COMPARISON BETWEEN SINGLE-STEP METHOD AND MULTISTEP METHOD USING TAYLOR'S SERIES METHOD MILNES METHOD PICARD'S METHOD AND ADAMS MOULTON METHOD

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Abstract : **This Paper mainly presents single-step method Taylor's series method, Picard's method and Multistep methods Milnes method and Adams Moulton Predictor-Corrector method, for solving initial value problems (IVP) for ordinary differential equations (ODE). The four proposed methods are quite efficient and practically well suited for solving these problems. To verify the accuracy, we compare numerical solutions with the exact solutions. The numerical solutions are in good agreement with the exact solutions. Numerical comparison between Taylor's series method, Adams Moulton Predictor-Corrector method, Milnes method and Picard's method have been presented. Also, we compare the performance and the computational effort of such methods to achieve higher accuracy in the solution, the step size needs to be very small. Finally, we investigate and compute the errors of the four proposed methods for different step sizes to examine superiority. Several numerical examples are given to demonstrate the reliability and efficiency.**

KEYWORDS

Ordinary differential equation, initial value problem, Adams Moulton Predictor-Corrector method, RK method, Taylor's series method, Picard's method, Milnes method and ERROR ANALYSIS.

1. INTRODUCTION

Numerical methods naturally find applications in all fields of engineering and physical sciences, but in 21st century, the life sciences and even the arts have adopted elements of scientific computations. Ordinary differential equation appears in the movement of heavenly bodies (planets, stars, and galaxies), optimization occurs in portfolio management. Numerical linear algebra is important for data analysis, stochastic differential equations and Markov chains are essential in simulating living cells medicine and biology.

To describe various numerical methods for the solution of ordinary differential equation, we consider the general first order differential equation $\frac{dy}{dx} = f(x,y)$ which the initial condition $y(x_0) = y_0$ and illustrate the theory with respect to this equation.

The solution of ordinary differential equation means to find an explicit expression for the dependent variable y interms of finite number of elementary function of x. Such a solution of differential equation is called closed or finite form of the solution. In most numerical methods we replace the differential equation by a difference equation and then solve it. The method developed and applied to solve ordinary differential equations of first order and first degree will yield the solution in one the following forms

(i) A power series in x for y, from which the values of y can be obtained by direct substitution.

(ii) A set of tabulated values of x and y.

The Taylor's series method, Picard method and Adams Moulton Predictor-Corrector method, Milnes method are discussed in this paper. These later methods are called step-by-step methods are marching methods because the values of y are computed by short steps ahead for equal intervals h of the independent variable. If, however the function values are described over a wider range, the methods due to Adams Moulton Predictor-Corrector method, Milnes method, etc... May be used. These methods use finite differences and require "STARTING VALUES" which are usually obtained by Taylor's series or RK methods.

It is well known that differential equations of the n^{th} order will have n arbitrary constants in its general solutions. Problems in which all the initial conditions are specified at the initial point only are called initial value problems.

The first problem considered in this research is $\frac{dy}{dx} = \frac{x+y}{2}$, with initial value $y(0) = 2$ and the interval 0.1 to 0.5. The second problem is $\frac{dy}{dx} = x - y,$ with initial value $y(0) = 1$ and the interval 0.1 to 0.5. All the above problems have the exact solutions also. Also, we find the errors this is helpful to compare the approximate values with the exact values. Hence the main purpose of this research is to compare the accuracy of Taylor's series method, Picard's method and Adams Moulton Predictor-Corrector method, Mines method for ordinary differential equation.

2. NUMERICAL METHODS

2.1 Taylor's series method

Derivation: Let us consider the initial value problem

$$y' = \frac{dy}{dx} = f(x,y); y(x_0) = y_0$$

Let $y = y(x)$ be the exact solution of such that $y(x_0) \neq 0$. Now expanding by Taylor's series about the point $x = x_0$, we get

$$y = y(x) = y_0 + (x - x_0)y'_0 + \frac{(x-x_0)^2}{2!}y''_0 + \frac{(x-x_0)^3}{3!}y'''_0 + \cdots \dots ....$$

In the expression, the derivatives $y'_0, y''_0, y'''_0, \dots \dots \dots$ are not explicitly known. However, if $f(x,y)$ is differentiable several times, the following expression in terms of $f(x,y)$ and its partial derivatives as the followings

$$y' = f(x,y) = f$$

$$y'' = f'(x,y) = f_x + y'f_y = f_x + ff_y$$

$$y''' = f''(x,y) = f_{xx} + 2ff\,f_{xy} + f_{yy}f^2 + f_x\,f_y + {f_y}^2 f$$

By similar manner a derivative of any order of y can be expressed in terms of $f(x,y)$ and its partial derivatives.

As the equation of higher order total derivatives creates a hard stage of computation, to overcome the problem we are to truncate the Taylor's expansion to a first few convenient terms of the series. This truncation in the series leads to a restriction on the value of for which the expansion is a reasonable approximation.

Now, for suitable small step length $h = x_i - x_{i-1,}$ the function $y = y(x)$ is evaluated at $x_1 = x_0 + h$. Then the Tailor's expansion becomes

$$y(x_0 + h) = y(x_0) + hy'(x_0) + \frac{h^2}{2!}y''(x_0) + \cdots \dots \dots ....$$

The derivatives $y'_0, y''_0, y'''_0, \dots \dots \dots$ are evaluated at $x_1 = x_0 + h,$ and then substituted in to obtain the value y of at $x_2 = x_0 + h,$ given by

$$y(x_0 + 2h) = y(x_0 + h) + hy'(x_0 + h) + \frac{h^2}{2!}y''(x_0 + h) + \cdots \dots \dots$$

By similar manner we get

$$y_3 = y_2 + hy_2' + \frac{h^2}{2}y_2'' + \frac{h^3}{6}y_2''' + \cdots \dots \dots \dots ..$$

$$y_4 = y_3 + hy_3' + \frac{h^2}{2}y_3'' + \frac{h^3}{6}y_3''' + \cdots \dots \dots \dots ..$$

Thus the general form obtained as

$$y_{n+1} = y_n + hy_n' + \frac{h^2}{2}y_n'' + \frac{h^3}{6}y_n''' + \cdots \dots \dots \dots ..$$

This equation can be used to obtain the value of y_{n+1}. which is the approximate value to the actual value of $y = y(x)$ at the value $x_{n+1} = x_0 + (n+1)h.$

2.2 PICARD'S METHOD

Derivation: let us consider the initial value problem

$$y' = \frac{dy}{dx} = f(x, y); y(x_0) = y_0$$

We have $dy = f(x, y)dx$

Integrating between corresponding limits x_0 to x and y_0 to y, then the above equation given as following

$$\int_{y_0}^{y} dy = \int_{x_0}^{x} f(x, y)dx$$

Or, y-$y_0 = \int_{x_0}^{x} f(x, y)dx$

Or, $y = y_0 + \int_{x_0}^{x} f(x, y)dx$

Here the integral term in the right-hand side represents the increment in y produced by an increment x-x_0 in x. The equation is complicated by the presence of y under the integral sign as well as outside it. An equation of this kind is called an integral equation and can be solved by a process of successive approximation or iteration, if the indicated integrations can be performed in the successive steps.

To solve by the Picard's method of successive approximation, we get a first approximation in y by putting $y = y_0 = y^{(0)}$, then

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y^{(0)})dx$$

The integral is now a function of x alone and the indicated integration can be performed at least for one time. Having first approximation to y, substitute it for y in the integrated and by integrating again we get the second approximation of y as following

$$y^{(2)} = y_0 + \int_{x_0}^{x} f(x, y^{(1)})dx$$

Proceeding in this way we obtain $y^{(3)}$, $y^{(4)}$&$y^{(5)}$ and so on. Thus, we get the n[th] approximation being given by the following equation

$$y^{(n)} = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)})dx$$

Then putting $y^{(n)}$ for y ,we get the next approximation as follows

$$y^{(n+1)} = y_0 + \int_{x_0}^{x} f(x, y^{(n)})dx$$

This process will be repeated in this way as many times as necessary or desirable. This process will be terminated when two consecutive values of y are same to the desired degree of accuracy.

2.3 ADAMS MOULTON PREDICTOR-CORRECTOR METHOD

Adams-Moulton method is a general approach to the predictor-corrector formula which developed for using the information of a function $y(x)$ and its first derivative given by $y' = f(x, y)$ at the past three points together with one more old value of derivatives.

2.4 MILNE'S PREDICTOR-CORRECTOR METHOD

Milne's method is a simple and reasonably accurate method of solving ordinary differential equations numerically. To solve the differential equation $y' = F(x, y)$ by this method, first we approximate the value of y_{n+1} by predictor formula at $x_{n+1,}$ and then improve this value of y_{n+1} by using a corrector formula.

"Predictor-corrector methods for solving ODEs. When considering the numerical solution of ordinary differential equations (ODEs), a Predictor-corrector method typically uses an explicit method for the predictor step and an implicit method for the corrector step"

3. ERROR ANALYSIS

Numerically computed solutions are subject to certain errors. Mainly there are three types of errors. They are inherent error, truncation errors and errors due to rounding

a. Inherent errors are experimental errors arise due to the assumptions made in the mathematical modelling of problem. It can also arise when the data is obtained from physical measurement of the parameters of the problem. *i.e.,* Errors arising from measurements.

b. Finite (or infinite) sequence of computational steps necessary to produce an exact result is "truncated" prematurely after a certain number of steps.

c. Round of errors are errors arising from the process of rounding off during computation. These are called *chopping*, *i.e.*, discarding all decimals from sum decimals on.

3.1 ERROR IN NUMERICAL COMPUTATION:

Due to errors that we have discussed, it can be seen that our numerical results is an approximate value of the (sometimes unknown) exact result, except for the rare case where the exact answer is sufficiently simple rational number.

If $\breve{a}$ is an approximate value of a quantity whose exact value is a, then the difference $\in = \tilde{a} - a$ is called the absolute error of $\tilde{a}$ or, briefly,the error of $\tilde{a}$. Hence, $a + \in = \tilde{a}$, *i.e.,*

Approximate value = exact value + error

Error =exact value – approximate value

4. NUMERICAL EXAMPLES

Example 1:

We consider the initial value problem $\frac{dy}{dx} = x - y$, with initial value $(0) = 1$. The exact solution of the given problem is given by $y = x - 1 + 2e^{-x}$ the approximate results and errors are obtained and shown in the table 1(i & ii) and the graph of the numerical solutions are displayed in Figures 1(i & ii)

TABLE:1(i)

X-VALUE	EXACT VALUE	MILNES VALUE	TAYLORS METHOD	PICARD METHOD	ADAMS MOULTON P-C METHOD
0.2	0.8374615	0.8374667	0.8374615	0.8374615	0.8374667
0.4	0.7406401	0.7406485	0.7406400	0.7406401	0.7406485
0.6	0.6976233	0.6943003	0.6984163	0.6976233	0.6943003
0.8	0.6986579	0.6994209	0.6993072	0.6986579	0.6959749
1.0	0.7357588	0.7322633	0.7362904	0.7357589	0.7334477
1.2	0.8023884	0.8032765	0.8047871	0.8023886	0.8022555
1.4	0.8931939	0.8893594	0.8951578	0.8931947	0.8930885
1.6	1.0037930	1.0057858	1.0054009	1.0037964	1.0038021
1.8	1.1305978	1.1262159	1.1319142	1.1306096	1.1304452
2.0	1.2706706	1.2741915	1.2670774	1.2707071	1.2715268

Table 1(i): List the numerical and analysis value of $y = x - 1 + 2e^{-x}$ the value for each y values, Taylor's series method, Picard's method and Adams Moulton Method and Milnes Method.

PLOT 1(i)

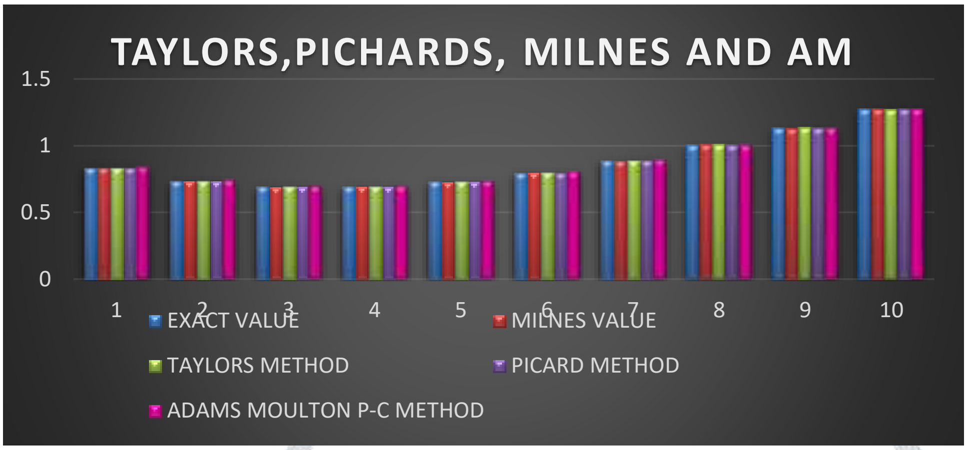
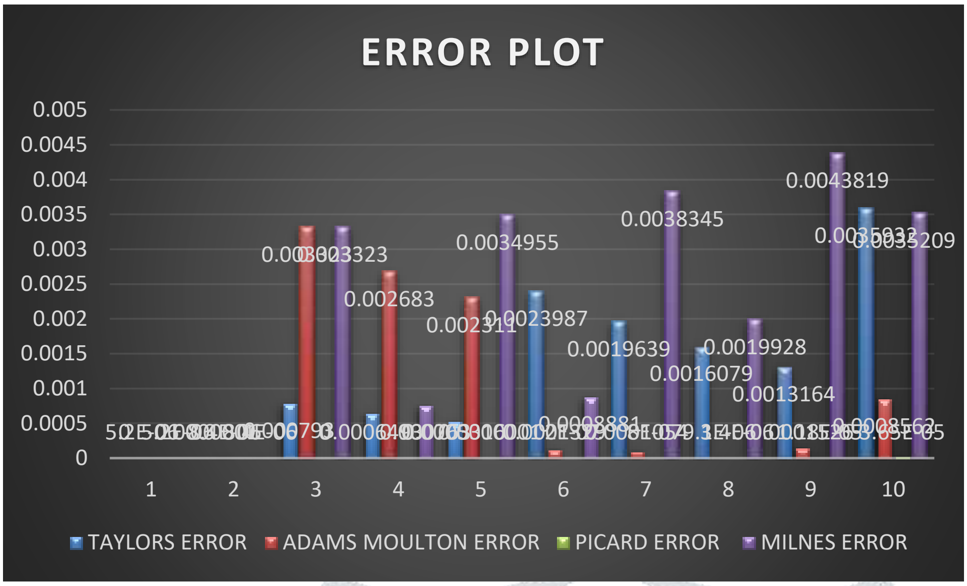


Table 1 (ii):

TAYLORS ERROR	ADAMS MOULTON ERROR	PICARD ERROR	MILNES ERROR
0.000000	0.0000052	0.000000	0.0000052
0.0000001	0.0000084	0.000000	0.0000084
0.000793	0.0033230	0.000000	0.003323
0.0006493	0.002683	0.000000	0.000763
0.0005316	0.002311	0.0000001	0.0034955
0.0023987	0.0001329	0.0000002	0.0008881
0.0019639	0.0001054	0.0000008	0.0038345
0.0016079	0.0000091	0.0000034	0.0019928
0.0013164	0.0001526	0.0000118	0.0043819
0.0035932	0.0008562	0.0000365	0.0035209

Table 1(ii): shows the errors of Taylor's series method, Picard's method and Adams Moulton Method and Milnes Method and for each y values.

PLOT 1(ii)



5. DISCUSSION OF RESULTS

The method is of general applicable and it is the standard to which we compare the accuracy of the various other numerical methods for solving an ordinary differential equation with initial values, we already compared the single-step Taylor's method, Picard's method, with multistep Milnes method and Moulton method with exact solution and error in this paper.

6. CONCLUSION

In this paper single-step Taylor's method, Picard's method, and multistep Milnes method and mouton method are used for solving ordinary differential equation (ODE). In Initial value problems (IVP). Finding more accurate results needs the step size smaller for all methods. From the figures we can see the accuracy of the method for decreasing the step size h and the graph of the approximate solution approaches to the graph of the exact solution. The numerical solution obtained by the four proposed methods are in good agreement with exact solutions. Comparing the results of the four methods under investigation, we observed that the single step Picard's method was found to be less accurate due to the inaccurate numerical results that were obtained from the approximate solution in comparison to the exact solution. Thus, we conclude that Single-step Picard's method is better than other method.

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