HEAT AND MASS TRANSFER ON INCLINED MAGNETIC FIELD MHD POLAR FLUID FLOW OVER AN INFINITE HORIZONTAL PLATE WITH DUFOUR EFFECT AND HEAT SOURCE/SINK

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Abstract: In this article, we investigate the unsteady free convective MHD flow a polar fluid over an infinite horizontal plate in heat source/sink with Dufour effect. The dimensional equations are transformed into a nondimensional equations are solved by regular perturbation method. The results are presented graphically to illustrate the influence of different physical parameters on the velocity, angular velocity, temperature and concentration profile are discussed.

Index Terms: MHD, Polar fluid, Dufour effect, Heat source, Convection.

I. INTRODUCTION

In recent years, the heat and mass transfer problems with chemical reaction are of importance in many practical processes such as distribution of temperature and moisture over agricultural field, energy transfer in a wet cooling tower, in the method of generating and extracting power from a moving fluid. So, many researchers have taken interest in studying the above-mentioned effects.

Chamkha [1] presented two-dimensional, laminar, boundary-layer flow of a viscous, incompressible, electrically conducting and heat-absorbing fluid along a semi-infinite vertical permeable moving plate in the presence of a uniform transverse magnetic field and thermal and concentration buoyancy effects is considered, solved analytically using two-term harmonic and nonharmonic functions. Hiranmoy Mondal et al. [2] investigated thermophoresis and Soret-Dufour on MHD mixed convective heat and mass transfer of a semi-infinite permeable inclined flat plate in the presence of Non uniform Heat source/sink and chemical reaction, solved using a numerical technique with appropriate boundary conditions for various physical parameters. Sreedevi et al. [3] analyzed effects of the magnetic field, Joule heating, thermal radiation absorption, viscous dissipation, Buoyancy forces, thermal-diffusion and diffusion thermion the convective heat and mass transfer flow of an electrically conducting fluid over a permeable vertically stretching sheet, using appropriate similarity transformation. Ibrahim et al. [4] presented slip and convective boundary condition is examined In this analysis, various effects such as velocity ratio, viscous dissipation, heat generation/absorption and chemical reaction are accentuated, using a homotopy analysis method (HAM). Shateyi et al. [5] presented free convection of combined heat and mass transfer toward an unsteady permeable stretching sheet with thermal radiation, viscous dissipation and chemical reaction, solved by an efficient Runge-Kutta-Fehlberg method. Raptis et al. [6] studied two-dimensional free convection and mass transfer flow, of an incompressible viscous fluid through a porous medium bounded by a vertical infinite limiting surface (plane wall) is considered by the temperature differences. Nadeem Ahmad Sheikh et al. [7] analyzed MHD flow of micro-polar fluid past an oscillating infinite vertical plate embedded in porous media, used in the mathematical formulation of a micro-polar fluid. Dulal Pal et al. [8] analyzed heat and mass transfer of an oscillatory viscous electrically conducting micro-polar fluid over an infinite moving permeable plate embedded in a saturated porous medium in the presence of transverse magnetic field, used to describe the radiative heat flux.

Convective fluids in the presence of a heat source within a porous medium are of great practical importance in geophysics and energy-related problem such as recovery of petroleum resources, cooling of underground electric cable, ground water pollution, fiber and granular insulation, chemical catalytic reactors and solidification of casting. Keeping the eye on the above applications authors, Pandit et al. [9] investigated effects of a chemical reaction and thermal radiation on unsteady MHD free convection heat and mass transfer flow of an electrically conducting, by the Laplace Transform technique. Ashish Paul [10] analyzed one-dimensional unsteady laminar boundary layer MHD flow of a viscous incompressible fluid past an exponentially accelerated infinite vertical plate in presence of transverse magnetic field, solved by Laplace transform technique. Adamu Gizachew Chanie et al. [11] investigated effects on MHD flow of a Casson fluid through a porous media due to a stretching surface with chemical reaction and suction, using Optimal Homotopy Asymptotic method (OHAM). Paras Ram et al. [12] presented free convective flow past a moving vertical porous plate in the presence of thermal radiation and first order chemical reaction with viscous dissipation, ordinary differential equations by using similarity transformation. Ramana Reddy et al. [13] studied frictional and irregular heat on MHD non-Newtonian fluid (Casson and Maxwell) flows due to stretched surface, solved using bvp5c MATLAB package. Sambath et al. [14] presented thermal radiation in laminar natural convective hydro-magnetic flow of viscous incompressible fluid past a vertical cone with mass transfer under the influence of chemical reaction with heat source/sink, solved by using well known Thomas algorithm. Praveena et al. [15] presented two dimensional free convective flow of a viscous electrically conducting micro-polar fluid through a highly porous medium over a semi-infinite moving vertical porous plate, using influence of uniform magnetic field. Babu Reddy et al. [16] investigated unsteady mixed convection and mass transfer flow past an infinite vertical porous plate embedded in a porous medium in the presence of radiation absorption, used for the momentum equation (porous media).

Electrically conducting polar fluid through a porous medium in the presence of magnetic field has attracted the researchers for its wide application in many engineering problems such as MHD generator, plasma studies, nuclear reactors, oil exploration, geothermal energy extraction and the boundary layer control in the field of aerodynamics. So, Gnaneswara Reddy et al. [17] analyzed heat transfer analysis on Carreau hydro magnetic fluid past a convectively nonlinear stretching surface, solved by numerically method. Suman Agarwalla et al. [18] presented unsteady MHD free convective mass transfer flow past an infinite inclined plate embedded in a saturated porous medium with variable plate velocity, temperature, and mass diffusion, solved by employing the Laplace transform technique. Singh et al. [19] studied unsteady MHD boundary layer flow of a rotating Walters'-B fluid (viscoelastic fluid) over an infinite vertical porous plate embedded in a uniform porous medium with fluctuating wall temperature and concentration taking Hal and ion-slip effects into consideration is discussed, solved analytically by using regular perturbation and variable separable methods. Pandya et al. [20] investigated effects of Soret-Dufour, radiation and chemical reaction on an unsteady MHD (Magneto hydro Dynamics) flow of an incompressible viscous and electrically conducting dusty fluid past a continuously moving inclined plate, solved numerically using Crank-Nicolson implicit finite difference method. Swapna et al. [21] investigated effect of injection/suction on the free convective flow through the porous medium bounded by two infinite vertical plates with chemical reaction, by considering the pressure gradient oscillating periodically. Yahaya Shagaiya Daniel et al. [22] presented effects of suction as well as thermal radiation, chemical reaction, viscous dissipation and Joule heating on a two-dimensional natural convective flow of unsteady electrical magneto hydro dynamics (MHD) Nano-fluid over a linearly permeable stretching sheet, solved by the Implicit finite difference. Beg et al. [23] studied heat and mass transfer characteristics of natural convection flow of a chemically-reacting Newtonian fluid along vertical and inclined plates in the presence of diffusionthermo (Dufour) and thermal-diffusion (Soret) effects, solved using the local non-similarity method. Vedavathi et al. [24] presented effects of heat and mass transfer on two-dimensional unsteady MHD free convection flow past a vertical porous plate in a porous medium in the presence of thermal radiation under the influence of Dufour and Soret effects, solved numerically using shooting method.

Radiating heat transfer in micro-polar fluid has also practical importance in the field of nuclear power plants, propulsion devices for aircraft, missiles, satellites and space vehicles. In space technology the convection coefficient is very small, hence, the radiation effect plays the leading role in heat transfer, so in this regards many authors have studied it. Umamaheswar et al. [25] investigated unsteady MHD free convection flow of a well-known non-Newtonian visco elastic second order Rivlin-Erickson fluid past an impulsively started semi-infinite vertical plate in the presence of homogeneous chemical reaction, solved numerically by using finite difference method. A.Sinha et al. [26] presented natural convective flow of an optically thin viscous incompressible electrically conducting fluid past a vertical plate in a porous medium with ramped wall temperature is obtained in presence of appreciable thermal radiation, solved analytically by adopting Laplace transform technique in closed form. Sarma et al. [27] investigated rotation and Soret effects on a unsteady MHD free convection heat and mass transfer flow of a viscous, incompressible and electrically conducting fluid past an infinite vertical plate embedded in a porous medium, by Laplace transform technique. Jyotsna Rani Pattnaik et al. [28] analyzed unsteady MHD free convection flow, heat and mass transfer past an exponentially accelerated inclined plate embedded in a saturated porous medium with uniform permeability, variable temperature and concentration, The Laplace transformation method has been used to solve the governing equations. Dastagiri Babu et al. [29] presented the unsteady MHD free convection flow of an incompressible electrically conducting fluid through porous medium bounded by an infinite vertical porous surface in the presence of heat source and chemical reaction in a rotating system, by Brinkman's model for the momentum equation. Ramesh Babu et al. [30] studied the effect unsteady MHD free convective flow of a viscoelastic incompressible electrically conducting fluid past a moving vertical plate through a porous medium with time dependent oscillatory permeability and suction in presence of a uniform transverse magnetic field and heat source and chemical reaction along with heat and mass transfer are reported, solved by applying multi parameter perturbation technique.

In the present analysis, the unsteady free convective MHD flow a polar fluid over an infinite horizontal plate in heat source/sink with Dufour effect. The dimensional equations are transformed into a nondimensional equations are solved by regular perturbation method. The results are presented graphically to illustrate the influence of different physical parameters on the velocity, angular velocity, temperature and concentration profile are discussed.

II. MATHEMATICAL FORMULATION

Consider the unsteady free convective MHD flow a polar fluid over an infinite horizontal plate in heat source/sink with Dufour effect. The *x* axis is taken along the plate and the *y* axis is perpendicular to it. A uniform magnetic field is applied in the direction normal to the *x* axis. Viscous and Darcy's resistance terms are taken into account with a variable permeability porous medium. The magnetic Reynolds number is assumed to be small enough so that the induced magnetic field can be neglected. The Hall effect, Joule heating, and viscous dissipation are all neglected in this study. The fluid is considered to be a gray, absorbing-emitting but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux. It is also assumed that there is no applied voltage, which implies the absence of an electric field. To allow the heat generation effect, we place a heat source in the flow. The governing equations for the flow under these assumptions are as follows:

$$\frac{\partial v}{\partial y} = 0,\tag{1}$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + (v + v_r) \frac{\partial^2 u}{\partial y^2} + 2v_r \frac{\partial \omega}{\partial y} + g\beta(T - T_{\infty})\cos\alpha + g\beta^*(C - C_{\infty})\cos\alpha - \frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$\frac{\partial \omega}{\partial t} + v \frac{\partial \omega}{\partial y} = \frac{\gamma}{I} \frac{\partial^2 \omega}{\partial y^2},$$
(3)
$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{\lambda}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho C_p} (T - T_{\infty}),$$
(4)
$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_c (C - C_{\infty}) + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2},$$
(5)

Here $\gamma = \frac{C_a + C_b}{I}$, where C_a, C_b are coefficients of couple stress viscosities and is a scalar constant of dimension

equal to that of the moment of inertia of unit mass

The boundary conditions for the problems are:

$$u = L_1 \frac{\partial u}{\partial y}, \quad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad T = T_{\omega}, \quad C = C_{\omega} \quad \text{at} \quad y = 0,$$

$$u = U(t) = U_0 (1 + \varepsilon e^{-nt}), \quad \omega \to 0, \quad T \to T_{\omega}, \quad C \to C_{\omega}, \quad \text{at} \quad y \to \infty,$$
 (6)

Where

$$L_1 = \left(\frac{2 - m_1}{m_1}\right)L,\tag{7}$$

L is the mean free path and the M_1 Maxwell's reflection coefficient. Outside the boundary layer, Eq. (2) gives

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial U_{\infty}}{\partial t} + \frac{v}{k}U_{\infty} + \frac{\sigma}{\rho}B_0^{2}U_{\infty},$$

Introducing the following non-dimensional parameters:

$$u^{*} = \frac{u}{U}, \ y^{*} = \frac{v_{0}y}{v}, \ t^{*} = \frac{tv_{0}^{2}}{v}, \ n^{*} = \frac{vn}{v_{0}^{2}}, \ \alpha = \frac{v_{r}}{v}, \ \beta = \frac{Iv}{\gamma}, \ M^{2} = \frac{\sigma B_{0}^{2}v}{\rho v_{0}^{2}}, \ \theta = \frac{T - T_{\infty}}{T_{\omega} - T_{\infty}}, \ \varphi = \frac{C - C_{\infty}}{C_{\omega} - C_{\infty}}, \ P_{r} = \frac{\rho v_{0}c_{p}}{\lambda}, \ h_{1} = \frac{L_{1}v_{0}}{v}, \ \omega^{*} = \frac{v\omega}{v_{0}U_{0}}, \ K_{0}^{*} = \frac{v_{0}^{2}k_{0}}{v^{2}}, \ R = \frac{16\sigma^{*}T_{\infty}^{3}}{3k_{p}\lambda}, \ S_{c} = \frac{v}{D}, \ G_{r} = \frac{vg\beta(T_{\omega} - T_{\infty})}{U_{0}v_{0}^{2}}, \ G_{c} = \frac{vg\beta(T_{\omega} - T_{\infty})}{U_{0}v_{0}^{2}}, \ K_{r} = \frac{vQ_{0}}{v_{0}^{2}\rho C_{p}}, \ S_{r} = \frac{D_{m}k_{T}(T_{\omega} - T_{\omega})}{T_{m}v(C_{\omega} - C_{\omega})}, \ U_{\omega}^{*} = \frac{U_{\omega}}{U_{0}}.$$
(8)

By using the Rosseland approximation for the thermal radiation the radiative heat flux is simplified as:

$$q_r = -\frac{4\sigma^*}{3k_p}\frac{\partial T^4}{\partial y}.\sigma^*k_p T^4$$
(9)

Here σ^* and k_p are the Stefan-Boltzmann constant and the Rossenland mean absorption coefficient, respectively. Assuming the temperature difference within the flow being sufficiently small, so that by Taylor series expansion truncating the higher-order terms we can express T^4 as a linear functions of temperature of the form:

$$T^4 \approx 4T_{\infty}^{\ 3}T - 3T_{\infty}^{\ 4}$$
 (10)

By using (6) and (10),

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_{\infty}^3}{3k_p} \frac{\partial^2 T}{\partial y^2}$$
(11)

Using (6) and (11) in the system of Eqs. (1)-(5), the reduced Non-dimensional equations are given

$$(1+\alpha)\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} + 2\alpha\frac{\partial \omega}{\partial y} + (M^2 + \frac{1}{K_0(1+\varepsilon Ae^{-nt})})U - \frac{\partial U}{\partial t} - M^2 u = \frac{\partial u}{\partial t} - G_r\theta\cos\alpha - G_c\varphi\cos\alpha$$
(12)

$$\frac{\partial^2 \omega}{\partial y^2} + \beta \frac{\partial \omega}{\partial y} = \beta \frac{\partial \omega}{\partial t},$$
(13)

$$\frac{\partial^2 \theta}{\partial y^2} + \frac{\mathbf{P}_r}{1+R} \frac{\partial \theta}{\partial y} = \frac{\mathbf{P}_r}{1+R} \frac{\partial \theta}{\partial t} - \frac{\mathbf{P}_r}{1+R} S\theta, \tag{14}$$

$$\frac{\partial^2 \varphi}{\partial y^2} + S_c \frac{\partial \varphi}{\partial y} - S_c K_r \varphi = S_c \frac{\partial \varphi}{\partial t} - S_r S_c \frac{\partial^2 \theta}{\partial y^2},$$
(15)

The corresponding boundary conditions are:

$$u = h_1 \frac{\partial u}{\partial y}, \quad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad \theta = 1, \quad \varphi = 1, \text{ at } y = 0,$$

$$k_0 \to K_0(t) = (1 + \varepsilon e^{-nt}), \quad u \to U(t) = (1 + \varepsilon e^{-nt}), \quad \omega \to 0, \quad \theta \to 0, \quad \varphi \to 0, \text{ at } y \to \infty.$$
(16)

Equations (12)–(15) are coupled nonlinear partial differential equations and these cannot be solved in a close form. First, these equations can be reduced to ordinary differential equations and then they can be solved by analytical methods. For this purpose we consider the velocity, angular momentum, temperature and concentration as follows:

$$u(y,t) = u_0(y) + \varepsilon e^{-nt} u_1(y), \quad \omega(y,t) = \omega_0(y) + \varepsilon e^{-nt} \omega_1(y),$$

$$\theta(y,t) = \theta_0(y) + \varepsilon e^{-nt} \theta_1(y), \quad \varphi(y,t) = \varphi_0(y) + \varepsilon e^{-nt} \varphi_1(y),$$
(17)

By substituting the above equations in Eqs. (12)-(15), equating the harmonic and non-harmonic terms, and truncating the higher powers of \mathcal{E} , we get the equations of zero orders and first orders.

Zero-order equations:

$$u_{0}^{''} + \frac{u_{0}^{'}}{1+\alpha} - \frac{M^{2}}{1+\alpha}u_{0} = -\frac{2\alpha}{1+\alpha}\omega_{0}^{'} - \frac{1}{1+\alpha}(M^{2} + \frac{1}{K_{0}}) - \frac{G_{r}\theta_{0}}{1+\alpha}\cos\alpha - \frac{G_{c}\varphi_{0}}{1+\alpha}\cos\alpha,$$
(18)

$$\theta_{0}^{''} + \frac{P_{r}}{1+R}\theta_{0}^{'} + \frac{P_{r}S}{1+R}\theta_{0} = 0,$$
(19)
$$\theta_{0}^{''} + \beta \theta_{0}^{'} = 0,$$
(20)

$$\varphi_{0}^{"} + S_{c}\varphi_{0}^{'} - S_{c}K_{r}\varphi_{0} = -S_{r}S_{c}\theta_{0}^{"}.$$
(21)

First-order equations:

$$u_{1}'' + \frac{1}{1+\alpha}u_{1}' - \frac{1}{1+\alpha}(M^{2}+n)u_{1} = -\frac{2\alpha}{1+\alpha}\omega_{1}' - \frac{1}{1+\alpha}(M^{2}+\frac{1}{K_{0}}+n) - \frac{1}{1+\alpha}G_{r}\theta_{1}\cos\alpha - \frac{1}{1+\alpha}G_{c}\phi_{1}\cos\alpha, \quad (22)$$

$$\theta_1^{''} + \frac{P_r}{1+R}\theta_1^{'} + \frac{P_r}{1+R}(n+s)\theta_1 = 0,$$

$$\omega_1^{''} + \beta\omega_1^{'} + \beta n\omega_1 = 0,$$
(23)
(24)

$$\varphi_{1}^{"} + S_{c}\varphi_{1}^{'} + S_{c}(n - K_{r})\varphi_{1} = -S_{r}S_{c}\theta_{1}^{"}.$$
(25)

The corresponding boundary conditions are:

$$u_{0} = h_{1} \frac{\partial u_{0}}{\partial y}, u_{1} = h_{1} \frac{\partial u_{1}}{\partial y}, \frac{\partial \omega_{0}}{\partial y} = -\frac{\partial^{2} u_{0}}{\partial y^{2}}, \frac{\partial \omega_{1}}{\partial y} = -\frac{\partial^{2} u_{1}}{\partial y^{2}}, \theta_{0} = \varphi_{0} = 1, \theta_{1} = \varphi_{1} = 0, \text{ at } y = 0,$$

$$u_{0} \rightarrow 1, u_{1} \rightarrow 1, \omega_{0} \rightarrow 0, \omega_{1} \rightarrow 0, \theta_{0} \rightarrow 0, \theta_{1} \rightarrow 0, \varphi_{0} \rightarrow 0, \varphi_{1} \rightarrow 0, at y \rightarrow \infty.$$
 (26)

By solving Eqs. (18)–(26) and substituting in Eqs. (11)–(14), the solutions are given by

$$u(y,t) = \left[c_7 e^{-m_6 y} + b_1 c_5 e^{-\beta y} - b_2 e^{-m_3 y} - b_3 c_3 e^{-m_1 y} - b_9 e^{-m_3 y} - m_8 - 1\right] + \varepsilon e^{-nt} \left[c_8 e^{-m_7 y} + b_8 c_6 e^{-m_6 y} + 1 + m_9\right], \quad (27)$$

$$\omega(y,t) = \left[c_5 e^{-\beta y}\right] + \varepsilon e^{-nt} \left[c_6 e^{-m_5 y}\right]$$
(28)

$$\varphi(y,t) = \left[1 - m_B\right] e^{-m_1 y} + m_B e^{-m_3 y}, \tag{29}$$

$$\theta(\mathbf{y}) = e^{-m_3 \mathbf{y}},$$

Where

$$\begin{split} a_{1} &= \frac{P_{r}}{1+R}, \ a_{2} = \frac{P_{r}S}{1+R}, \ a_{4} = \frac{1}{1+\alpha}, \\ a_{5} &= \frac{M^{2}}{1+\alpha}, \ a_{6} = \frac{2\alpha}{1+\alpha}, \ m_{1} = \frac{S_{c} + \sqrt{S_{c}^{2} + 4S_{c}K_{r}}}{2}, \\ m_{2} &= \frac{S_{c} - \sqrt{S_{c}^{2} + 4S_{c}K_{r}}}{2}, \ m_{3} = \frac{a_{1} + \sqrt{a_{1}^{2} - 4a_{2}}}{2}, \ m_{4} = \frac{a_{1} - \sqrt{a_{1}^{2} - 4a_{2}}}{2}, \ m_{5} = \frac{\beta + \sqrt{\beta^{2} - 4n\beta}}{2}, \\ m_{6} &= \frac{a_{4} + \sqrt{a_{4}^{2} + 4a_{5}}}{2}, \ m_{7} = \frac{a_{4} + \sqrt{a_{4}^{2} + 4(a_{5} + na_{5})}}{2}, \ m_{8} = \frac{1}{M^{2}K_{0}}, \ m_{9} = \frac{1}{K_{0}[M^{2} + n]}, \\ m_{8} &= \frac{-S_{r}S_{c}m_{3}^{2}}{m_{3}^{2} - S_{c}m_{3} - S_{c}K_{r}}, \ m_{c} = \frac{S_{c} + \sqrt{S_{c}^{2} - 4S_{c}(n - K_{r})}}{2}, \ b_{1} = \frac{a_{6}\beta}{\beta^{2} - a_{4}\beta - a_{5}}, \\ b_{2} &= \frac{a_{4}G_{r}\cos\alpha}{m_{3}^{2} - m_{3}a_{4} - a_{5}}, \ b_{3} = \frac{a_{4}G_{c}\cos\alpha}{m_{1}^{2} - m_{1}a_{4} - a_{5}}, \ b_{9} = \frac{a_{4}m_{B}G_{c}\cos\alpha}{m_{3}^{2} - m_{3}a_{4} - a_{5}}, \ c_{3} = 1 - m_{B}, \\ c_{5} &= \frac{m_{6}^{2}\left[b_{2}(1 + h_{1}m_{3}) + b_{9}(1 + h_{1}m_{3}) + b_{3}c_{3}(1 + h_{1}m_{1}) + m_{8} + 1\right] - \left[m_{3}^{2}b_{2} + m_{1}^{2}b_{3}c_{3} + m_{3}^{2}b_{9}\right](1 + m_{6}h_{1})}{\beta(1 + m_{6}h_{1}) - \beta^{2}b_{1}(1 + m_{6}h_{1}) + m_{6}^{2}b_{1}(1 + h_{6}\beta)}, \\ c_{7} &= \frac{-b_{1}c_{5}(1 + h_{1}\beta) + (b_{2} + b_{9})(1 + h_{1}m_{3}) + b_{3}c_{3}(1 + h_{1}m_{1}) + m_{8} + 1}{1 + m_{6}h_{1}}, \\ c_{8} &= \frac{-b_{8}c_{6}(1 + h_{1}m_{6}) - 1 - m_{9}}{(1 + h_{0}r_{7})}, \end{split}$$

III. RESULTS AND DISCUSSION

In the present analysis, analytical solution for the free convective flow through a porous medium with variable permeability in the slip flow regime with couple stress in the presence of a heat source was carried out by the perturbation method. The variations of fluid behaviors with various physical parameters are shown graphically in Figs. 1–12. Computations have been carried out for various values of $\varepsilon = 0.01$, A = 0.5, $h_1 = 0.4$, $\beta = 2$, $\alpha = 0.2$, $\varepsilon = 0.01$, Pr=0.71, Sc = 0.5, S = 1, R = 1, Kr = 1, $\eta = 0.1$, t = 1, M = 2, and $K_0 = 1$.

Velocity, Temperature, and Concentration Distribution

The variation of M, Gr, Gc, Kr, α , and K_0 on the velocity field is represented in Figs. 1-4. The influence of Gc on the velocity field is plotted in Fig. 1, it is observed that the velocity profile increases with a increase in Gc. The magnitude of the chemical reacting fluid velocity profile is less as compared to the clear fluid. It is also observed that the fluid velocity near wall is higher and these velocities recede gradually to the free stream. Variation of β versus velocity profile was studied in Fig. 2. The results show that initially the velocity profile increases, but the reverse effect was seen from a certain point on increasing the value of β . Figure 3 depicts that translational fluid velocity that is shown in Fig. 3. In Fig. 4, it is observed that the velocity profile increases with a increase with a increase in thermal diffusivity α and attains a peak value near the wall.

Figures 5-8 illustrate the influence of emerging parameters on the angular velocity profiles. Figures 5 and 6 describe the effect of Gc and Gr on the angular velocity profile. From Figs. 5 and 6, it seen that the angular profiles decreases with increase in Gc and Gr the magnitude of angular velocity. That happens because when Gr and Gc increases, the thermal and mass buoyancy effect increases, which leads a lower angular velocity. Figure 7 shows that the angular velocity increase with increase in the

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(30)

values of the magnetic field parameter M, Figure 8 shows the angular velocity profiles for various values of the chemical reaction parameter (K_0). The results display that with an increase of K_0 , the magnitude of angular velocity profiles decrease.



Fig. 2.Velocity profile for variation of β



Fig. 4.Velocity profile for variation of α



Fig.6. Angular velocity profile for variation of Gr



Fig.8. Angular velocity profile for variation of K_0



Fig.10. Temperature profile for variation of Pr

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Fig. 12.Concentration profile for variation of Sc

Variation of temperature profile versus heat source parameter is displayed in Fig. 9, it is observed that temperature of the fluid increases with decrease in heat source due to the decrease in the thermal boundary layer. Figure 10 shows that with decreasing Prandtl number there is a increase in the heat generation parameter of the micro-polar fluid, which agrees with the physical nature of the fluid. It is observed that the thermal boundary layer decreases with increasing Pr and a higher temperature is observed near the boundary layer. Figs.11-12 it is observed that the concentration profiles. Figures 11 and 12 show the dimensionless concentration profiles for different values of Schmidt number (Sc) and chemical reaction parameter Kr. From Fig.12 we see that

the concentration decreases as the molecular diffusivity increase and that the concentration values are higher at the wall with decreasing Schmidt number.

IV. CONCLUSIONS

In this paper the unsteady free convective MHD flow polar fluids over an infinite horizontal plate in heat source/sink with Dufour effect. The important findings are summarized as below:

- > Velocity profiles increase with an increase in the thermal Grashof number (Gr), coefficient of volume expansion (β) and Soret number (Sr)
- Angular profiles decrease with an increase in the Grashof number (Gr), Thermal Grashof number (Gc) and Nondimensional permeability parameter (K_0),
- Temperature of the fluid decreases with an increase in the Chemical reaction parameter (Kr) and Schmidt number (Sc) due to increasing thermal boundary layer.
- > Velocity profiles decrease with an increase in Thermal diffusivity (α),
- > Angular profiles increase with an increase in the Magnetic field parameter (M).
- > The concentration profile decrease with an increase in the Heat source parameter (S) and Prandtl number (Pr).
- The enhancement of magnetic field parameter decreases the translational velocity as well as the Micro-rotational velocity.
- Radiation parameter enhances the fluid temperature, which agrees with the physical properties of the fluid.

NOTATIONS

u and v Velocity components along x and y axes, respectively

- u Non-dimensional velocity component
- B_0 Magnetic field strength
- g Acceleration due to gravity
- V Kinematic viscosity
- V_r Kinematic rotational viscosity
- \mathcal{O} Mean angular velocity
- P_r Prandtl number
- S_c Schmidt number
- S_r Soret number
- *S* Heat source parameter
- M Magnetic field parameter
- D Mass diffusivity
- α Thermal diffusivity
- T Temperature
- T_{ω} Fluid temperature near the wall
- T_{∞} Fluid temperature away from the wall
- T_m Mean fluid temperature
- θ Non-dimensional temperature
- *C* Concentration
- C_{ω} Fluid concentration near the wall

- C_{∞} Fluid concentration away from the wall
- G_r Thermal Grashof number
- C Non-dimensional concentration
- G_c Mass Grashof number

 K_r Chemical reaction parameter

- k_T Thermal diffusion ratio
- K_0 Non-dimensional permeability parameter
- R Radiation parameter
- q_r Radiative heat flux
- ho Fluid density
- σ Fluid electrical conductivity
- C_n Specific heat at constant pressure
- K(t) Variable permeability of the porous medium
- V_0 Suction velocity
- C_{∞} Fluid concentration away from the plate

 C_{ω} Fluid concentration near the wall

- β Coefficient of volume expansion
- β * Coefficient of volume expansion with concentration
- ω Small reference parameter 1

V. REFERENCES

- [1] Ali J. Chamkha, 2004. Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. International Journal of Engineering Science, 42: 217-230.
- [2] Hiranmoy Mondala, Dulal Palb, Sewli Chatterjeec and Precious Sibandaa, 2017. Thermophoresis is and Soret-Dufour on MHD mixed convection mass transfer over an inclined plate with non- uniform heat source/sink and chemical reaction. Ain Shams Engineering Journal.
- [3] Sreedevia, G. Prasada Raob, DRV. Makindec, O D. and Venkata Ramana Reddya, G. 2017. Soret and Dufour effects on MHD flow with heat and mass transfer past a permeable stretching sheet in presence of thermal radiation. Indian Journal of Pure & Applied Physics, 55: 551-563.
- [4] Ibrahim, S. M. Kumar, P.V. Lorenzini, G. Lorenzini, E. F. 26(2): 256-271.
- [5] Shateyi, S. Mabood, F. and Lorenzini, G. 2017. Casson Fluid Flow: Free Convective Heat and Mass Transfer over an Unsteady Permeable Stretching Surface Considering Viscous Dissipation. Journal of Engineering Thermophysics, 26(1): 39-52.
- [6] Raptis, A. Tzivanidis, G. and Kafousias, N. 1981. Free Convection And Mass Transfer Flow Through A Porous Medium Bounded By An Infinite Vertical Limiting Surface With Constant Suction. Letters In Heat And M~Ss Transfer, 8: 417-424.
- [7] Nadeem Ahmad Sheikh, Farhad Ali, Ilyas Khan, Muhammad Saqib and Arshad Khan, 2017. MHD Flow of Micro-polar Fluid over an Oscillating Vertical Plate Embedded in Porous Media with Constant Temperature and Concentration. Hindawi Mathematical Problems in Engineering, Article ID 9402964, 20 pages.
- [8] Dulal Pal, Sukanta Biswas, 2016. Perturbation analysis of magneto hydrodynamics oscillatory flow on Convective Radiative heat and mass transfer of micro-polar fluid in a porous medium with chemical reaction. Engineering Science and Technology, an International Journal, 19: 444-462.
- [9] Pandit, K.K. Sarma, D. and Singh, S.1. 2017. A Study Of Chemically Reactive Species And Thermal Radiation Effects On An Unsteady MHD Free Convection Flow Through A Porous Medium Past A Flat Plate With Ramped Wall Temperature. Int. J. Of Applied Mechanics and Engineering, 22(4): 945-964.
- [10] Ashish Paul. 2017. Transient Free Convective MHD Flow Past an Exponentially Accelerated Vertical Porous Plate with Variable Temperature through a Porous Medium. International Journal of Engineering Mathematics, Article ID 2981071, 9 pages.

- [11] Adamu Gizachew Chanie, Bandari Shankar, Mahantesh and Nandeppanavar, 2017. Mass Transfer on MHD Flow of a Casson Fluid Through a Porous Media Due to a Stretching Sheet With the Effect of Chemical Reaction and Suction. Imperial Journal of Interdisciplinary Research (IJIR), 3(10).
- [12] Paras Ram, HawaSingh, Rakesh Kumar, Vikas Kumar and Vimal Kumar Joshi, 2017. Free Convective Boundary Layer Flow of Radiating and Reacting MHD Fluid Past a Cosinusoidally Fluctuating Heated Plate. Int. J. Appl. Comput. Math.
- [13] Ramana Reddya, J.V. Anantha Kumara, K. Sugunamma, V. and Sandeep, N. 2017. Effect of cross diffusionon MHD non-Newtonian fluids flow past a stretchin gsheet with Non Uniform heat source/sink: A comparative study. Alexandria Engineering Journal.
- [14] Sambatha, P. Bapuji Pullepua, Hussainb, T. and Sabir Ali Shehzad, 2018. Radiated chemical reaction impacts on natural convective MHD mass transfer flow induced by a vertical cone. Results in Physics, 8: 304-315.
- [15] Praveena, D. Varma, S. V. K. Veeresh, C. and Raju, M. C. 2017. Chemical reaction and heat source effect on MHD free convection flow of a micro-polar fluid through a porous medium over a Semi-infinite moving plate with constant heat and mass flux. Journal of Information and Optimization Sciences.
- [16] Babu Reddy, D. and Raju, G.S.S. 2017. Radiation and Chemical Reaction Effects on Unsteady MHD Mixed Convection Flow over a Vertical Porous Plate with Radiation Absorption. Global Journal of Pure and Applied Mathematics, 13(8): 4015-4034.
- [17] Gnaneswara Reddy, M. Sudha Rani, M.V.V.N.L. Makinde, O.D. 2017. Effects of Nonlinear Thermal Radiation and Thermo-Diffusion on MHD Carreau Fluid Flow Past a Stretching Surface with Slip. Diffusion Foundations, ISSN: 2296-3642, 11:57-71.
- [18] Suman Agarwalla and Nazibuddin Ahmed, 2017. MHD Mass transfer flow past an inclined plate with variable temperature and plate velocity embedded in a porous medium. Department of Mathematics, Gauhati University, Guwahati, 781014.
- [19] Singh, J.K. Joshi, N. and Rohidas, P. 2017. Unsteady MHD Natural Convective Flow of a Rotating Walters-B Fluid Over An Oscillating Plate With Fluctuating Wall Temperature And Concentration. Journal of Mechanics.
- [20] Pandya, N. RaviKantYadav and Shukla, A.K. 2017. Combined effects of Soret-Dufour, Radiation and Chemical reaction on Unsteady MHD flow of Dusty fluid over inclined porous plate embedded in porous medium. Int. J. Adv. Appl. Math. and Mech. 5(1): 49-58.
- [21] Swapna, Y. Raju, M.C. Ram Prakash Sharma and Varma, S.V.K. 2017. Chemical Reaction, Thermal Radiation and Injection/Suction Effects on MHD Mixed Convective Oscillatory Flow through a Porous Medium Bounded ByTwo vertical Porous Plates. Bull. Cal. Math. Soc., 109(3): 189-210.
- [22] Daniel, Y.S. Aziz, Z.A. Ismail, Z. Salah, F. 2017. Thermal radiation on unsteady electrical MHD flow of Nano-fluid over stretching sheet with chemical reaction. Journal of King Saud University Science.
- [23] Beg, O.A. Beg, T. A. Bakier, A. Y. and Prasad, V. R. 2009. Chemically-Reacting Mixed Convective Heat And Mass Transfer Along Inclined And Vertical Plates With Soret And Dufour Effects: Numerical Solutions. Int. J. of Appl. Math and Mech. 5(2): 39-57.
- [24] Vedavathia, N. Ramakrishna, K. and K.Jayarami Reddy, 2015. Radiation and mass transfer effects on unsteady MHD convective flow past an infinite vertical plate with Dufour and Soret effects. Ain Shams Engineering Journal, 6: 363-371.
- [25] Umamaheswara, M. Rajua, M.C. Varma, S.V.K. and Gireeshkumar, J. 2016. Numerical investigation of MHD free convection flow of a non-Newtonian fluid past an impulsively started vertical plate in the presence of thermal diffusion and radiation absorption. Alexandria Engineering Journal, 55: 2005-2014.
- [26] Sinha, A. Ahmed, N. and Agarwalla, S. 2017. MHD Free Convective Flow through a Porous Medium Past a Vertical Plate with Ramped Wall Temperature. Applied Mathematical Sciences, 11(20): 963 974.
- [27] Sarma, D. and Pandit, K.K. 2016. Effects of Hall current, rotation and Soret effects on MHD free convection heat and mass transfer flow past an accelerated vertical plate through a porous medium. Ain Shams Engineering Journal.
- [28] Jyotsna Rani Pattnaika, Gouranga Charan Dash and Suprava Singh, 2017. Radiation and mass transfer effects on MHD flow through porous medium past an exponentially accelerated inclined plate with variable temperature. Ain Shams Engineering Journal, 8: 67-75.
- [29] Dastagiri Babu, D. Venkateswarlu, S. and Keshava Reddy, E. 2017. Heat and Mass transfer on Unsteady MHD Free Convection Rotating flow through a porous medium over an infinite vertical plate with Hall effects. AIP Conference Proceedings, 1859, 020077.
- [30] Ramesh Babu, K. Parandhama, A. Venkateswara Raju, K. Raju, M. C. and Satya Narayana, P. V. 2017. Unsteady MHD Free Convective Flow of a Visco-Elastic Fluid Past an Infinite Vertical Porous Moving Plate with Variable Temperature and Concentration. Int. J. Appl. Comput. Math.