# PERTURBATION ANALYSIS OF MHD CASSON FLOW OF A VISCO-ELASTIC FLUID PAST AN INFINITE VERTICAL POROUS PLATE WITH THERMAL RADIATION AND CHEMICAL REACTION EFFECTS

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***Abstract:*** The present study, we considered the effects of unsteady MHD non-Newtonian flow of a Visco-Elastic fluid past an infinite vertical porous plate and the effects of thermal radiation and chemical reaction along with heat and mass transfer are reported. The governing equations are transformed into nonlinear ordinary differential equations using suitable transformation and then solved analytically by using perturbation technique. The velocity, temperature and concentration are presented graphically with help of various physical parameters.

***Index Terms*:** MHD, Casson fluid, Visco-Elastic fluid, Thermal radiation, Chemical reaction.

**I.INTRODUCTION**

Numerical applications of Visco-elastic fluid in several manufacturing processes have led scientists to investigate Visco-elastic flow on the boundary layer. The study of viscoelastic fluid flowing over a vertical surface immersed in porous media in presence of magnetic field has attracted the researchers because of its applications in geophysics, astrophysics, geo-hydrology, chemical engineering, biological system, soil physics and filtration of solid from liquids.

Srinivasa Raju et al. [1] observed Analytical and Numerical study of Unsteady MHD free convective flow over an exponentially moving vertical plate with Heat absorption solved by Analytic method. Satya et al. [2] has investigated the chemical reaction and heat source effects on MHD oscillatory flow in an irregular channel, the governing equations are solved by analytically. Yigid aksoy [3] Studied, effects of couple stresses on the heat transfer and entropy generation rates of a flow between parallel plates with constant heat flux solved by analytically. Raju et al. [4] discussed Heat and mass transfer in magneto-hydrodynamic casson fluid over an exponentially permeable stretching surface solved by numerical method. Manoj kumar nayak et al. [5] have investigated heat and mass transfer effects on MHD viscoelastic fluid over a stretching sheet through porous medium in presence of chemical reaction solved by numerical method. Prashant G.metri et al. [6] has reported heat transfer in MHD mixed convection viscoelastic fluid flow over a stretching sheet embedded in a porous medium with viscous dissipation and non-uniform heat source/sink solved by numerically. Jayachandra babu et al. [7] considered MHD non-Newtonian fluid flow over a slandering stretching sheet in the presence of casson diffusion effects solved by numerical. Dual pal et al. [8] obtained perturbation analysis of magneto-hydrodynamics oscillatory flow on convective radiation heat and mass transfer of micro polar fluid in a porous medium with chemical reaction solved by analytically. Raju et al. [9] have discussed unsteady three-dimensional flow of casson-carreau fluids past a stretching surface solved by numerical method. Joaquin zeuco et al. [10] studied 2D unsteady free convective heat and mass transfer Newtonian Hartmann flow with thermal diffusion and soret effects; network model and finite differences solved by numerical method. Meraj Mustafa [11] investigated an analytical treatment for MHD mixed convection boundary layer flow of Oldroyd -B fluid utilizing non-Fourier heat flux model solved by analytically. Bhuvana vijaya et al. [12] to study combined influence of thermal radiation, Soret, Duffer effects on non-Darcy mixed convective heat and mass transfer flow with dissipation in a vertical channel solved by numerical method. Nadem et al. [13] observed comparison and analysis of the Atangana-Baleanu and Caputo-Febrizio fractional derivatives for generation and chemical reaction solved by analytically. Kumaran et al. [14] studied computational analysis of magneto-hydrodynamic casson and Maxwell flows over a stretching sheet with cross diffusion solved by numerically. Sahin Ahmed et al. [15] have investigated the effects of chemical reaction, heat and mass transfer and viscous dissipation over a MHD flow in a vertical porous wall using perturbation method. Us Rajput et al. [16] have discussed effects of hall current and chemical reaction on MHD flow through porous medium past on oscillating inclined plate with variable temperature and mass diffusion solved by analytical method. Hari Krishna et al. [17] have studied effects of radiation and chemical reaction on MHD flow past on oscillating inclined porous plate with variable temperature and mass diffusion solved by perturbation method. Imran ullah et al. [18] to study effects of slip condition and Newtonian heating on MHD flow of casson fluid over a non-linear stretching sheet saturated in a porous medium solved by numerical method. Parida et al. [19] have reported free convective flow through porous medium with variable permeability in slip flow regime with couple stress in the presence of heat source solved by perturbation. Ibrahim et al. [20] studied influence of chemical reaction and heat source on dissipation MHD mixed convective flow of a casson Nano fluid over a non-linear permeable stretching sheet solved by analytically. Sunita rani [21] observed Jeffrey fluid performance on MHD convective flow past a semi-infinite vertically inclined permeable moving plate in presence of heat and mass transfer; a finite difference technique solved by numerically. Kartini Ahmed et al. [22] have investigated magneto-hydrodynamic (MHD) Jeffery fluid over a stretching vertical surface in a porous medium, solved by numerical method. Wubshet Ibrahim [23] discussed magneto-hydrodynamic (MHD) boundary layer stagnation point flow and heat and mass transfer of a Nano fluid past a stretching sheet with meeting solved by numerically. Das et al. [24] studied MHD convective mass transfer flow of a polar fluid past a semi-infinite vertical porous flat moving plate embedded on a porous medium solved by analytically. Srinivas reddy et al. [25] have discussed MHD flow and heat transfer characteristics of Williamson Nano fluid over a stretching sheet with variable thickness and variable thermal conductivity solved by numerically. Bala anki reddy [26] observed numerical study of magneto-hydrodynamic (MHD) boundary layer slip flow of a Maxwell Nano-fluid over an exponentially stretching surface with convective boundary condition. Mohamed Abd El-aziz [27] to study perturbation analysis of unsteady layer slip flow and heat transfer of casson fluid past a vertical permeable plate with hall current. Sharif Uddin et al. [28] investigated thermal boundary layer in stagnation point flow past a permeable shrinking sheet with variable surface temperature solved by numerically. Ramesh et al. [29] studied three dimensional flow of Maxwell fluid with suspended nanoparticles past a bidirectional porous stretching surface with thermal radiation solved by numerically. Macha madhu et al. [30] discussed unsteady flow of a Maxwell Nano-fluid over a stretching surface in the presence of magneto-hydrodynamic and thermal radiation effects solved by numerically. Sumit gupta et al. [31] have investigated MHD mixed convective stagnation point flow and heat and mass transfer of an incompressible Nano fluid over an inclined stretching sheet with chemical reaction solved by analytically.

In the present study, the effects of unsteady MHD non-Newtonian flow of a Visco-Elastic fluid past an infinite vertical porous plate and the effects of thermal radiation and chemical reaction along with heat and mass transfer are reported. The governing equations are transformed into nonlinear ordinary differential equations using suitable transformation and then solved analytically by using perturbation technique. The velocity, temperature and concentration are presented graphically with help of various physical parameters.

## II. MATHEMATICAL FORMULATION

Consider unsteady two-dimensional flow of a laminar, viscoelastic, electrically conducting and heat absorbing fluid past a semi-infinite vertical permeable moving plate embedded in a uniform porous medium and subjected to a uniform transverse magnetic field in the presence of thermal and concentration buoyancy effects. It is assumed that there is no applied voltage which implies the absence of an electrical field. The fluid properties are assumed to be constant except that the influence of density variation with temperature. The concentration of diffusing species is very small in comparison to other chemical species, the concentration of species far from the wall$C_\infty$, is infinitesimally small and hence the Soret and Dufour effects are neglected. The chemical reaction takes place in the flow and all thermo physical properties which are assumed to be constant on the linear momentum equation is approximated according to the Bossiness approximation. Due to the semi-infinite plane surface assumption, the flow variables are functions of $y'$ and the time $t'$ only.

Under these assumptions, the equations that the physical situation are given by

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

**MOMENTUM**

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma\beta_0^2}{\rho} + \frac{\nu}{k'}\right)u' + g\beta_T\left(T'-T_\infty\right) + g\beta_c\left(C'-C_\infty\right) - k_0\left(\frac{\partial^3 u'}{\partial t'\partial y'^2} + v'\frac{\partial^3 u'}{\partial y'^3}\right) \tag{2}$$

**TEMPERATURE**

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0}{\rho c_p}\left(T'-T_\infty\right) + \frac{Q_1'}{\rho c_p}\left(C'-C_\infty\right) - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} \tag{3}$$

**CONCENTRATION**

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}-K_1\left(C'-C_\infty\right) \tag{4}$$

Where $x'$ is the dimensional distances, $y'$ is perpendicular to the plate and $t'$ is the dimensional time. $u'$ and $v'$ are the components of dimensional velocities along $x'$ and $y'$ directions, respectively. $t'$ is the dimensional temperature, $C'$ is the dimensional concentration, $C_W$ and $T_W$ are the concentration and temperature at the wall, respectively. $C_\infty$ and $T_\infty$ are the free stream dimensional concentration and temperature, respectively. $\rho$ is the fluid density, v is the kinematic viscosity, $c_p$ is the specific heat at constant pressure, $\sigma$ is the fluid electrical conductivity, $B_0$ is the magnetic induction, $k_0$ is the Visco-elastic parameter, $k'$ is the permeability of the porous medium, $Q_0'$ is the dimensional heat absorption coefficient , $Q_1'$ is the coefficient of proportionality for the absorption of radiation, k is the thermal conductivity parameter, D is the mass diffusivity, g is the gravitational acceleration, $\beta_T$ and $\beta_c$ are the thermal and concentration expansion coefficient, respectively and $K_1$ is the chemical reaction coefficient. The magnetic and viscous dissipations are neglected in this study. The third and fourth terms on the RHS of the momentum Eq. (2) denote the thermal and concentration buoyancy effects, respectively. Also the second and third terms on the RHS of the energy Eq. (3) represent the heat and radiation absorption effects, respectively. It is assumed that the permeable plate moves with the variable velocity in the direction of fluid flow. In addition, it is assumed that the temperature and the concentration at the wall as well as the suction velocity are exponentially varying with time.

Under these assumptions, the appropriate boundary conditions for the velocity, temperature and concentration fields are

$$\begin{aligned}&u'=u_p',\ T'=T_W+\varepsilon(T_W-T_\infty)e^{n't'},\ C'=C_W+\varepsilon(C_W-C_\infty)e^{n't'}\ \text{at}\ y'=0\\&u'=0, T'\to T_\infty, C'\to C_\infty,\ \text{at}\ y'\to\infty\end{aligned} \tag{5}$$

Where $u_p'$ is the wall dimensional velocity, $n'$ is constant. It is clear from Eq. (1) that the suction velocity at the plate surface is a function of time only. Assuming that the following exponential form:

$$v'=-V_0(1+\varepsilon A e^{n't'}), \tag{6}$$

Where **A** is a suction parameter, $\varepsilon$ and **A** are small such that $\varepsilon$<<1, **A**<<1 and $V_0$ is a scale of suction velocity which has non-zero position constant. Now the following non-dimensional variables are introduced.

$$\begin{aligned}&F=\frac{4vI'}{\rho C p V_0^2},\ u=\frac{u'}{V_0},\ v=\frac{v'}{V_0},\ y=\frac{V_0 y'}{v},\ t=\frac{V_0^2 t'}{v},\\&u_p=\frac{u_p'}{V_0},\ n=\frac{n'v}{V_0^2},\ \theta=\frac{T'-T_\infty}{T_W-T_\infty},\ C=\frac{C'-C_\infty}{C_W-C_\infty}\end{aligned} \tag{7}$$

In view of the above non-dimensional variables, the basic field Eqn. (2)-(4) can be expressed in the non-dimensional form as

$$\frac{\partial u}{\partial t}-(1+\varepsilon A e^{nt})\frac{\partial u}{\partial y}=\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\left(M+\frac{1}{K}\right)u+G_r\theta+G_m C-E\left[\frac{\partial^3 u}{\partial t\partial y^2}-(1+\varepsilon A e^{nt})\frac{\partial^3 u}{\partial y^3}\right] \tag{8}$$

$$\frac{\partial \theta}{\partial t}-(1+\varepsilon A e^{nt})\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}-(\phi+F)\theta+Q_1 C \tag{9}$$

$$\frac{\partial C}{\partial t}-(1+\varepsilon e^{nt})\frac{\partial c}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-\gamma C \tag{10}$$

$$u = u_p, \theta = 1 + \varepsilon e^{nt}, C = 1 + \varepsilon e^{nt} \text{ at } y = 0 \tag{11}$$
$$u \to 0, \theta \to 0, C \to 0 \text{ at } y \to \infty$$

$$Gr = \frac{vg\beta_t(T_W - T_\infty)}{V_0^3}, \quad Gm = \frac{vg\beta_c(C_W - C_\infty)}{V_0^3} \; Q_1 = \frac{vQ_1'(C_W - C_\infty)}{\rho C_P V T_0^2(_W - T_\infty)}, \Pr = \frac{v\rho c_p}{k}, K = \frac{K' v_0^2}{v^2},$$
$$M = \frac{\sigma\beta_0^2 v}{\rho V_0^2}, \; \gamma = \frac{K_1 v}{V_0^2}, \; Sc = \frac{v}{D}, \; \phi = \frac{vQ_0}{\rho c_p v_0^2}, \; E = \frac{k_0 v_0^2}{v^2} \tag{12}$$

M is the magnetic field parameter, K is the permeability parameter, $\gamma$ is the Chemical reaction parameter, Sc is the Schmidt number, $\phi$ is the heat source parameter and $Q_1$ is the absorption of radiation parameter.

The mathematical statement of the problem is now complete and embodies the solution of Eqs. (8)-(10) Subject to boundary condition (11).

## III. SOLUTION OF THE PROBLEM

Solutions of Eqns. (8)-(10) are obtained by regular and multi parameter perturbation technique. The parameter Visco elastic parameter (E), constant $\varepsilon$ and $A$ are assumed small such that E<<1 and $\varepsilon$ <<1.The velocity u, temperature C within the boundary layer region can be expressed as:

$$u(y,t) = u_0(y) + \varepsilon e^{nt} u_1(y) + O(\varepsilon^2), \; \theta(y,t) = \theta_0(y) + \varepsilon e^{nt}\theta_1(y) + O(\varepsilon^2), \; C(y,t) = C_0(y) + \varepsilon e^{nt} C_1(y) + O(\varepsilon^2) \tag{13}$$

Where $u_0$, $\theta_0$ and $C_0$ are the mean velocity, mean temperature and mean concentration respectively. Using Eq.(13) in Eqs. (8)-(10). Equation harmonic and non-harmonic terms for mean velocity, mean temperature and mean concentration, after neglecting coefficient of $\varepsilon^2$, we get Zero order

$$Eu_0''' + \left(1 + \frac{1}{\beta}\right)u_0'' + u_0' - \left(M + \frac{1}{K}\right)u_0 = -Gr\theta_0 - GmC_0 \tag{14}$$

$$\theta_0'' + \Pr\theta_0' - \Pr\phi\theta_0 = -Q_1 \Pr C_0 \tag{15}$$

$$C_0'' + ScC_0' - Sc\gamma C_0 = 0 \tag{16}$$

With corresponding boundary conditions

$$u_0 = u_p, \theta_0 = 1, C_0 = 1 \text{ at } y = 0$$
$$u_0 \to 0, \theta_0 \to 0, C_0 \to 0 \text{ as } y \to \infty \tag{17}$$

First order

$$Eu_1''' + \left[\left(1 + \frac{1}{\beta}\right) - En\right]u_1'' + u_1' - \left[\left(M + \frac{1}{K}\right) + n\right]u_1 = -Gr\theta_1 - GmC_1 - AEu_0''' - Au_0' \tag{18}$$

$$\theta_1'' + \Pr\theta_1' - \Pr(\phi + F + n)\theta_1 = -\Pr A\theta_0' - \Pr Q_1 C_1 \tag{19}$$

$$C_1'' + ScC_1' - Sc(\gamma + n)C_1 = -AScC_0' \tag{20}$$

With corresponding boundary conditions

$$u_1 = 0\,, \theta_1 = 0\,,\; C_1 = 0\,, \text{ on } \; y = 0$$
$$u_1 \to 0\,, \theta_1 \to 0\,, C_1 \to 0 \;\text{ as } y \to \infty \tag{21}$$

Equations (14) and (18) are third order differential equations due to presence of viscoelastic parameter. There are only two boundary conditions. Therefore, it needs one boundary condition more for unique solution. Thus, to avoid this difficulty, we adopted perturbation method and expanded following Beard and Walters [1].

$$u_0(y) = u_{00}(y) + Eu_{01}(y) + O(E^2)$$
$$u_1(y) = u_{10}(y) + Eu_{11}(y) + O(E^2) \tag{22}$$

Substitute Eq. (22) in Eq. (14) and equating coefficient of zero order and first order of E,

We get

$$\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u_{00}}{\partial y^2} + \frac{\partial u_{00}}{\partial y} - \left(M + \frac{1}{K}\right)u_{00} = -Gr\theta_o - GmC_0 \tag{23}$$

$$\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u_{01}}{\partial y^2} + \frac{\partial u_{01}}{\partial y} - \left(M + \frac{1}{K}\right)u_{01} = -\frac{\partial^3 u_{00}}{\partial y^3} \tag{24}$$

The corresponding boundary conditions are

$$u_{00} = u_p\,, \theta_{01} = 0\,, \text{ on } \; y = 0$$
$$u_{01} \to 0\,, \theta_{01} \to 0\,, \text{ as } \; y \to \infty \tag{25}$$

Substituting Eq. (22) in Eq. (18) and equating coefficient of zero order and first order of E, we get

$$\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u_{10}}{\partial y^2} + \frac{\partial u_{10}}{\partial y} - \left(M + \frac{1}{K} + n\right)u_{10} = -Gr\theta_1 - GmC_1 - A\frac{\partial u_{00}}{\partial y} \tag{26}$$

$$\left[\left(1+\frac{1}{\beta}\right) - n\right]\frac{\partial^2 u_{11}}{\partial y^2} + \frac{\partial u_{11}}{\partial y} - \left(M + \frac{1}{K} + n\right)u_{11} = -A\frac{\partial^3 u_{00}}{\partial y^3} - A\frac{\partial u_{01}}{\partial y} - \frac{\partial^3 u_{10}}{\partial y^3} + n\frac{\partial^2 u_{10}}{\partial y^2} \tag{27}$$

Where,

$$B = 1 + \frac{1}{\beta}, \quad S = M + \frac{1}{K}$$

The corresponding boundary conditions are

$$u_{10} = 0\,, \theta_{11} = 0\,, \text{ on } \; y = 0$$
$$u_{10} \to 0\,, \theta_{11} \to 0\,, \text{ as } \; y \to \infty \tag{28}$$

Solving Eqs. (23), (24), (26) and (27) by using the boundary conditions (25) and (28), we get

$$u_{00} = A_{16}e^{-m_5 y} + A_{12}e^{-m_3 y} + A_{15}e^{-m_1 y}$$

$$u_{01} = A_{20}e^{-m_6 y} + A_{17}e^{-m_5 y} + A_{18}e^{-m_1 y} + A_{19}e^{-m_1 y}$$

$$u_{10} = A_{33}e^{-m_7 y} + A_{27}e^{-m_5 y} + A_{30}e^{-m_3 y} + A_{31}e^{-m_2 y} + A_{32}e^{-m_1 y}$$

$$u_{11} = A_{59}e^{-m_8 y} + A_{53}e^{-m_7 y} + A_{37}e^{-m_6 y} + A_{54}e^{-m_5 y} + A_{55}e^{-m_4 y} + A_{56}e^{-m_3 y} + A_{57}e^{-m_2 y} + A_{58}e^{-m_1 y}$$

$$u_0(y) = u_{00}(y) + Eu_{01}(y)$$

$$u_0(y) = (A_{16}e^{-m_5 y} + A_{12}e^{-m_3 y} + A_{15}e^{-m_1 y}) + E(A_{20}e^{-m_6 y} + A_{17}e^{-m_5 y} + A_{18}e^{-m_3 y} + A_{19}e^{-m_1 y})$$

$$u_1(y) = u_{10}(y) + Eu_{11}(y)$$

$$u_1(y) = (A_{33}e^{-m_7 y} + A_{27}e^{-m_5 y} + A_{21}e^{-m_4 y} + A_{30}e^{-m_3 y} + A_{31}e^{-m_2 y} + A_{32}e^{-m_1 y}) + E(A_{59}e^{-m_8 y} + A_{53}e^{-m_7 y} + A_{37}e^{-m_6 y} + A_{54}e^{-m_5 y} + A_{55}e^{-m_4 y} + A_{56}e^{-m_3 y} + A_{57}e^{-m_2 y} + A_{58}e^{-m_1 y})$$

$$u(y,t) = u_0(y) + \varepsilon e^{nt} u_1(y) \tag{29}$$

$$C(y,t) = C_0(y) + \varepsilon e^{nt} C_1(y)$$

$$C_0(y) = e^{-m_1 y}$$

$$C_1(y) = (A_2 e^{-m_2 y} + A_1 e^{-m_1 y})$$

$$C(y,t) = e^{-m_1 y} + \varepsilon e^{nt}(A_2 e^{-m_2 y} + A_1 e^{-m_1 y}) \tag{30}$$

$$\theta(y,t) = \theta_0(y) + \varepsilon e^{nt}\theta_1(y)$$

$$\theta_0(y) = (A_4 e^{-m_3 y} + A_3 e^{-m_1 y})$$

$$\theta_1(y) = (A_{11}e^{-m_4 y} + A_5 e^{-m_3 y} + A_7 e^{-m_2 y} + A_9 e^{-m_1 y})$$

$$\theta(y,t) = (A_4 e^{-m_3 y} + A_3 e^{-m_1 y}) + \varepsilon e^{nt}(A_{11}e^{-m_4 y} + A_5 e^{-m_3 y} + A_7 e^{-m_2 y} + A_9 e^{-m_1 y}) \tag{31}$$

### 3.1 Skin Friction

Very important physical parameter at the boundary is the skin friction which is given in the non-dimensional form and derived a

$$\tau = \frac{\tau_W'}{\rho V_0^2} = \left.\frac{\partial u}{\partial y}\right]_{y=0}$$
$$= -(m_5 A_{16} + m_3 A_{12} + m_1 A_{15}) - E(m_6 A_{20} + m_5 A_{17} + m_3 A_{18} + m_1 A_{19})$$
$$-\varepsilon e^{nt}\begin{bmatrix}(m_7 A_{33} + m_5 A_{27} + m_4 A_{21} + m_3 A_{30} + m_2 A_{31} + m_1 A_{32}) \\ +E(m_8 A_{59} + m_7 A_{53} + m_6 A_{37} + m_5 A_{54} + m_4 A_{55} + m_3 A_{56} + m_2 A_{57} + m_1 A_{58})\end{bmatrix} \tag{32}$$

### 3.2 Rate of Heat Transfer

Physical parameter like rate of heat and mass transfer in the form of Nusselt number derived is given below.

$$Nu = X\frac{\left.\frac{\partial T'}{\partial y'}\right|_{y'=0}}{T_W' - T_\infty'} = Nu\,\mathrm{Re}_X^{-1} = -\left.\frac{\partial\theta}{\partial y}\right|_{y=0}$$
$$= \mathrm{Re}_x(m_3 A_4 + m_1 A_3) + \varepsilon e^{nt}(m_4 A_{11} + m_3 A_5 + m_2 A_7 + m_1 A_9) \tag{33}$$

Where $\mathrm{Re}_X$ is the Reynolds number.

### 3.3 Sherwood Number

Another physical parameter like rate of mass transfer in the form of Sherwood number derived is given below.

$$Sh = X\,\frac{\left.\dfrac{\partial C'}{\partial y'}\right|_{y'=0}}{C'_W - C'_\infty} = Sh\,\mathrm{Re}_X^{-1} = -\left.\frac{\partial C}{\partial y}\right|_{y=0}$$

$$= \mathrm{Re}_x\left[m_1 + \varepsilon\exp(nt)(m_1A_1 + m_2A_2)\right] \qquad (34)$$

## IV. RESULT AND DISCUSSION

In this section, the effects of various physical parameters such as Visco-elastic parameter (E), Grashof number (Gr), modified Grashof number (Gm), Magnetic parameter (M), Prandtl number (Pr), Heat absorption coefficient ($Q_1$), Heat source parameter (∅), Schmidt number (Sc), Chemical reaction coefficient ($\gamma$), Permeability parameter (K) are analyzed. The analytical results obtained in the previous section are studied numerically and the variations in velocity u(y, t), temperature $\theta$(y, t), concentration ∅(y, t) are discussed through graphs. Also the variation of Skin friction, Nusselt number and Sherwood number are discussed for various values of M, K, E, A, $\gamma$, Gr, Gm, Sc, $Q_1$, ∅.

**Fig. 1** describes the variations in velocity distribution with respect to the magnetic parameter M. This figure describes that the velocity decreases as usual with an increase in M.

**Fig. 2** and **Fig. 10** Describes the effect of Prandtl number Pr on the velocity and temperature distributions. This figure shows that the velocity and temperature decrease with an increase in Pr.

**Fig. 3** Display the effect of Grashof number Gr on velocity distribution. This figure shows that velocity increases with an increase on Gr.

**Fig. 4** depicts the effects of Magnetic parameter F on velocity distribution. It describes that velocity increases with an increase in F.

**Fig. 5** represents the effects of modified Grashof number Gm on velocity distribution. This figure shows that velocity increases with an increase Gm.

**Fig. 6** describes the effects of Casson parameter $\beta$ on velocity distribution. The figure shows that velocity increase with a decrease in $\beta$.

**Fig. 7** Display the effects of Heat source parameter ∅ on the velocity distribution. This figure describe that velocity increases with a decrease in∅.

**Fig. 8** and **Fig. 12** Display the effect of Sc on temperature and concentration distributions. It shows that temperature and concentration increase with a decrease in Sc.

**Fig. 9** and **Fig. 10** Illustrates the effects of $\gamma$ on the temperature and concentration distributions. It describes that the temperature and concentration increase with a decrease in $\gamma$.

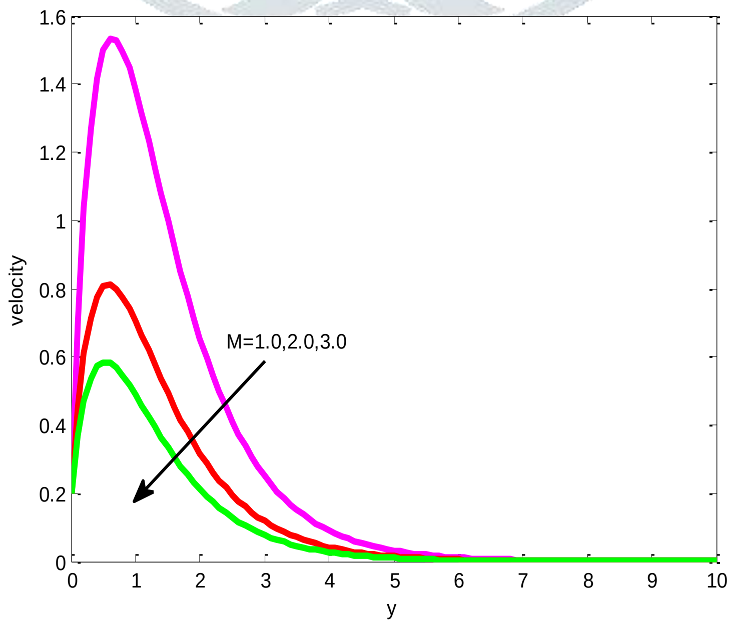


**Fig. 1** Effects of M on velocity profiles.

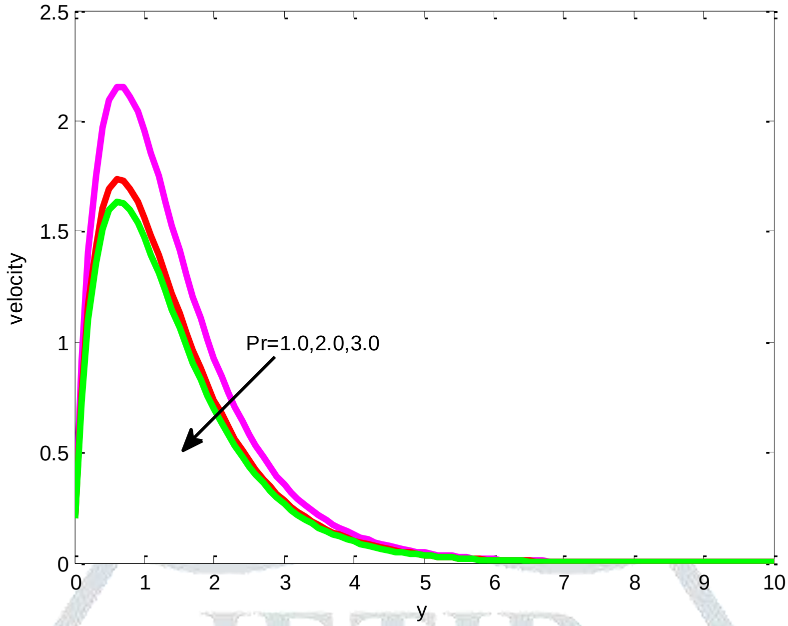


**Fig. 2** Effects of Pr on velocity profiles.

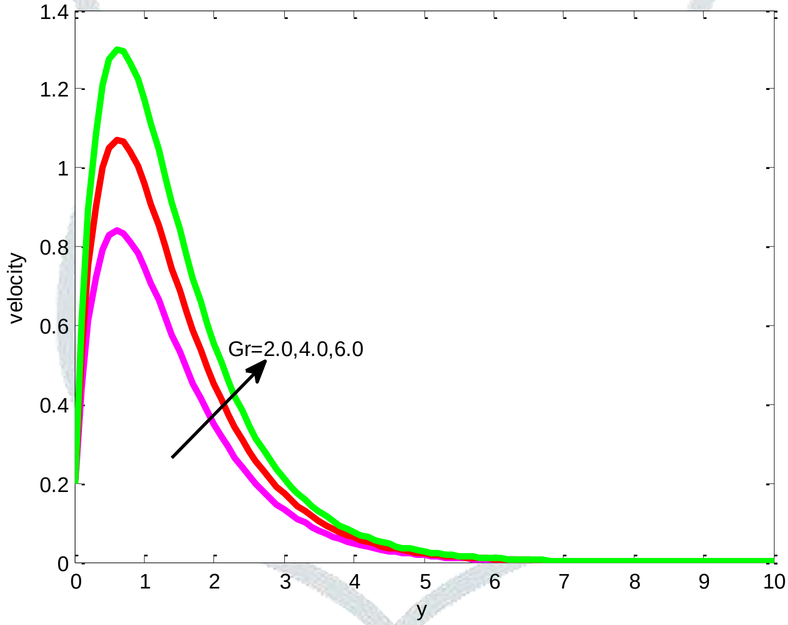


**Fig. 3** Effects of Gr on velocity profiles.

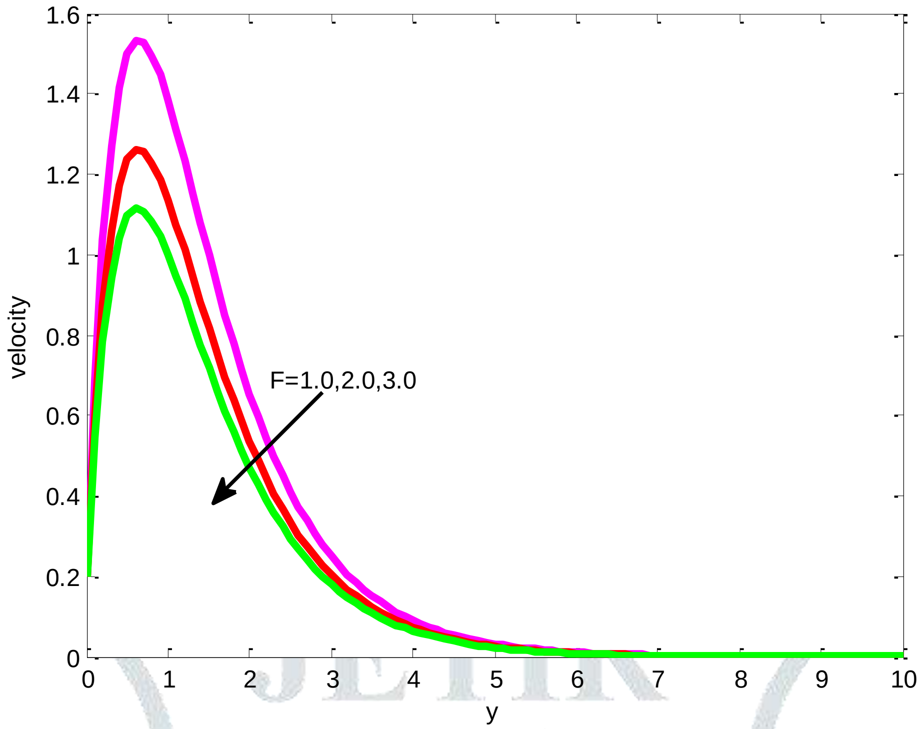


**Fig. 4** Effects of F on velocity profiles.

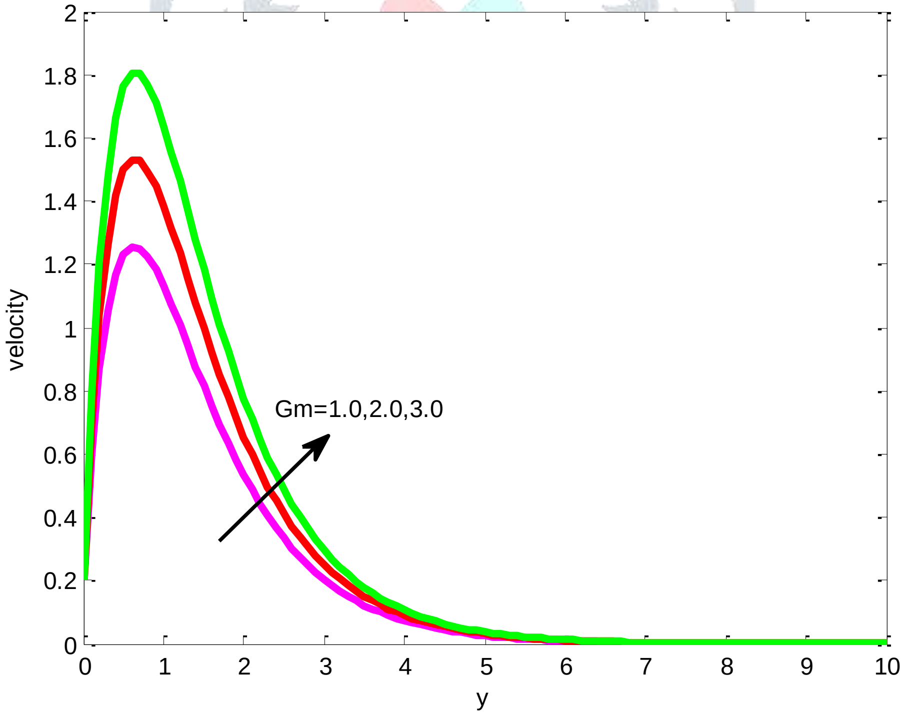


**Fig. 5** Effects of Gm on velocity profiles.

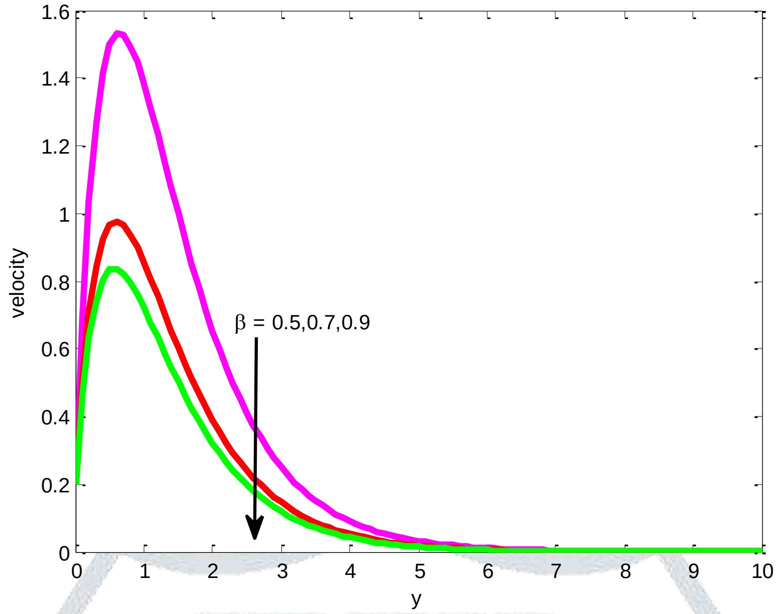


**Fig. 6** Effects of $\beta$ on velocity profiles.

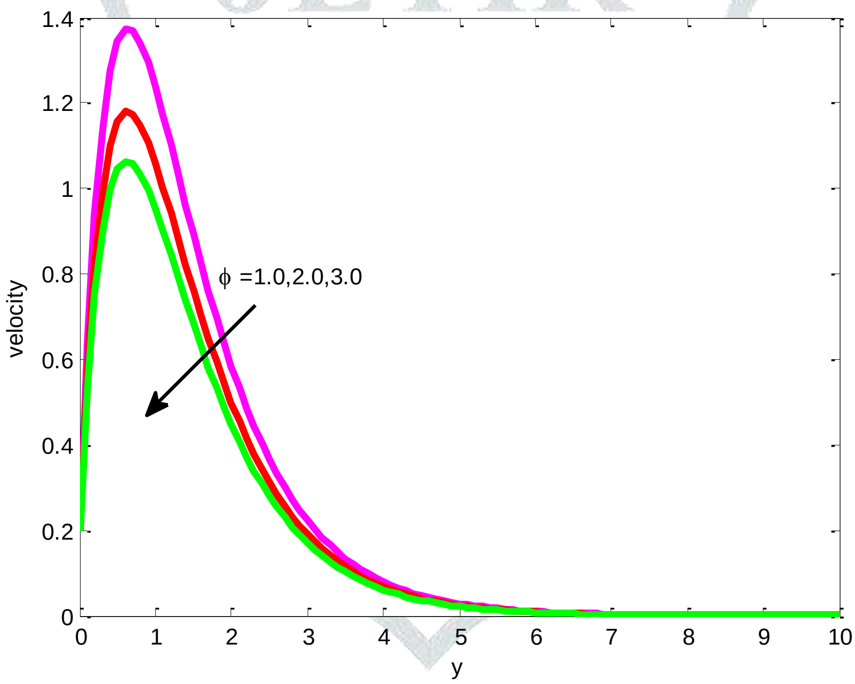


**Fig. 7** Effects of $\emptyset$ on velocity profiles.

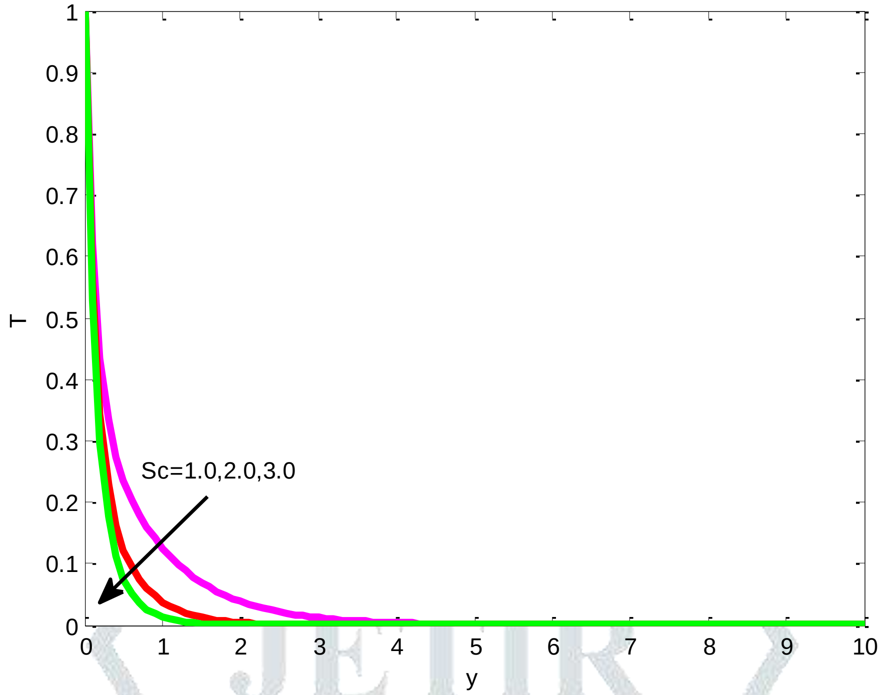


**Fig. 8** Effects of Sc on temperature profiles.

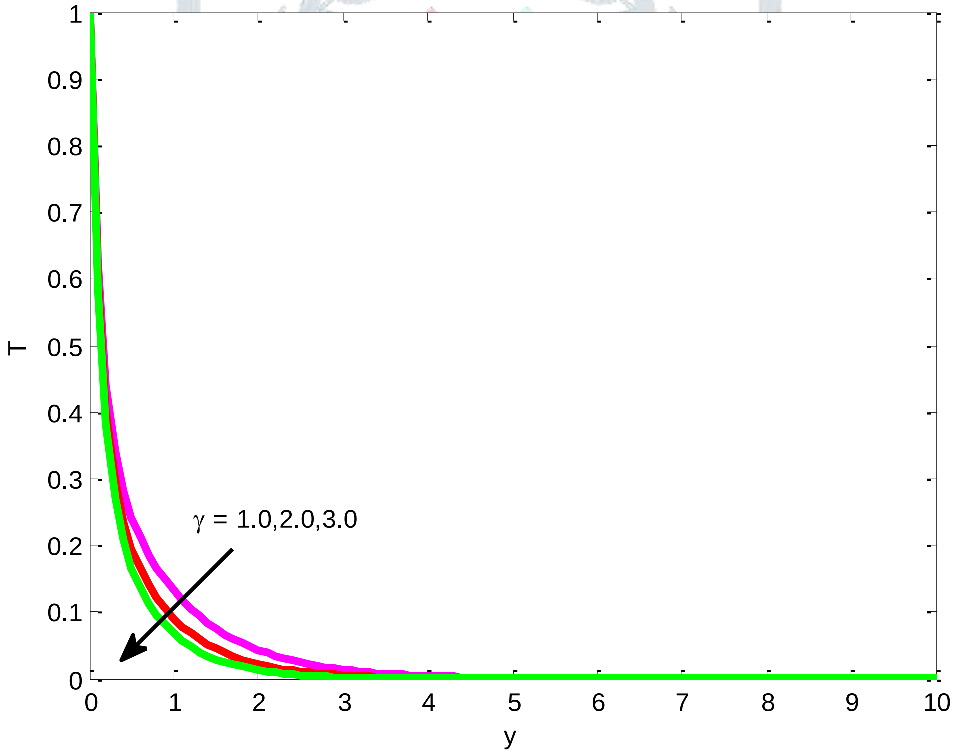


**Fig. 9** Effects of $\gamma$ on temperature profiles.

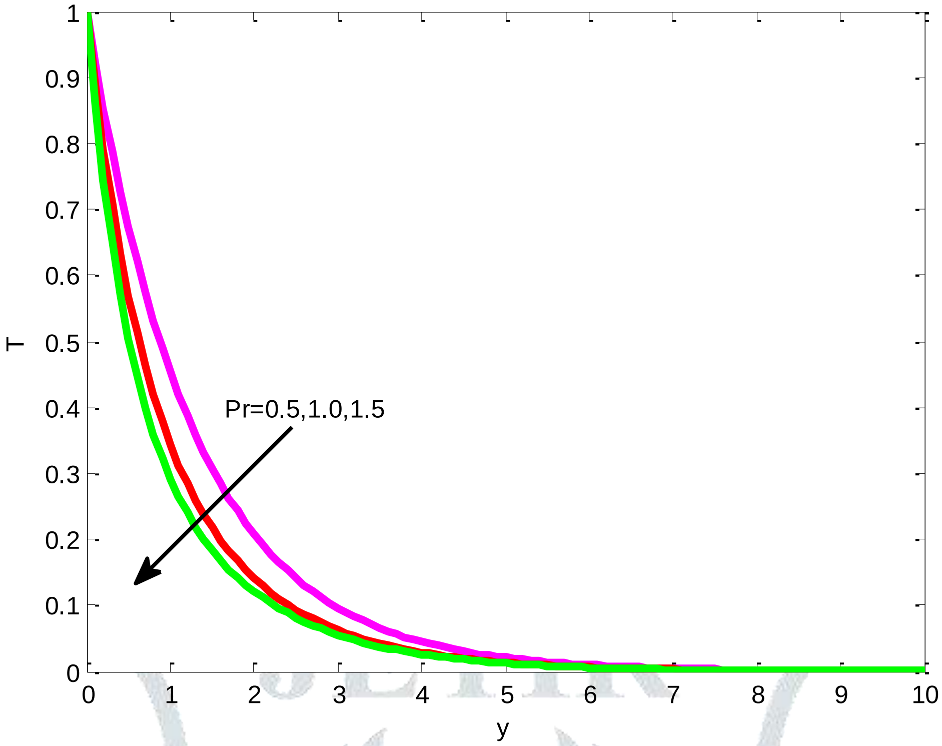


**Fig. 10** Effects of Pr on temperature profiles.

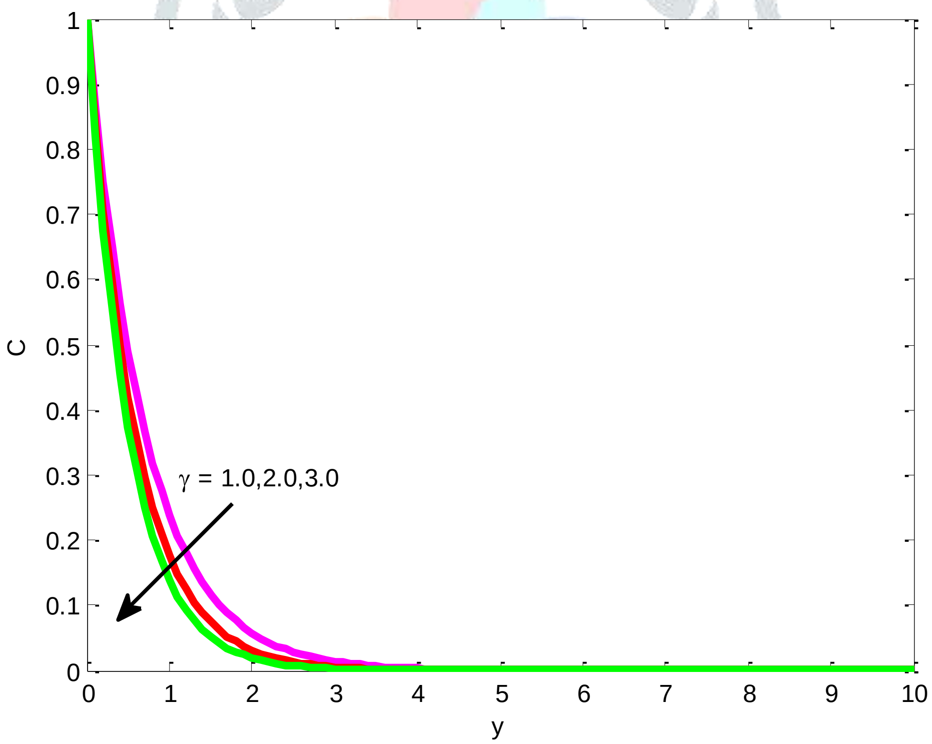


**Fig. 11** Effects of $\gamma$ on concentration profiles.

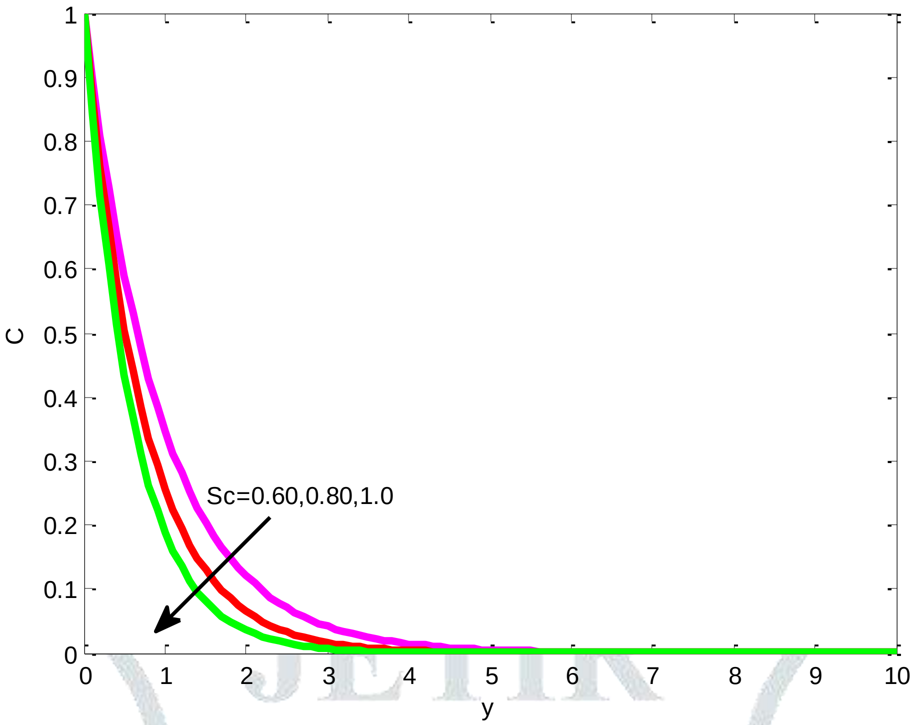


**Fig 12** Effects of Sc on concentration profiles.

## V. CONCLUSIONS

- Velocity increases for increasing values of Grashof number (Gr), Magnetic parameter (F), solutal Grashof number (Gm).
- Velocity increases for decreasing values of magnetic field parameter (M), Prandtl number (Pr), Casson parameter ($\beta$), Heat source parameter ($\emptyset$).
- Temperature increases for increasing decreases of Schmidt number (Sc), Chemical reaction parameter ($\gamma$), Pradtl number (Pr).
- Concentration increases for decreases the value of Chemical reaction parameter ($\gamma$), Schmidt number (Sc).

## NOMENCLATURE

$x'$ Dimensional distances
$y'$ Perpendicular to the plate
$t'$ Dimensional time
$u'$ Components of dimensional velocity $x'$
$v'$ Components of dimensional velocity $y'$
$T'$ Dimensional temperature
$C'$ Dimensional concentration
$C_w$ Concentration at the wall
$T_w$ Temperature at the wall
$C_\infty$ Free stream dimensional concentration
$T_\infty$ Free stream dimensional temperature
$\rho$ Fluid density
$\nu$ Kinematic viscosity

$C_P$ Specific heat at constant pressure
$\sigma$ Fluid electrical conductivity
$B_0$ Magnetic induction
$k_0$ Visco-elastic parameter
$k'$ Permeability of the porous medium
$Q'_0$ Dimensional heat absorption coefficient
$Q'_1$ Coefficient of proportionality for the absorption of radiation
$K$ Thermal conductivity parameter
$D$ Mass diffusivity
$\gamma$ Chemical reaction parameter
$g$ Gravitational acceleration
$\beta$ Casson parameter
$\emptyset$ Heat source of parameter
$\beta_T$ Thermal expansion coefficient
$\beta_C$ Concentration expansion coefficient
$K_1$ Chemical reaction coefficient
F Magnetic parameter
M Magnetic field parameter

**APPENDIX**

$$m_1 = \frac{1}{2}\left(Sc + \sqrt{Sc^2 + 4Sc\gamma}\right), \qquad m_2 = \frac{1}{2}\left(Sc + \sqrt{Sc^2 + 4Sc(\gamma + n)}\right)$$

$$m_3 = \frac{1}{2}\left(\Pr + \sqrt{\Pr^2 + 4\Pr(\phi + F)}\right), \quad m_4 = \frac{1}{2}\left(\Pr + \sqrt{\Pr^2 + 4\Pr(\phi + F + n)}\right),$$

$$m_5 = \frac{1}{2B}\left(1 + \sqrt{1 + 4BS}\right), \quad m_6 = \frac{1}{2B}\left(1 + \sqrt{1 + 4BS}\right)$$

$$m_7 = \frac{1}{2B}\left(1 + \sqrt{1 + 4B\left(M + \frac{1}{K} + n\right)}\right), \qquad m_8 = \frac{1}{2(B-n)}\left(1 + \sqrt{4(B-n)\left(M + \frac{1}{K} + n\right)}\right)$$

$$A_1 = \frac{AScm_1}{m_1^2 - Scm_1 - Sc(\gamma + n)}, \quad A_2 = 1 - A_1, \quad A_3 = \frac{-Q_1 \Pr}{m_1^2 - \Pr m_1 - \Pr(\phi + F)}, \quad A_4 = 1 - A_3$$

$$A_5 = \frac{A \Pr A_4 m_3}{m_3^2 - \Pr m_3 - \Pr(\phi + F + n)}, \quad A_6 = \frac{A \Pr A_3 m_1}{m_1^2 - \Pr m_1 - \Pr(\phi + F + n)}$$

$$A_7 = \frac{-Q_1 \Pr A_2}{m_2^2 - \Pr m_2 - \Pr(\phi + F + n)}, \quad A_8 = \frac{-Q_1 \Pr A_1}{m_1^2 - \Pr m_1 - \Pr(\phi + F + n)}$$

$$A_9 = A_6 + A_8, \quad A_{10} = A_5 + A_7 + A_9, \quad A_{11} = 1 - A_{10},$$

$$A_{12} = \frac{-GrA_4}{Bm_3^2 - m_3 - \left(M + \frac{1}{K}\right)}, \qquad A_{13} = \frac{-GrA_3}{Bm_1^2 - m_1 - \left(M + \frac{1}{K}\right)}$$

$$A_{14} = \frac{-Gm}{Bm_1^2 - m_1 - \left(M + \frac{1}{K}\right)}, \qquad A_{15} = A_{13} + A_{14}, \qquad A_{16} = u_p - (A_{12} + A_{15})$$

$$A_{17} = \frac{A_{16}(m_5)^3}{Bm_5^2 - m_5 - \left(M + \frac{1}{K}\right)}, \qquad A_{18} = \frac{A_{12}(m_3)^3}{Bm_3^2 - m_3 - \left(M + \frac{1}{K}\right)},$$

$$A_{19} = \frac{A_{15}(m_1)^3}{Bm_1^2 - m_1 - \left(M + \frac{1}{K}\right)}, \qquad A_{20} = -(A_{17} + A_{18} + A_{19}),$$

$$A_{21} = \frac{-GrA_{11}}{Bm_4^2 - m_4 - \left(M + \frac{1}{K} + n\right)}, \quad A_{22} = \frac{-GrA_5}{Bm_3^2 - m_3 - \left(M + \frac{1}{K} + n\right)},$$

$$A_{23} = \frac{-GrA_7}{Bm_2^2 - m_2 - \left(M + \frac{1}{K} + n\right)}, \quad A_{24} = \frac{-GrA_9}{Bm_1^2 - m_1 - \left(M + \frac{1}{K} + n\right)},$$

$$A_{25} = \frac{-GmA_2}{m_2^2 - m_2 - \left(M + \frac{1}{K} + n\right)}, \quad A_{26} = \frac{-GmA_1}{Bm_1^2 - m_1 - \left(M + \frac{1}{K} + n\right)},$$

$$A_{27} = \frac{AA_{16}m_5}{Bm_5^2 - m_5 - \left(M + \frac{1}{K} + n\right)}, \quad A_{28} = \frac{AA_{12}m_3}{Bm_3^2 - m_3 - \left(M + \frac{1}{K} + n\right)},$$

$$A_{29} = \frac{AA_{15}m_1}{Bm_1^2 - m_1 - \left(M + \frac{1}{K} + n\right)}, \quad A_{30} = A_{22} + A_{28},$$

$$A_{31} = A_{23} + A_{25}, \qquad A_{32} = A_{24} + A_{26} + A_{29}, \quad A_{33} = -(A_{27} + A_{21} + A_{30} + A_{31} + A_{32})$$

$$A_{34} = \frac{AA_{16}(m_5)^3}{m_5^2 - m_5 - \left(M + \frac{1}{K} + n\right)}, \quad A_{35} = \frac{AA_{12}(m_3)^3}{m_3^2 - m_3 - \left(M + \frac{1}{K} + n\right)},$$

$$A_{36} = \frac{AA_{15}(m_1)^3}{m_1^2 - m_1 - \left(M + \frac{1}{K} + n\right)}, \quad A_{37} = \frac{AA_{20}m_6}{m_6^2 - m_6 - \left(M + \frac{1}{K} + n\right)}$$

$$A_{38} = \frac{AA_{17}m_5}{m_5^2 - m_5 - \left(M + \frac{1}{K} + n\right)}, \quad A_{39} = \frac{AA_{18}m_3}{m_3^2 - m_3 - \left(M + \frac{1}{K} + n\right)},$$

$$A_{40} = \frac{AA_{19}m_1}{m_1^2 - m_1 - \left(M + \frac{1}{K} + n\right)}, \quad A_{41} = \frac{A_{33}(m_7)^3}{m_7^2 - m_7 - \left(M + \frac{1}{K} + n\right)}$$

$$A_{42} = \frac{A_{27}(m_5)^3}{m_5^2 - m_5 - \left(M + \frac{1}{K} + n\right)}, \quad A_{43} = \frac{A_{21}(m_4)^3}{m_4^2 - m_4 - \left(M + \frac{1}{K} + n\right)}$$

$$A_{44} = \frac{A_{30}(m_3)^3}{m_3^2 - m_3 - \left(M + \frac{1}{K} + n\right)}, \quad A_{45} = \frac{A_{31}(m_2)^3}{m_2^2 - m_2 - \left(M + \frac{1}{K} + n\right)}$$

$$A_{46} = \frac{A_{32}(m_1)^3}{m_1^2 - m_1 - \left(M + \frac{1}{K} + n\right)}, \quad A_{47} = \frac{nA_{33}(m_7)^2}{m_7^2 - m_7\left(M + \frac{1}{K} + n\right)}$$

$$A_{48} = \frac{nA_{27}(m_5)^2}{m_5^2 - m_5 - \left(M + \frac{1}{K} + n\right)}, \quad A_{49} = \frac{nA_{21}(m_4)^2}{m_4^2 - m_4 - \left(M + \frac{1}{K} + n\right)}$$

$$A_{50} = \frac{nA_{30}(m_3)^2}{m_3^{\,2} - m_3 - \left(M + \frac{1}{K} + n\right)}, \quad A_{51} = \frac{nA_{31}(m_2)^2}{m_2^{\,2} - m_2 - \left(M + \frac{1}{K} + n\right)}$$

$$A_{52} = \frac{nA_{32}(m_1)^2}{m_1^{\,2} - m_1 - \left(M + \frac{1}{K} + n\right)}, \quad A_{53} = A_{41} + A_{47}$$

$$A_{54} = A_{34} + A_{38} + A_{42} + A_{48}, \; A_{55} = A_{43} + A_{49}, \quad A_{56} = A_{35} + A_{39} + A_{44} + A_{50}$$

$$A_{57} = A_{45} + A_{51}, \quad A_{58} = A_{36} + A_{40} + A_{46} + A_{52}, \; A_{59} = -(A_{53} + A_{37} + A_{54} + A_{55} + A_{56} + A_{57} + A_{58})$$

## VI. REFERENCES


**[1]** Srinivasa Raju, R. Jithender Reddy, G. Anand Rao, J. rashidi, M.M and Rama Subba Reddy Gorla. 2016. Analytical and Numerical study of Unsteady MHD free convective flow over an exponentially moving vertical plate with Heat absorption. International Journal of Thermal Science, 107:303-315.

**[2]** Satya Narayana, P.V. Venkateswarlu, B. and Devika, B. 2016. Chemical reaction and heat source effects on MHD oscillatory flow in an irregular channel. Ain shams Engineering Journal, 7: 1079-1088.

**[3]** Yigit Aksoy. 2016. Effects of couple stresses on the heat transfer and entropy generation rates of a flow between parallel plates with constant heat flux. International Journal of Thermal Science, 107: 1-12.

**[4]** Raju, C.S.K. Sandeep, N. Sugunamma, V. Jayachandra Babu, M. and Ramana Reddy, J.V. 2016. Heat and mass transfer in magneto-hydrodynamic casson fluid over an exponentially permeable stretching surface. Engineering science and Technology, an International Journal, 19: 45-52.

**[5]** Manojkumar Nayak, Gauranga charan Dash, and Lambodar prased Singh. 2016. Heat and mass transfer effects on MHD viscoelastic fluid over a stretching sheet through porous medium in presence of chemical reaction. Propulsion and Power Research, 5(1): 70-80.

**[6]** Prashant G.metri, Pushpajali G.metri, subhas Abel, and Sergei silvestrov. 2016. Heat transfer in MHD mixed convection viscoelastic fluid flow over a stretching sheet embedded in a porous medium with viscous dissipation and non-uniform heat source/sink. Procedia Engineering, 157: 309-316.

**[7]** Jayachandra Babu, M. and Sandeep, N. 2016. MHD non-Newtonian fluid flow over a slandering stretching sheet in the presence of Casson diffusion effects. Alexandria Engineering Journal, 55: 2193-2201.

**[8]** Dual Pal and Sukanta Biswas. 2016. Perturbation analysis of magneto-hydrodynamics oscillatory flow on convective radiation heat and mass transfer of micro polar fluid in a porous medium with chemical reaction. Engineering science and Technology, an International Journal, 19: 444-462.

**[9]** Raju, C.S.K. and Sandeep, N. 2016. Unsteady three-dimensional flow of casson-carreau fluids past a stretching surface. Alexandria Engineering Journal, 55: 115-1126.

**[10]** Joaquin Zeuco, Sahin Ahmed and Luis Maria Lopez-Gonzelez. 2017. 2D unsteady free convective heat and mass transfer Newtonian Hartmann flow with thermal diffusion and Soret effects. Network model and finite differences. International Journal of Heat and Mass Transfer, 110:467-475.

**[11]** Meraj Mustafa. 2017. An analytical treatment for MHD mixed convection boundary layer flow of Oldroyd -B fluid utilizing non-Fourier heat flux model. International Journal of Heat and Mass Transfer, 113:1012-1020.

**[12]** Bhuzvana Vijaya, R. Haritha, B. and Prasada rao, D.R.V. 2017. Combined influence of thermal radiation, Soret, Duffer effects on non-Darcy mixed convective heat and mass transfer flow with dissipation in a vertical channel. International Journal of Mathematical Archive, 8(1):161-189.

**[13]** Nadem Ahmed Sheikh, Farhad Ali, Muhammad Saqib, ilayas khan, Syed Aftab Alam Jan, Ali Saleh Alshomrani, and Metib said Alghamdi, 2017. Comparison and analysis of the Atangana-Baleanu and Caputo-Febrizio fractional derivatives for generation and chemical reaction. Result in Physics, 7:789-800.

**[14]** Kumaran, G. Sandeep, N. and Ali, M.E. 2017. Computational analysis of magneto-hydrodynamic casson and Maxwell flows over a stretching sheet with cross diffusion. Result in Physics, 7:147-155.

**[15]** Sahin Ahmed, Joaquin Zueco and Luis M. Lopez-Gonzalez, 2017. Effects of chemical reaction, heat and mass transfer and viscous dissipation over a MHD flow in a vertical porous wall using perturbation method. International Journal of Heat and Mass Transfer, 104: 409 – 418.

**[16]** Us Rajput and Gaurav Kumar, 2017. Effects of Hass current and chemical reaction on MHD flow through porous medium past on oscillating inclined plate with variable temperature and mass diffusion. European Journal of Advances in Engineering and technology, 4(1): 56-63.

**[17]** Hari Krishna, y. Ramana Murthy, mv. Bhikshu NL and Venkata Ramana, G. 2017. Effects of radiation and chemical reaction on MHD flow past an oscillating Inclined porous plate with variable temperature and mass diffusion. international journal of chemical sciences, 15: 1853.

**[18]** Imran ullah, Sharidan shafe, Ilyas khan, 2017. Effects of slip condition and Newtonian heating on MHD flow of casson fluid over a nonlinearly stretching sheet saturated on a porous medium. Journal of King Saud university – science, 29: 250-259.
**[19]** Parida, S.k. and Rout, B.R. 2017. Free convective flow through porous medium with variable permeability in slip flow regime with couple stress in the presence of heat source. Journal of Engineering Thermo physics, 26(1): 125-138.
**[20]** Ibrahim, S.M. lorenzini, G. Vijaya kumar, P. and Raju, C.S.K. 2017. Influence of chemical reaction and heat source on dissipative MHD mixed convection flow of a casson Nano fluid over a nonlinear permeable stretching sheet. International Journal of heat and Mass Transfer, 3: 346-355.
**[21]** Sunita ran, Y. 2017. Jeffrey fluid performance on MHD convective flow past a semi-infinite vertically inclined permeable moving plate in presence of heat and mass transfer a finite difference technique. International Journal of Dynamics of fluids, ISSN 0973-1784. 13(2): 173-195.
**[22]** Kartini Ahamad, and Anclar Ishak, 2017. Magneto hydrodynamic (MHD) Jeffrey fluid over a stretching vertical surface n a porous medium. Propulsion and Power Research, 6(4): 269-276.
**[23]** Wubshet Ibrahim, 2017. Magneto hydrodynamic (MHD) boundary layer stagnation point flow and heat transfer of a Nano fluid past a stretching sheet with melting. Propulsion and Power Research, 6(3): 214-22.
**[24]** Das, S.S. saran, M.R. and Pradhan, B. 2017. MHD convective mass transfer flow of a polar fluid past a semi-infinite vertical porous flat moving plate embedded in a porous medium. International Journal of pure and Applied Physics, ISSN 0973-1776, 13(3):387- 402.
**[25]** Srinivas Reddy, C. and Krishna naikoti mohammad Mehdi Rashidi. 2017. MHD flow and heat transfer characteristics of Williamson Nano fluid over a stretching sheet with variable thickness and variable thermal conductivity. Transaction of A. Razmadze mathematical institute, 171: 195-2011.
**[26]** Bala Anki Reddy, P. Suneetha, S. and Bhaskar Reddy, N. 2017. Numerical studies of magneto-hydrodynamic (MHD) boundary layer slip flow of a Maxwell Nano-fluid over an exponentially stretching surface with convective boundary condition. Propulsion and Power Research, 6(4):259-268.
**[27]** Mohamed Abd EL-Aziz and Aishah S.Yahya. 2017. Perturbation analysis of unsteady layer slip flow and heat transfer of casson fluid past a vertical permeable plate with hall current. Applied Mathematics and computation, 307:146-164.
**[28]** Sharif Uddin, M.D. and Krishnendu Bhattacharya. 2017. Thermal boundary layer in stagnation point flow past a permeable shrinking sheet with variable surface temperature. Propulsion and Power Research, 6(3): 186-194.
**[29]** Ramesh, G.K. Prasannakumar, B.C. Gireesha, B.J. Shehzed, S.A. and Abbasi, F.M. 2017. Three dimensional flow of Maxwell fluid with suspended nanoparticles past a bidirectional porous stretching surface with thermal radiation. Thermal Science and Engineering progress, 1:6-14.
**[30]** Macha Madhu, Naikoti Krishna, Ali J.Chamkha. 2017. Unsteady flow of a Maxwell Nano-fluid over a stretching surface in the presence of magneto-hydrodynamic and thermal radiation effects. Propulsion and Power Research, 6(1):31-40.
**[31]** Simit Gupta, Devendra Kumar, Jagdev Singh, 2018.MHD mixed convective stagnation point flow and heat and mass transfer of an incompressible Nano fluid over an inclined stretching sheet with chemical reaction. International Journal of Heat and Mass Transfer, 118: 378-387.