# Difference cordial labeling of some graphs related to square graph

[1]V.Mohan, [2]A.Sugumaran
[1, 2]Department of mathematics, Government Arts College, Thiruvannamalai-606603,
Tamilnadu, India.

**Abstract.** A difference cordial labeling of a graph G with p vertices is a bijective assignment of labels from {1, 2, …, p} to the vertices of G so that each edge e = uv is assigned the label | f(u) – f(v)| (mod 2), if the number of edges labeled with 1 and the number of edges labeled with 0 are differ by at most 1. In this paper we show that the graphs P(r . $P_n^2$), C(r . $P_n^2$), S(r . $P_n^2$), $C_n^2$ $\Theta v_n$ $P_k$ and $C_n^2$ $\Theta v_n$ $P_k^2$ admit difference cordial labeling.

**Mathematics subject classification.** 05C78

**Keywords.** Path union of graphs; cycle union of graphs; open star of graphs; difference cordial graphs.

**1. Introduction.** All graphs considered in this paper are simple, finite and undirected graphs. By the expression G = (p, q) we mean a simple undirected graph with order p and size q. Labeling of graphs have been used in several fields such as, astronomy, radar and circuit design and database management[1]. In [3, 4, 5, 6, 7, 8], the difference cordial labeling behavior of Path, Cycle, Complete graph, Complete bipartite graph, Bistar, Wheel and Web graphs have been investigated. Seoud and Salman [9, 10] have studied the difference cordial labeling behavior of the families of graphs such as Ladder, Triangular ladder, Grid, Step ladder and Two sided step ladder graphs. In this paper we investigate the difference cordial labeling behavior of P(r . $P_n^2$), C(r . $P_n^2$), S(r . $P_n^2$), $C_n^2$ $\Theta v_n$ $P_k$ and $C_n^2$ $\Theta v_n$ $P_k^2$.

**2. Basic definitions and notations.** In this section we recall some of the basic definitions and notations.

Let f : V(G)→{1, 2, 3, … , p} be any bijective vertex labeling map. We denote $e_f(i)$ is the number of edges of G labeled with i under f.

**Definition 2.1.** Let G = (V(G), E(G)) be a (p, q) graph and let f : V(G)→{1, 2, 3, … , p} be bijective. The mapping f is called a *difference cordial labeling* of graph G, if each edge e = uv is assigned the label |f(u) – f(v)| (mod 2) and satisfying $|e_f(0) - e_f(1)| \leq 1$. A graph which admits difference cordial labeling is called a *difference cordial graph.*

The concept of difference cordial labeling has been introduced by R. Ponraj et al.[3].

**Definition 2.2.** Let G be a graph and let $G_1, G_2, G_3, \ldots, G_n$, be n copies of graph G. For each i = 1, 2, 3, … , n-1, if we joining the graphs $G_i$ and $G_{i+1}$ by an edge, then the graph is called a *path union of graph* G and it is denoted by P(n.G).

**Definition 2.3.** Let G be any graph with n copies. For each i = 1, 2, 3, … , n, if any one of the vertices of the $i^{th}$ copy of G is append with the $i^{th}$ vertex of cycle $C_n$, then the resulting graph is called a *cycle union of graph* G and it is denoted by C(n.G).

**Definition 2.4.** Let G be any graph with n copies. For each i = 1, 2, 3, … , n, if any one of the vertices of the $i^{th}$ copy of G is append with the $i^{th}$ pendent vertex of cycle $K_{1, n}$, then the graph obtained is called an *open star of graph* G and it is denoted by S(n.G).

**Definition 2.5.** Let G be any simple connected graph. The *square of graph* G is denoted by $G^2$, defined as the graph with the same vertex set as that of G and any two vertices are adjacent in $G^2$ if they are at a distance 1 or 2 apart in G.

**Definition 2.6.** Let G, H be any two graphs and let v be a vertex of G. Suppose we append any one of the vertices of the graph H at vertex v, then the graph is denoted by $G\ O_v\ H$.

**3. Main results.**

**Theorem 3.1.** P(r . $P_n^2$) is difference cordial graph for n, r ≥ 2.

**Proof.** Let G = P(r. $P_n^2$). We denote $v_i^k$ is the i^th vertex of k^th copy of $P_n^2$, where $1 \le k \le r$ and $1 \le i \le n$. Note that |V(G)| = nr and |E(G)| = 2r(n-1)-1.

We define a vertex labeling function f : V(G)→{1, 2, 3, … , nr} as follows:

$f(v_i^k) = i+4(k-1)$ for $1 \le i \le n$ and $1 \le k \le r$.

Also we interchange the labels of the vertices $v_1^k$ and $v_2^k$ for each $2 \le k \le r$.

In view of the above labeling pattern, we have $|e_f(0) - e_f(1)| = 1$. Hence G is a difference cordial graph.

**Example 3.2.** Figure 1 illustrates that the path union of 3 copies of $P_4^2$ admits difference cordial labeling.

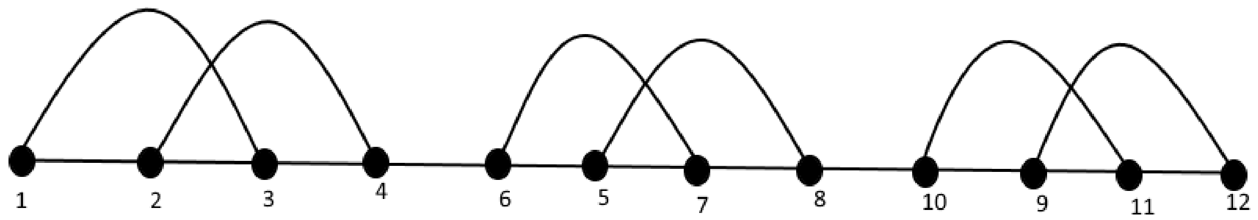


**Figure 1.** The graph P(3. $P_4^2$).

**Theorem 3.3.** Cycle union of r copies of the graph $P_n^2$ is a difference cordial graph.

**Proof.** Let G = P(r. $P_n^2$). We denote $v_i^k$ is the i^th vertex of k^th copy of $P_n^2$, where $1 \le k \le r$ and $1 \le i \le n$. Note that |V(G)| = nr and |E(G)| = 2(n-1)r. We define a vertex labeling function f : V(G)→{1, 2, 3, … , nr} as follows:

$f(v_i^k) = i+4(k-1)$ for $1 \le i \le n$ and $1 \le k \le r$.

Also we interchange the labels of the vertices $v_1^k$ and $v_2^k$ assigned by f, for each $2 \le k \le r$.

In view of the above labeling pattern, we have $|e_f(0) - e_f(1)| = 0$. Hence G is a difference cordial graph.

**Example 3.4.** Figure 2 illustrates that the cycle union of 4 copies of $P_4^2$ admits difference cordial labeling.

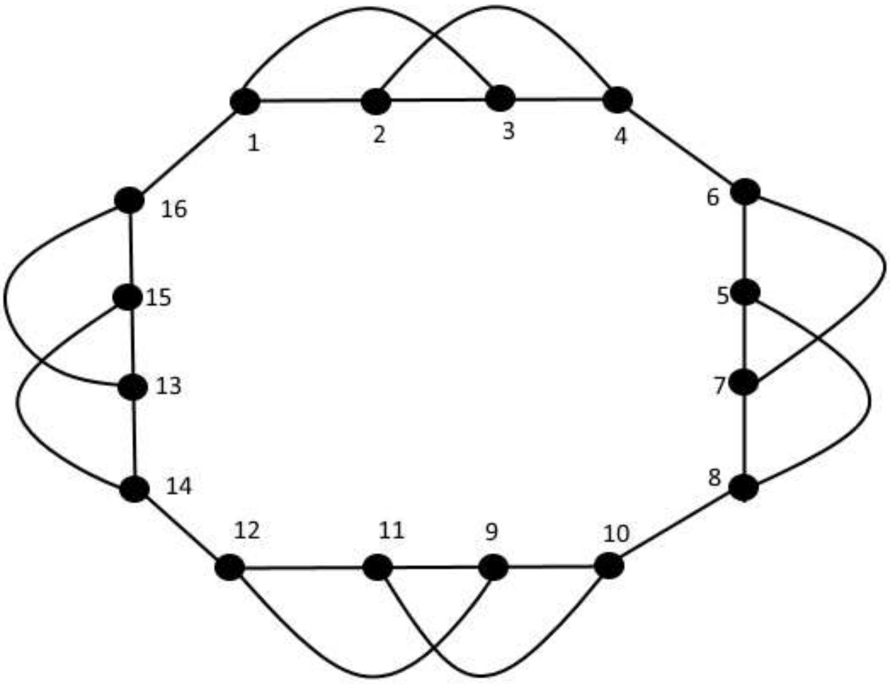


**Figure 2.** The graph C(4. $P_4^2$).

**Theorem 3.5.** Open star of r copies of the square graph $P_n^2$ is a difference cordial graph.

**Proof.** Let G = S(r. $P_n^2$). Let v be the apex vertex of star graph. Let G = S(r. $P_n^2$). We denote $v_i^k$ is the i$^{th}$ vertex of k$^{th}$ copy of $P_n^2$, where $1\leq k \leq r$ and $1\leq i \leq n$. Note that |V(G)| = nr + 1 and |E(G)| = 2(n-1)r. We define a vertex labeling function f : V(G)→{1, 2, 3, … , nr + 1} as follows:

f(v) = 1,

f($v_i^k$) = (i + 1) + 4(k-1) for $2\leq i \leq n$ and $1\leq k \leq r$.

Also we interchange the labels of the vertices v and $v_1^1$ which are assigned by f. In view of the above labeling pattern, we have $|e_f(0) - e_f(1)| = 0$. Hence G is a difference cordial graph.

**Example 3.6.** Figure 3 shows that the difference cordial labeling of the graph S(4. $P_4^2$).

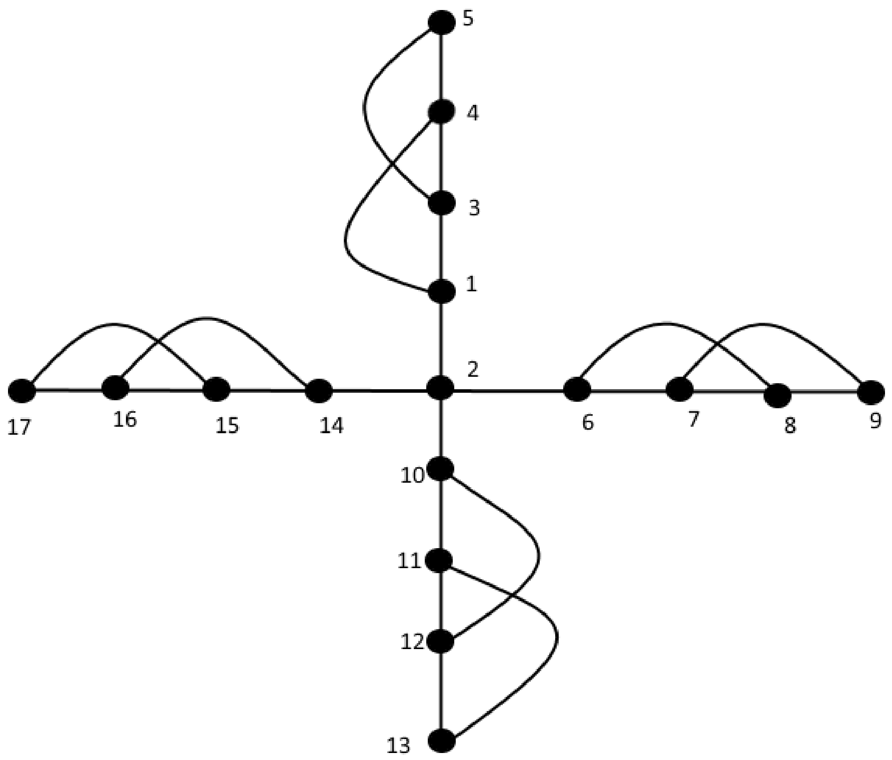


**Figure 3.** The graph S(4. $P_4^2$).

**Theorem 3.7.** The graph $C_n^2$ ⊙$v_n$ P$_k$, (k = 2, 3, 4) is a difference cordial graph, (n > 4).

**Proof.** Let G =$C_n^2$ ⊙$v_n$ P$_k$. Let $v_1$, $v_2$, $v_3$, … , $v_n$ be the vertices of $C_n^2$. We distinguish the difference cordial labeling into three cases.

**Case 1**. When k = 2.

Let $u_1$, $u_2$ be the vertices of path $P_2$ and let $u_1$ = $v_n$. Note that |V(G)| = n + 1 and |E(G)| = 2n + 1. We define a vertex labeling function f : V(G)→{1, 2, 3, … , n + 1} as follows:

f($v_i$) = i for 1≤ $i \leq n$;

f($u_2$) = n + 1.

**Case 2.** When k = 3.

Let $u_1$, $u_2$, $u_3$ be the vertices of path $P_3$ and let $u_1$ = $v_n$. Note that |V(G)| = n + 2 and |E(G)| = 2n + 2. We define a vertex labeling function f : V(G)→{1, 2, 3, … , n + 2} as follows:

f($v_i$) = i for 1≤ $i \leq n$;

f($u_2$) = n + 1;

f($u_3$) = n + 2.

**Case 3.** When k = 4.

Let $u_1$, $u_2$, $u_3$, $u_4$ be the vertices of path $P_4$ and let $u_1$ = $v_n$. Note that |V(G)| = n + 3 and |E(G)| = 2n + 3. We define a vertex labeling function f : V(G)→{1, 2, 3, … , n + 3} as follows:

f($v_i$) = i for 1≤ $i \leq n$;

f($u_2$) = n + 1;

f($u_3$) = n + 2;

f($u_4$) = n + 3.

In view of the above labeling pattern from all the cases, we have $|e_f(0) - e_f(1)| \leq 1$. Hence G is a difference cordial graph.

**Example 3.8.** Figure 4 shows that the difference cordial labeling of the graph $C_6^2$ $\Theta v_6$ $P_3$.

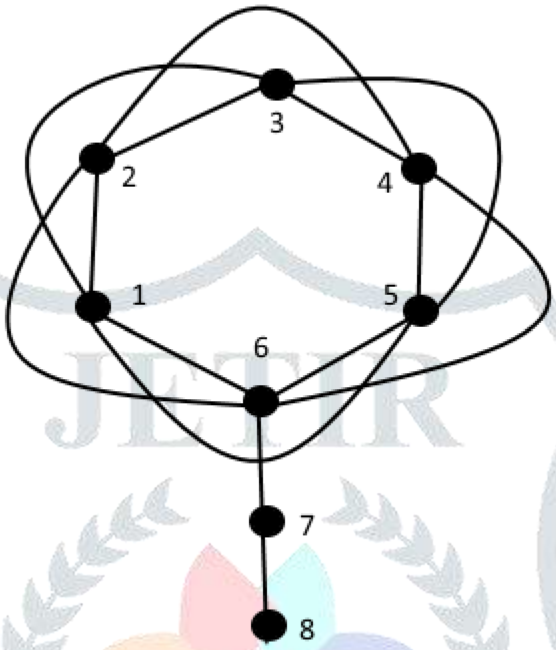


**Figure 4.** The graph $C_6^2$ $\Theta v_6$ $P_3$.

**Theorem 3.9.** The graph $C_n^2$ $\Theta v_n$ $P_k^2$ is a difference cordial graph, where n > 4, m > 2.

**Proof.** Let G = $C_n^2$ $\Theta v_n$ $P_k^2$. Let $v_1, v_2, v_3, \dots, v_n$ be the vertices of $C_n^2$ and let $u_1, u_2, u_3, \dots, u_k$ be the vertices of $P_k^2$ and let $u_1 = v_n$. Note that |V(G)| = n + k – 1 and |E(G)| = 2n + 2k – 3. We define a vertex labeling function f : V(G)→{1, 2, 3, … , n + k - 1} as follows:

f($v_i$) = i for $1 \leq i \leq n$;

f($u_{i+1}$) = i + n for $1 \leq i \leq k - 1$.

In view of the above labeling pattern, we have $|e_f(0) - e_f(1)| = 1$. Hence G is a difference cordial graph.

**Example 3.10.** Figure 5 shows that the difference cordial labeling of the graph $C_6^2$ $\Theta v_6$ $P_3^2$.

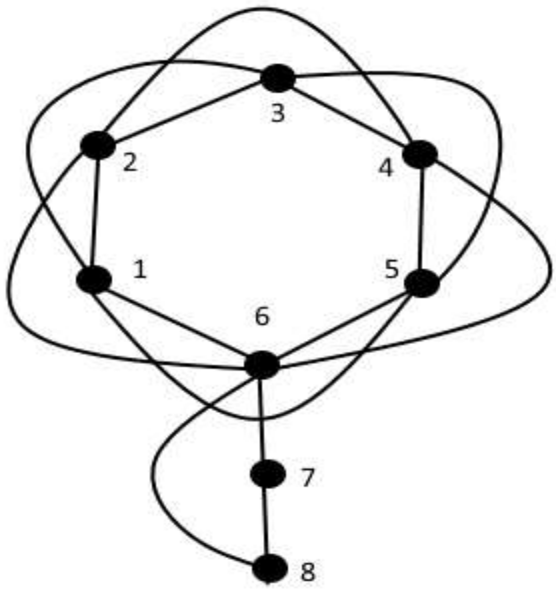


**Figure 5.** The graph $C_6^2 \, \Theta v_6 \, P_3^2$.

**4. Conclusion.** Difference cordial labeling concept has been investigated by many researchers. In this paper, we proved that the graphs P(r . $P_n^2$), C(r . $P_n^2$), S(r . $P_n^2$), $C_n^2 \, \Theta v_n$ P$_k$ and $C_n^2 \, \Theta v_n \, P_k^2$ are difference cordial graphs.

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