{t}{\sigma}\right)^\eta}\right]; 0 < t < \infty, \eta > 0, \sigma > 0, |\lambda| \leq 1 \text{ ----- (1)}$$

$$F(t; \sigma) = \left[1 - e^{-\left(\frac{t}{\sigma}\right)^\eta}\right] \left[1 + \lambda e^{-\left(\frac{t}{\sigma}\right)^\eta}\right]; 0 < t < \infty, \eta > 0, \sigma > 0, |\lambda| \leq 1 \text{ ----- (2)}$$

where σ is the scale parameter and η, λ are the shape parameters.

The probability density function (pdf) and the cumulative distribution function (cdf) of the Transmuted Size Biased Exponential Distribution are given by

$$f(t; \sigma) = \sigma^2 t e^{-\sigma t} [1 - \lambda + 2\lambda(1 + \sigma t)e^{-\sigma t}]; 0 < t < \infty, \sigma > 0, |\lambda| \leq 1 \text{ ----- (3)}$$

$$F(t; \sigma) = (1 - e^{-\sigma t}(1 + \sigma t))(1 + \lambda(1 + \sigma t)e^{-\sigma t}); 0 < t < \infty, \sigma > 0, |\lambda| \leq 1 \text{ ----- (4)}$$

Where σ is the scale parameter and λ is the shape parameter. Therefore it is assumed that the distribution function depends on time only through the ratio of t/σ_0 . The suggested double acceptance sampling plan for life tests is presented in section II. Description of tables with examples are given in section III. The comparative study is presented in section IV. And finally the conclusion is given in section V.

II. The Double Acceptance Sampling Plan for Life Tests

It is known that the double acceptance sampling plan (DASP) is more efficient than the single sampling plan in terms of the sample size required. Further, a DASP is expected to reduce the producer's risk when specifying the consumer's risk, because it provides another opportunity for acceptance of the product. In the DASP a sample with n_1 items is taken from the lot which is called the first sample. This first sample is put on test. Let c_1 and c_2 be the acceptance numbers for the first and the second sample, respectively. The single sampling plan is a special case of DASP when $c_1 = c_2$. We terminate the experiment if no more than c_1 failures occur during the experiment time t , i.e. we reject or accept the lot on the basis of sample 1 if more than c_2 failures occur or the time of experiment has ended (whichever occurs earlier). If $(c_1 + 1)$ failures occur in the first sample then all possibilities for the second sample are given as:

First Sample	Second Sample
$(c_1 + 1)$ failures occur in sample 1	$< (c_2 - 1)$ failures in sample 2 are required to accept
$(c_1 + 2)$ failures occur in sample 1	$< (c_2 - 2)$ failures must occur in sample 2 to accept

and so on.

Let σ represent the true average life of a product and σ_0 denote the specified life of an item, under the assumption that the lifetime of an item follows the Transmuted Weibull Distribution and Transmuted Size Biased Exponential Distribution. A product is considered as good and accepted for consumer's use if the sample information supports the hypothesis $H_0: \sigma \geq \sigma_0$. Otherwise the lot of the product is rejected. In Acceptance Sampling schemes, this hypothesis is tested based on the number of failures from a sample in a pre-fixed time. If the number of failures exceeds the action limit c we reject the lot. We accept the lot if there is enough evidence that $\sigma \geq \sigma_0$ at a certain level of consumer's risk. Otherwise we reject the lot. In order to determine the parameters of the proposed sampling plan we use the consumer's risk. Often the consumer's risk is expressed by the consumer's confidence level. If the confidence level is P^* , then the consumer's risk is $\beta = 1 - P^*$. In this study we fix the consumer's risk such that

$$\Pr(\text{number of failures} \leq c | p) = \sum_{i=0}^c \binom{n}{i} p^i (1 - p)^{n-i} \leq 1 - P^* \text{ ----- (5)}$$

where p is the probability that an item fails before termination time.

Consider a life testing experiment having n_1 items in the first sample put on test, with an acceptance number of $c_1 = 0$ in this first sample, and n_2 items in the second sample put on test and we accept the lots if no more than two failures occur in the second sample, i.e. $c_2 = 2$. If no failure occurs in the first sample of n_1 items put on test, we accept the lot. If the true but unknown lifetime of the product deviates from the specified lifetime of the product it should result in a considerable change in the probability of acceptance of the lot based on the sampling plan. Hence the probability of acceptance can be regarded as a function of the deviation of the specified average from the true average. This function is called the operating characteristic (OC) function of the sampling plan. Values of the probability of acceptance for the first sample using the Transmuted Weibull Distribution and

Transmuted Size Biased Exponential Distribution are given in Table 1.1 and Table 1.2. Denote the probability of acceptances as $L(P_1)$ and $L(P_2)$ for sampling plans $(n_1, c_1, t/\sigma_0)$ and $(n_2, c_2, t/\sigma_0)$, respectively, then

$$L(P_1) = \sum_{i=0}^{c_1-1} \binom{n_1}{i} P^i (1 - P)^{n_1-i} \text{ -----(6)}$$

$$L(P_2) = \sum_{i=0}^{c_2-2} \binom{n_2}{i} P^i (1 - P)^{n_2-i} \text{ -----(7)}$$

where, $P = F_T(t; \sigma) = F_T((t/\sigma_0).(\sigma_0/\sigma))$ is given in (2) and (4)

The Probability of acceptance for a DASP can be obtained by using (6) and (7) and is

$$L(P) = \text{Pr}(\text{no failure occurs in sample 1})$$

$$+ \text{Pr}(1 \text{ failure occurs in sample 1 and } 0 \text{ or } 1 \text{ failure occurs in sample 2)}$$

$$+ \text{Pr}(2 \text{ failures occur in sample 1 and } 0 \text{ failure occurs in sample 2}).$$

$$L(P) = \binom{n_1}{0} P^0 (1 - P)^{n_1-0} + \binom{n_1}{1} P^1 (1 - P)^{n_1-1} \left[\sum_{i=0}^1 \binom{n_2}{i} P^i (1 - P)^{n_2-i} \right] + \binom{n_1}{2} P^2 (1 - P)^{n_1-2} \left[\binom{n_2}{0} P^0 (1 - P)^{n_2-0} \right]$$

Values of the probability of acceptance for a DASP are determined at $P^* = 0.75, 0.90, 0.95, 0.99$ and $t/\sigma_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.142, 3.927, 4.712$ and are given in Table 2.1 and 2.2. It is important to note that in the first sample and in the second sample, P is a function of the cdf of the transmuted weibull distribution and transmuted size biased exponential distribution respectively. These choices are consistent with Gupta and Groll (1961), Gupta (1962), Kantam et al. (2001), Baklizi and El Masri (2004), Balakrishnan et al. (2007).

Table 1.1: Operating characteristic values of the sampling plan for transmuted weibull distribution $(n_1, c_1, t/\sigma_0)$ when $c_1=0$

P^*	t/σ_0	n_1	σ/σ_0					
			2	4	6	8	10	12
0.75	0.628	5	0.6790	0.9077	0.9579	0.9761	0.9846	0.9893
	0.912	3	0.6127	0.8847	0.9470	0.9698	0.9806	0.9865
	1.257	2	0.5377	0.8563	0.9334	0.9620	0.9755	0.9829
	1.571	2	0.3794	0.7848	0.8979	0.9412	0.9620	0.9734
	2.356	2	0.1131	0.5799	0.7849	0.8726	0.9165	0.9412
	3.141	2	0.0208	0.3797	0.6502	0.7850	0.8565	0.8980
	3.927	2	0.0023	0.2201	0.5103	0.6849	0.7849	0.8452
	4.712	2	0.0002	0.1131	0.3796	0.5799	0.7056	0.7849
0.90	0.628	8	0.5383	0.8565	0.9335	0.9620	0.9755	0.9829
	0.912	4	0.5204	0.8493	0.9300	0.9600	0.9472	0.9820
	1.257	2	0.5377	0.8563	0.9334	0.9620	0.9755	0.9829
	1.571	2	0.3794	0.7848	0.8979	0.9412	0.9620	0.9734
	2.356	2	0.1131	0.5799	0.7849	0.8726	0.9165	0.9412
	3.141	2	0.0208	0.3797	0.6502	0.7850	0.8565	0.8980
	3.927	2	0.0023	0.2201	0.5103	0.6849	0.7849	0.8452
	4.712	2	0.0002	0.1131	0.3796	0.5799	0.7056	0.7849
0.95	0.628	10	0.4611	0.8240	0.9176	0.9528	0.9695	0.9787
	0.912	5	0.4420	0.8154	0.9133	0.9502	0.9679	0.9776
	1.257	3	0.3943	0.7924	0.9018	0.9435	0.9635	0.9745
	1.571	2	0.3794	0.7848	0.8979	0.9412	0.9620	0.9734
	2.356	2	0.1131	0.5799	0.7849	0.8726	0.9165	0.9412
	3.141	2	0.0208	0.3797	0.6502	0.7850	0.8565	0.8980
	3.927	2	0.0023	0.2201	0.5103	0.6849	0.7849	0.8452
	4.712	2	0.0002	0.1131	0.3796	0.5799	0.7056	0.7849

0.99	0.628	15	0.3131	0.7480	0.8789	0.9299	0.9546	0.9682
	0.912	7	0.3189	0.7514	0.8807	0.9310	0.9553	0.9687
	1.257	4	0.2892	0.7333	0.8712	0.9254	0.9516	0.9661
	1.571	3	0.2337	0.6953	0.8509	0.9132	0.9435	0.9604
	2.356	2	0.1131	0.5799	0.7849	0.8726	0.9165	0.9412
	3.141	2	0.0208	0.3797	0.6502	0.7850	0.8565	0.8980
	3.927	2	0.0023	0.2201	0.5103	0.6849	0.7849	0.8452
	4.712	2	0.0002	0.1131	0.3796	0.5799	0.7056	0.7849

Table 1.2: Operating characteristic values of the sampling plan for transmuted size biased exponential distribution ($n_1, c_1, t/\sigma_0$) when $c_1=0$

P^*	t/σ_0	n_1	σ/σ_0					
			2	4	6	8	10	12
0.75	0.628	4	0.6119	0.8724	0.9389	0.9645	0.9768	0.9837
	0.912	2	0.6214	0.8720	0.9378	0.9635	0.9761	0.9832
	1.257	2	0.4389	0.7820	0.8895	0.9339	0.9562	0.9689
	1.571	2	0.3038	0.6934	0.8381	0.9014	0.9340	0.9528
	2.356	2	0.1034	0.4780	0.6935	0.8035	0.8646	0.9014
	3.141	2	0.0300	0.3040	0.5467	0.6935	0.7821	0.8382
	3.927	2	0.0077	0.1816	0.4142	0.5824	0.6935	0.7675
	4.712	2	0.0018	0.1034	0.3039	0.4780	0.6044	0.6935
0.90	0.628	6	0.4786	0.8149	0.9098	0.9472	0.9655	0.9757
	0.912	2	0.6214	0.8720	0.9378	0.9635	0.9761	0.9832
	1.257	2	0.4389	0.7820	0.8895	0.9339	0.9562	0.9689
	1.571	2	0.3038	0.6934	0.8381	0.9014	0.9340	0.9528
	2.356	2	0.1034	0.4780	0.6935	0.8035	0.8646	0.9014
	3.141	2	0.0300	0.3040	0.5467	0.6935	0.7821	0.8382
	3.927	2	0.0077	0.1816	0.4142	0.5824	0.6935	0.7675
	4.712	2	0.0018	0.1034	0.3039	0.4780	0.6044	0.6935
0.95	0.628	8	0.3744	0.7611	0.8816	0.9302	0.9542	0.9677
	0.912	2	0.6214	0.8720	0.9378	0.9635	0.9761	0.9832
	1.257	2	0.4389	0.7820	0.8895	0.9339	0.9562	0.9689
	1.571	2	0.3038	0.6934	0.8381	0.9014	0.9340	0.9528
	2.356	2	0.1034	0.4780	0.6935	0.8035	0.8646	0.9014
	3.141	2	0.0300	0.3040	0.5467	0.6935	0.7821	0.8382
	3.927	2	0.0077	0.1816	0.4142	0.5824	0.6935	0.7675
	4.712	2	0.0018	0.1034	0.3039	0.4780	0.6044	0.6935
0.99	0.628	12	0.2291	0.6640	0.8277	0.8972	0.9321	0.9520
	0.912	4	0.3862	0.7604	0.8795	0.9284	0.9528	0.9666
	1.257	2	0.4389	0.7820	0.8895	0.9339	0.9562	0.9689
	1.571	2	0.3038	0.6934	0.8381	0.9014	0.9340	0.9528
	2.356	2	0.1034	0.4780	0.6935	0.8035	0.8646	0.9014
	3.141	2	0.0300	0.3040	0.5467	0.6935	0.7821	0.8382
	3.927	2	0.0077	0.1816	0.4142	0.5824	0.6935	0.7675
	4.712	2	0.0018	0.1034	0.3039	0.4780	0.6044	0.6935

Table 2.1: Operating characteristic values of the double sampling plan for transmuted weibull distribution ($n_2, c_2, t/\sigma_0$) when $c_1=0, c_2=2$.

P^*	t/σ_0	n_1	n_2	σ/σ_0					
				2	4	6	8	10	12

0.75	0.628	5	14	0.8905	0.9966	0.9996	0.9999	1.0000	1.0000
	0.912	3	8	0.8420	0.9944	0.9994	0.9999	1.0000	1.0000
	1.257	2	5	0.7866	0.9916	0.9991	0.9998	0.9999	1.0000
	1.571	2	4	0.6386	0.9815	0.9979	0.9996	0.9999	1.0000
	2.356	2	3	0.2474	0.9161	0.9887	0.9977	0.9993	0.9998
	3.141	2	3	0.0369	0.7316	0.9516	0.9888	0.9967	0.9988
	3.927	2	3	0.0031	0.4752	0.8673	0.9646	0.9887	0.9958
	4.712	2	3	0.0002	0.2474	0.7315	0.9161	0.9711	0.9887
0.90	0.628	8	19	0.7621	0.9900	0.9989	0.9998	0.9999	1.0000
	0.912	4	10	0.7397	0.9885	0.9988	0.9998	0.9999	1.0000
	1.257	2	6	0.7423	0.9885	0.9988	0.9998	0.9999	1.0000
	1.571	2	4	0.6386	0.9815	0.9979	0.9996	0.9999	1.0000
	2.356	2	3	0.2474	0.9161	0.9887	0.9977	0.9993	0.9998
	3.141	2	3	0.0369	0.7316	0.9516	0.9888	0.9967	0.9988
	3.927	2	3	0.0031	0.4752	0.8673	0.9646	0.9887	0.9958
	4.712	2	3	0.0002	0.2474	0.7315	0.9161	0.9711	0.9887
0.95	0.628	10	22	0.6728	0.9837	0.9982	0.9996	0.9999	1.0000
	0.912	5	11	0.6570	0.9825	0.9980	0.9996	0.9999	1.0000
	1.257	3	7	0.5861	0.9757	0.9972	0.9994	0.9998	0.9999
	1.571	2	5	0.5653	0.9733	0.9969	0.9994	0.9998	0.9998
	2.356	2	3	0.2474	0.9161	0.9887	0.9977	0.9993	0.9998
	3.141	2	3	0.0369	0.7316	0.9516	0.9888	0.9967	0.9988
	3.927	2	3	0.0031	0.4752	0.8673	0.9646	0.9887	0.9958
	4.712	2	3	0.0002	0.2474	0.7315	0.9161	0.9711	0.9887
0.99	0.628	15	28	0.4782	0.9628	0.9955	0.9991	0.9997	0.9999
	0.912	7	14	0.4805	0.9628	0.9955	0.9991	0.9997	0.9999
	1.257	4	8	0.4459	0.9579	0.9949	0.9989	0.9997	0.9999
	1.571	3	6	0.3618	0.9422	0.9926	0.9985	0.9996	0.9998
	2.356	2	4	0.1695	0.8742	0.9815	0.9960	0.9989	0.9996
	3.141	2	3	0.0369	0.7316	0.9516	0.9888	0.9967	0.9988
	3.927	2	3	0.0031	0.4752	0.8673	0.9646	0.9887	0.9958
	4.712	2	3	0.0002	0.2474	0.7315	0.9161	0.9711	0.9887

Table 2.2: Operating characteristic values of the double sampling plan for transmuted size biased exponential distribution ($n_2, c_2, t/\sigma_0$) when $c_1=0, c_2=2$.

P^*	t/σ_0	n_1	n_2	σ/σ_0					
				2	4	6	8	10	12
0.75	0.628	4	11	0.8300	0.9917	0.9990	0.9998	0.9999	1.0000
	0.912	2	7	0.8118	0.9892	0.9986	0.9997	0.9999	1.0000
	1.257	2	5	0.6559	0.9722	0.9961	0.9991	0.9991	0.9999
	1.571	2	4	0.5168	0.9485	0.9920	0.9981	0.9994	0.9998
	2.356	2	3	0.2250	0.8394	0.9674	0.9914	0.9972	0.9989
	3.141	2	3	0.0563	0.6231	0.8947	0.9674	0.9883	0.9952
	3.927	2	3	0.0118	0.3979	0.7736	0.9176	0.9674	0.9858
	4.712	2	3	0.0024	0.2250	0.6230	0.8394	0.9299	0.9674
0.90	0.628	6	15	0.6741	0.9784	0.9972	0.9994	0.9998	0.9999
	0.912	2	9	0.7608	0.9837	0.9978	0.9995	0.9999	0.9999
	1.257	2	6	0.6019	0.9630	0.9946	0.9988	0.9996	0.9999
	1.571	2	5	0.4417	0.9282	0.9881	0.9972	0.9991	0.9997
	2.356	2	4	0.1523	0.7714	0.9485	0.9858	0.9952	0.9981
	3.141	2	3	0.0563	0.6231	0.8947	0.9674	0.9883	0.9952
	3.927	2	3	0.0118	0.3979	0.7736	0.9176	0.9674	0.9858
	4.712	2	3	0.0024	0.2250	0.6230	0.8394	0.9299	0.9674
0.95	0.628	8	17	0.5529	0.9636	0.9951	0.9990	0.9997	0.9999
	0.912	2	10	0.7396	0.9807	0.9974	0.9994	0.9998	0.9999
	1.257	2	7	0.5579	0.9534	0.9929	0.9984	0.9995	0.9998
	1.571	2	5	0.4417	0.9282	0.9881	0.9972	0.9991	0.9997
	2.356	2	4	0.1523	0.7714	0.9485	0.9858	0.9952	0.9981
	3.141	2	3	0.0563	0.6231	0.8947	0.9674	0.9883	0.9952
	3.927	2	3	0.0118	0.3979	0.7736	0.9176	0.9674	0.9858
	4.712	2	3	0.0024	0.2250	0.6230	0.8394	0.9299	0.9674

0.99	0.628	12	22	0.3398	0.9192	0.9880	0.9973	0.9992	0.9997
	0.912	4	13	0.4784	0.9399	0.9909	0.9979	0.9994	0.9998
	1.257	2	8	0.5274	0.9436	0.9911	0.9979	0.9994	0.9998
	1.571	2	6	0.3911	0.9076	0.9839	0.9961	0.9988	0.9995
	2.356	2	5	0.1223	0.7109	0.9283	0.9794	0.9929	0.9972
	3.141	2	4	0.0357	0.5171	0.8446	0.9485	0.9808	0.9920
	3.927	2	3	0.0118	0.3979	0.7736	0.9176	0.9674	0.9858
	4.712	2	3	0.0024	0.2250	0.6230	0.8394	0.9299	0.9674

Table 3.1: Producer’s risk with respect to time of experiment for double acceptance sampling plan based on transmuted weibull distribution ($P^* = 0.95$)

σ/σ_0	$c_1 = 0, c_2 = 2, n_1 = 10, n_2 = 22, t/\sigma_0=0.628$	$c_1 = 0, c_2 = 2, n_1 = 2, n_2 = 3, t/\sigma_0=1.571$
2	0.3272	0.4347
4	0.0163	0.0267
6	0.0018	0.0031
8	0.0004	0.0004
10	0.0001	0.0002
12	0.0000	0.0001

Table 3.2: Producer’s risk with respect to time of experiment for double acceptance sampling plan based on transmuted size biased exponential distribution ($P^* = 0.95$)

σ/σ_0	$c_1 = 0, c_2 = 2, n_1 = 10, n_2 = 22, t/\sigma_0=0.628$	$c_1 = 0, c_2 = 2, n_1 = 2, n_2 = 3, t/\sigma_0=1.571$
2	0.4471	0.5583
4	0.0364	0.0718
6	0.0049	0.0119
8	0.0010	0.0028
10	0.0003	0.0009
12	0.0001	0.0003

III. DESCRIPTION OF TABLES WITH EXAMPLES

Suppose that the lifetime of a product follows the Transmuted Weibull Distribution and an experimenter wants to establish that its true unknown mean life is at least 1000 hours with confidence level 0.90 respectively. The acceptance numbers for this experiment are $c_1 = 0$ and $c_2 = 2$ with sample sizes $n_1=8$ and $n_2=19$. The lot is accepted if during 628 hours no failure is observed in a sample and $\sigma/\sigma_0 = 2$. The probability of acceptance for DASP from Table 2.1 is 0.7621. In a DASP scheme as σ/σ_0 increases the probability of acceptance also increases. For the above sampling plan, the probability of acceptance is 1.0000 when the ratio of the unknown average lifetime to the specified average lifetime is 12. When the time of the experiment increases, the probability of acceptance for a DASP decreases. From Table 2.1 it is clear that when the time of the experiment is 4712 hours and the ratio $\sigma/\sigma_0 = 2$, the probability of acceptance is 0.0002. For the same experiment time, when σ/σ_0 increases the probability of acceptance also increases. It is important to note that a DASP minimizes the producer’s risk, but this scheme also exerts the pressure on the producer to improve the quality level of his product. At 4712 hours and with $\sigma/\sigma_0 = 12$ and $P^* = 0.90$, the probability of acceptance is 0.9887. The producer’s risk for the first sample for $P^* = 0.95$ are given in Table 3.1. For $\sigma/\sigma_0 = 2$ (if the unknown average lifetime is twice the specified average lifetime) the producer’s risks when the times of an experiment are 628 and 1571 hours are 0.3272 and 0.4347 respectively. The producer’s risk decreases as the quality level of the product increases with $P^* = 0.95$.

Suppose that the lifetime of a product follows the Transmuted size biased exponential distribution and an experimenter wants to establish that its true unknown mean life is at least 1000 hours with confidence level 0.90 respectively. The acceptance numbers for this experiment are $c_1 = 0$ and $c_2 = 2$ with sample sizes $n_1=6$ and $n_2=15$. The lot is accepted if during 628 hours no failure is observed in a sample and $\sigma/\sigma_0 = 2$. The probability of acceptance for DASP from Table 2.2 is 0.6741. In a DASP scheme as σ/σ_0 increases the probability of acceptance also increases. For the above sampling plan, the probability of acceptance is 0.9999 when the ratio of the unknown average lifetime to the specified average lifetime is 12. When the time of the experiment increases, the probability of acceptance for a DASP decreases. From Table 2.2 it is clear that when the time of the experiment is 4712 hours and the ratio $\sigma/\sigma_0 = 2$, the probability of acceptance is 0.0024. For the same experiment time, when σ/σ_0 increases the probability of acceptance also increases. It is important to note that a DASP minimizes the producer’s risk, but this scheme also exerts the pressure on the producer to improve the quality level of his product. At 4712 hours and with $\sigma/\sigma_0 = 12$ and $P^* = 0.90$, the probability of acceptance is 0.9674. The producer’s risk for the first sample for $P^* = 0.95$ are given in Table 3.2. For $\sigma/\sigma_0 = 2$ (if the unknown average lifetime is twice the specified average lifetime) the producer’s risks when the times of an

experiment are 628 and 1571 hours are 0.4471 and 0.5583 respectively. The producer’s risk decreases as the quality level of the product increases with $P^* = 0.95$.

IV. COMPARATIVE STUDY

In this study, a DASP is proposed for Transmuted Weibull Distribution and Transmuted Size Biased Exponential Distribution based on truncated life test. From Table 4, it can be easily observed that the DASP based on Transmuted Weibull Distribution perform better than the DASP based on Transmuted Size Biased Exponential Distribution.

Table 4: At $P^* = 0.95$ and $t/\sigma_0 = 0.628$, OC values of the DASP with $c_1 = 0$ and $c_2 = 2$.

σ/σ_0	2	4	6	8	10	12
OC values for Transmuted Weibull Distribution	0.6728	0.9837	0.9982	0.9996	0.9999	1.0000
OC values for Transmuted Size-Biased Exponential Distribution	0.5529	0.9636	0.9951	0.9990	0.9997	0.9999

Here the probability of acceptance for proposed DASP based on transmuted weibull distribution is higher than the probability of acceptance for proposed DASP based on transmuted size biased exponential distribution. Also the producer risk tends to zero for most of the values. This shows that in case of double acceptance sampling plan using transmuted weibull distribution will be profitable and provides more satisfaction for the producer. This is clearly shown in the following figure 1:

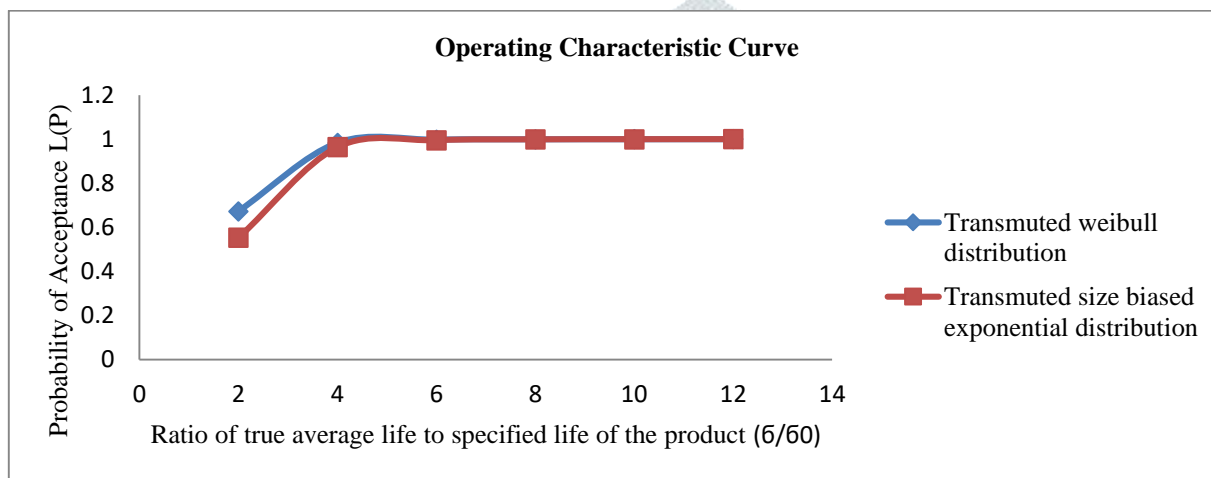


Figure 1: Operating characteristic curve for the double acceptance sampling plan with $P^* = 0.95$, $t/\sigma_0 = 0.628$, $c_1 = 0$ and $c_2 = 2$.

V. CONCLUSION

In this paper, the double acceptance sampling plan for the truncated life test is presented. We find the acceptance sampling plans for various values of σ/σ_0 and different experiment times assuming that the life test follows the Transmuted Weibull Distribution and Transmuted size biased exponential distribution. The minimum sample sizes n_1 and n_2 are determined for the predetermined acceptance numbers c_1 and c_2 . The operating characteristic values were tabulated. We can see that these distributions provides the high probability when σ/σ_0 is greater than 4 and we can conclude that the operating characteristic values increases when the quality improves. Among the two lifetime distributions used, the transmuted weibull distribution is better as its probability of acceptance is higher when compared with the other distribution and can be used conveniently in practical situations to save the time and cost of life test experiments.

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