COMPARISON BETWEEN LAPLACE ,SUMUDU AND MAHGOUB TRANSFORMS FOR SOLVING SYSTEM OF FIRST ORDER FIRST DEGREE DIFFERENTIAL EQUATIONS

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Abstract : In this paper we discuss some relationship between Laplace transform , Sumudu transform and Mahgoub Transforms .We solve first order ordinary differential equations using both transforms and show that Sumudu transform and Mahgoub transform are closely connected with the Laplace transform.

IndexTerms - **Laplace transform , Sumudu transform, Elzaki Transform , Mahgoub Transforms, Differential equations.**

I. INTRODUCTION

Recently, In 2016 Mahgob introduced a useful technique for solving ordinary & partial differential equations in the time domain [1]. Hassan Eltayeb introduced some relationship between Sumudu and Laplace transforms, further; for the comparison purpose, he apply both transforms to solve differential equations to see the differences and similarities[2]. In 2016 , P.R.Bhadane observed that the new method using Elzaki transform was presented to solve system of homogeneous and non-homogeneous linear differential equations of first order and first degree with constant coefficients and satisfying some initial conditions. The solution obtained for the system of homogeneous and non-homogeneous linear differential equations of first order and first degree is also discussed. These results prove that the Elzaki transform new method is quite capable, well appropriate to solve such types of problems[5].

Recently In 2016 Abdelbagy A. Alshikh, Mohand M. Abdelrahim Mahgob discussed some relationship between Laplace transform and the new two transform called ELzaki transform and Aboodh transform and solved first and second order ordinary differential equations using both transforms, and show that ELzaki transform and Aboodh transform are closely connected with the Laplace transform[3]. In 2007, Jun Zhang discussed An algorithm based on Sumudu transform was developed. The algorithm can be implemented in computer algebra systems like Maple. It can be used to solve differential equations of special form[7]. In 2010, Hassan Eltayeb and Adem Kılıcman introduced some relationship between Sumudu and Laplace transforms, further; for the comparison purpose and applied both transforms to solve differential equations to see the differences and similarities. Finally provided some examples regarding to second order differential equations with non constant coefficients as special case[8].

1. Definitions and Standard Results :

2.1 *The Laplace Transform* :

Definition : If f(t) is a function defined for all positive values of t,then the Laplace Transform is defined as

L[f(t)] =F(s)= $\int_0^\infty e^{-st} f(t)dt$ (1)

Provided that the integral exists.Here the parameter s is a real or complex number.The corresponding inverse Laplace transform is that the integral exists.Here the parameter s is a real or complex number.The corresponding inverse Laplace transform is $L^{-1}[F(s)] = f(t)$.Here f(t) and F(s) are called as pair of Laplace transforms.

2.1.1 Laplace Transform of some functions :

(i) $L(1)=\frac{1}{s} = F(s)$

Inversion Formula : $L^{-1}\left(\frac{1}{s}\right) = 1 = f(t)$

(ii) $L(t^n)=\frac{n!}{s^{n+1}} = F(s)$

Inversion Formula : $L^{-1}\left(\frac{1}{s^{n+1}}\right) = \frac{t^n}{n!} = f(t)$

(iii) $L(e^{at})=\frac{1}{s-a} = F(s)$

Inversion Formula : $L^{-1}\left(\frac{1}{S-a}\right) = e^{at} = f(t)$

(iv) $L(sin(at)) = \frac{a}{s^2+a^2} = F(s)$

Inversion Formula : $L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{\sin(at)}{a} = f(t)$

(v) $L(cos(at)) = \frac{s}{s^2+a^2} = F(s)$

Inversion Formula : $L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos(at) = f(t)$

2.1.2 Laplace Transform of derivatives :

(i) $L[f'(t)] = sF(s) - f(0)$

(ii) $L[f''(t)] = s^2F(s) - sf(0) - f'(0)$

2.2 *The Sumudu Transform* :

Definition :

Over the set of functions, $= \{f(t)/\, \exists M, \tau_1, \tau_2 > 0, |f(t)| < Me^{\frac{|t|}{\tau_j}}, if\ t \in (-1)^j \times [0,\infty)\}$, the sumudu transform is defined by $G(u) = S[f(t)] = \int_0^\infty e^{-t} f(ut)dt,\ u \in (\tau_1, \tau_2)$. (2)

2.2.1 Sumudu Transform of some functions :

i) $S(1)$=1=$G(u)$

Inversion Formula : $S^{-1}(1) = 1 = f(t)$

ii) $S(\frac{t^n}{n!})$=u^n=$G(u)$

Inversion Formula : $S^{-1}(n!\, u^n) = t^n = f(t)$

iii) $S(e^{at})$=$\frac{1}{1-au} = G(u)$

Inversion Formula : $S^{-1}\left(\frac{1}{1-au}\right) = e^{at} = f(t)$

iv) $S(sinat)$=$\frac{au}{1+a^2u^2}$=$G(u)$

Inversion Formula : $S^{-1}(\frac{u}{1+a^2u^2}) = \frac{sinat}{a} = f(t)$

v) $S(\cos(at))$=$\frac{1}{1+a^2u^2} = G(u)$

Inversion Formula : $S^{-1}\left(\frac{1}{1+a^2u^2}\right) = \cos(at) = f(t)$

2.2.2 Sumudu Transform of derivatives :

(i) $S[f'(t)]$=$\frac{G(u)-f(0)}{u}$

(ii) $S[f''(t)]$=$\frac{G(u)}{u^2} - \frac{f(0)}{u^2} - \frac{f'(0)}{u}$

2.3 *Mahgoub Transform* :

Definition :

A new transform called the Mahgoub transform defined for function of exponential order we consider functions in the set A defined by :

$$A = \{f(t)/\, \exists M, \tau_1, \tau_2 > 0, |f(t)| < Me^{\frac{|t|}{\tau_j}}, if\ t \in (-1)^j \times [0,\infty)\}$$

For a given function in the set A,the constant M must be finite number ,k_1, k_2 may be finite or infinite.The Mahgoub transform denoted by the operator M(.) defined by the integral equations

M[f(t)] = H (v)=$v \int_0^\infty e^{-vt} f(t)dt.\ t \geq 0\ , k_1 \leq v \leq k_2$. (3)

2.3.1 Mahgoub Transform of some functions :

i) M (1)=1=$H(v)$

Inversion Formula : $M^{-1}(1) = 1 = f(t)$

ii) M ($\frac{t^n}{n!}$)=$\frac{1}{v^n}$=$H(v)$

Inversion Formula : $M^{-1}(\frac{n!}{v^n}) = t^n = f(t)$

iii) M (e^{at})=$\frac{v}{v-a} = H(v)$

Inversion Formula : $M^{-1}\left(\frac{v}{v-a}\right) = e^{at} = f(t)$

iv) M $(sinat)$=$\frac{av}{v^2+a^2}$=$H(v)$

Inversion Formula : $M^{-1}(\frac{v}{v^2+a^2}) = \frac{sinat}{a} = f(t)$

v) M (cos(at))=$\frac{v^2}{v^2+a^2} = H(v)$

Inversion Formula : $M^{-1}\left(\frac{v^2}{v^2+a^2}\right) = \cos(at) = f(t)$

2.3.2 Mahgoub Transform of derivatives :

(i) $M[f'(t)]$=$vH(v) - vf(0)$

(ii) $M\,[f''(t)]=v^2H(v)-vf'(0)-v^2f(0)$

3. *Application* :

In this section, the effectiveness and the usefulness of Laplace, Sumudu and Mahgoub transform technique are demonstrated by finding exact solution of a system of homogeneous and non homogeneous Linear differential equations of first order and first degree with constant coefficients and satisfying some initial conditions.

Example : (1) Find the solutions of the system of equations

$$dx/dt + y = 2cost \quad (4)$$

$$dy/dt - x = 1 \quad (5)$$

With initial conditions x(0) = -1 and y(0) = 1

Solution: Applying the Laplace transform of both sides of Eq. (4) and (5)

$$L[dx/\,dt] + L[y] = 2L[cost]$$

$$L[dy/dt] - L[x] = L[1]$$

Since $L[x(t)] = F_1(s)\ and\ L[y(t)] = F_2(s)$

$$sF_1(s) - x(0) + F_2(s) = 2/(s^2+1\,) \quad (6)$$

$$sF_2(s) - y(0) + F_1(s) = 1/s \quad (7)$$

Solving these equations for $F_1(s) and\ F_2(s)$;

$$F_1(s) = 2s^2/(s^2+1\,)^2\ - 1/s(s^2+1\,) - 1/(s^2+1\,) - s/(s^2+1\,) \quad (8)$$

$$F_2(s) = 2s/(s^2+1\,)^2\ +\ s/(s^2+1\,) \quad (9)$$

Appling Inverse Laplace transforms.

Thus required solution of given differential equations are

$$x(t) = tcost - 1\ \ and\ y(t) = tsint + cost \quad (10)$$

2: Applying the Sumudu transform of both sides of Eq. (4) and (5)

$$S[dx/\,dt] + S[y] = 2S[cost] \quad (11)$$

$$S[dy/dt] - S[x] = S[1] \quad (12)$$

Since $S[x(t)] = G_1(u)\ and\ S[y(t)] = G_2(u)$

$$(G_1(u) - x(0))/u + G_2(u) = 2/(u^2+1\,) \quad (13)$$

$$(G_2(u) - y(0))/u + G_1(u) = 1 \quad (14)$$

Solving these equations for $G_1(u) and\ G_2(u)$ Then Appling Inverse Sumudu transforms.

Thus required solution of given differential equations are

$$x(t) = tcost - 1\ \ and\ y(t) = tsint + cost \quad (15)$$

3: Applying the Mahgoub transform of both sides of Eq. (4) and (5)

$$M[dx/\,dt] + M[y] = 2M[cost] \quad (16)$$

$$M[dy/dt] - M[x] = M[1] \quad (17)$$

Since $M[x(t)] = H_1(v)\ and\ M[y(t)] = H_2(v)$

$$vH_1(v) - vx(0) + H_2(v) = 2v^2/(v^2+1\,) \quad (18)$$

$$vH_2(v) - vy(0) + H_1(v) = 1 \quad (19)$$

Solving these equations for $H_1(v)\ and\ H_2(v)$ Then Appling Inverse Mahgoub transforms.

Thus required solution of given differential equations are

$$x(t) = tcost - 1\ \ and\ y(t) = tsint + cost \quad (20)$$

Example : (2) Find the solutions of the system of equations

$$dx/\,dt + \alpha y = 0 \quad (21)$$

$$dy/dt - \alpha x = 0 \quad (22)$$

With initial conditions x(0) = c_1 and y(0) = c_2**,**where c_1, c_2 are arbitrary constants.

Solution: Applying the Laplace transform of both sides of Eq. (21) and (22),

$$L[dx/\,dt] + \alpha L[y] = 0 \quad (23)$$

$$L[dy/dt] - \alpha L[x] = 0 \quad (24)$$

Since $L[x(t)] = F_1(s)$ and $L[y(t)] = F_2(s)$

$$sF_1(s) - c_1 + \alpha F_2(s) = 0 \quad (25)$$

$$sF_2(s) - c_2 - \alpha F_1(s) = 0 \quad (26)$$

Solving these equations for $F_1(s)$ and $F_2(s)$;

$$F_1(s) = \frac{c_1 s - c_2\alpha}{s^2 + \alpha^2} \quad and \; F_2(s) = \frac{c_1\alpha + c_2 s}{s^2 + \alpha^2}$$

Appling Inverse Laplace transforms, we get general solution of given differential equations are

$$x(t) = c_1 cos\alpha t - c_2 sin\alpha t \quad and \; y(t) = c_1 sin\alpha t + c_2 cos\alpha t \quad (27)$$

Squaring and adding ,we get

$$x^2 + y^2 = {c_1}^2 + {c_2}^2 \quad \text{which represents a circle.} \quad (28)$$

2: Applying the Sumudu transform of both sides of Eq. (21) and (22),

$$S[dx/\,dt] + \alpha S[y] = 0 \quad (29)$$

$$S[dy/dt] - \alpha S[x] = 0 \quad (30)$$

Since $S[x(t)] = G_1(u)$ and $S[y(t)] = G_2(u)$

$$(G_1(u) - x(0))/u + \alpha G_2(u) = 0 \quad (31)$$

$$(G_2(u) - y(0))/u - \alpha G_1(u) = 0 \quad (32)$$

Solving these equations for $G_1(u)$ and $G_2(u)$ Then Appling Inverse Sumudu transforms.

Thus required solution of given differential equations are

$$x(t) = c_1 cos\alpha t - c_2 sin\alpha t \quad and \; y(t) = c_1 sin\alpha t + c_2 cos\alpha t \quad (33)$$

3: Applying the Mahgoub transform of both sides of Eq. (21) and (22),

$$M[dx/\,dt] + \alpha M[y] = 0 \quad (34)$$

$$M[dy/dt] - \alpha M[x] = 0 \quad (35)$$

Since $M[x(t)] = H_1(v)$ and $M[y(t)] = H_2(v)$

$$vH_1(v) - vx(0) + \alpha H_2(v) = 0 \quad (36)$$

$$vH_2(v) - vy(0) - \alpha H_1(v) = 0 \quad (37)$$

Solving these equations for $H_1(v)$ and $H_2(v)$ Then Appling Inverse Mahgoub transforms.

Thus required solution of given differential equations are

$$x(t) = c_1 cos\alpha t - c_2 sin\alpha t \quad and \; y(t) = c_1 sin\alpha t + c_2 cos\alpha t \quad (38)$$

Example : (3) Find the solutions of the system of equations

$$\frac{dx}{dt} + y = e^t \quad (39)$$

$$\frac{dy}{dt} - x = -t \quad (40)$$

With initial conditions x(0) = 0 and y(0) = 0

Solution: Applying the Laplace transform of both sides of Eq. (39) and (40),

$$L[dx/\,dt] + L[y] = L[e^t\,] \quad (41)$$

$$L[dy/dt] - L[x] = -L[t] \quad (42)$$

Since $L[x(t)] = F_1(s)$ and $L[y(t)] = F_2(s)$

$$sF_1(s) - x(0) + F_2(s) = 1/(s-1\,) \quad (43)$$

$$sF_2(s) - y(0) - F_1(s) = -1/s^2 \quad (44)$$

Solving these equations for $F_1(s)$ and $F_2(s)$;

$$F_1(s) = (s^3 + s - 1)/s^2(s-1\,)^2(s+1) \quad (45)$$

$$F_2(s) = 1 - 2s/s(s-1\,)^2(s+1) \quad (46)$$

Appling Inverse Laplace transforms.

Thus required solution of given differential equations are

$$x(t) = e^t/2 - 1/2cost - 1/2sint \quad and \quad y(t) = -1 + e^t/2 + 1/2cost - 1/2sint \qquad (47)$$

2: Applying the Sumudu transform of both sides of Eq. (39) and (40),

$$S[dx/dt] + S[y] = S[e^t] \qquad (49)$$

$$S[dy/dt] - S[x] = -S[t] \qquad (50)$$

Since $S[x(t)] = G_1(u)$ and $S[y(t)] = G_2(u)$

$$(G_1(u) - x(0))/u + G_2(u) = 1/(1-u) \qquad (50)$$

$$(G_2(u) - y(0))/u - G_1(u) = -u \qquad (51)$$

Solving these equations for $G_1(u)$ and $G_2(u)$ Then Appling Inverse Sumudu transforms.

Thus required solution of given differential equations are

$$x(t) = e^t/2 - 1/2cost - 1/2sint \quad and \quad y(t) = -1 + e^t/2 + 1/2cost - 1/2sint \qquad (52)$$

3: Applying the Mahgoub transform of both sides of Eq. (39) and (40),

$$M[dx/dt] + M[y] = M[e^t] \qquad (53)$$

$$M[dy/dt] - M[x] = -M[t] \qquad (54)$$

Since $M[x(t)] = H_1(v)$ and $M[y(t)] = H_2(v)$

$$vH_1(v) - vx(0) + H_2(v) = v/(v-1) \qquad (55)$$

$$vH_2(v) - vy(0) - H_1(v) = -1/v \qquad (56)$$

Solving these equations for $H_1(v)$ and $H_2(v)$ Then Appling Inverse Mahgoub transforms.

Thus required solution of given differential equations are

$$x(t) = e^t/2 - 1/2cost - 1/2sint \quad and \quad y(t) = -1 + e^t/2 + 1/2cost - 1/2sint \qquad (57)$$

Example : (4) Find the solutions of the system of equations

$$\frac{dx}{dt} = x + y \qquad (58)$$

$$\frac{dy}{dt} = 2x + 4y \qquad (59)$$

With initial conditions x(0) = 1 and y(0) = 2

Solution: Applying the Laplace transform of both sides of Eq. (58) and (59),

$$L[dx/dt] = L[x] + L[y] \qquad (60)$$

$$L[dy/dt] = 2L[x] + 4L[y] \qquad (61)$$

Since $L[x(t)] = F_1(s)$ and $L[y(t)] = F_2(s)$

$$sF_1(s) - x(0) = F_1(s) + F_2(s) \qquad (62)$$

$$sF_2(s) - y(0) = 2F_1(s) + 4F_2(s) \qquad (63)$$

Solving these equations for $F_1(s)$ and $F_2(s)$ and Appling Inverse Laplace transforms,we get

Thus required solution of given differential equations are

$$x(t) = e^{2t} - 2e^{-t} - 2t + 1 \; and \; y(t) = e^{2t} + 4e^{-t} + 2t - 3 \qquad (64)$$

2: Applying the Sumudu transform of both sides of Eq. (58) and (59),

$$S[dx/dt] = S[x] + S[y] \qquad (65)$$

$$S[dy/dt] = 2S[x] + 4S[y] \qquad (66)$$

Since $S[x(t)] = G_1(u)$ and $S[y(t)] = G_2(u)$

$$(G_1(u) - x(0))/u = G_1(u) + G_2(u) \qquad (67)$$

$$(G_2(u) - y(0))/u = 2G_1(u) + 4G_2(u) \qquad (69)$$

Solving these equations for $G_1(u)$ and $G_2(u)$ Then Appling Inverse Sumudu transforms.

Thus required solution of given differential equations are

$$x(t) = e^{2t} - 2e^{-t} - 2t + 1 \; and \; y(t) = e^{2t} + 4e^{-t} + 2t - 3 \qquad (70)$$

3: Applying the Mahgoub transform of both sides of Eq. (58) and (59),

$$M[dx/dt] = M[x] + M[y] \qquad (71)$$

$$M[dy/dt] = 2M[x] + 4M[y] \qquad (72)$$

Since $M[x(t)] = H_1(v)$ and $M[y(t)] = H_2(v)$

$$vH_1(v) - vx(0) = H_1(u) + H_2(u) \quad (73)$$

$$vH_2(v) - vy(0) = 2H_1(u) + 4H_2(u) \quad (74)$$

Solving these equations for $H_1(v)$ and $H_2(v)$ Then Appling Inverse Mahgoub transforms.

Thus required solution of given differential equations are

$$x(t) = e^{2t} - 2e^{-t} - 2t + 1 \text{ and } y(t) = e^{2t} + 4e^{-t} + 2t - 3 \quad (75)$$

4. *Conclusion*

The main goal of this paper is to conduct Comparison between Laplace, Sumudu and Mahgoub Transforms for Solving system of First order First Degree Differential Equations .The three methods are powerful and efficient.Sumudu and Mahgoub Transforms is a convenient tool for solving Solving system of First order First Degree differential equations in the time domain without the need for performing an inverse Sumudutransform and inverse a Mahgoub transform and the connection of Sumudu transform and a Mahgoub transform with Laplace transform goes much deeper.

REFERENCES

[1] Abdelbagy A. Alshikh, Mohand M. Abdelrahim Mahgob. A Comparative Study Between Laplace Transform and Two New Integrals "ELzaki" Transform and "Aboodh" Transform. *Pure and Applied Mathematics Journal*. Vol. 5, No. 5, 2016, pp. 145-150.

[2] Hassan Eltayeb and AdemKilicman, (2010), A Note on the Sumudu Transforms and differential Equations,Applied Mathematical Sciences, VOL, 4, no. 22, 1089-1098

[3] Mohand M. Abdelrahim Mahgoub., "The New Integral Transform Mahgoub Transform", Advances in Theoretical and Applied Mathematics, Vol.11, No.4, pp. 391 – 398, 2016

[4] G.K.watugala , simudu transform- a new integral transform to Solve differential equation and control engineering problems .Math .Engrg Induct .6 (1998), no 4,319-329.

[5] P.R.Bhadane and K.P.Ghadle(2016), Application of Elzaki Transform to System of Linear Differential Equations,the international Journal ,RJSITM, Volume: 06

[6] A. Kilicman and H. E. Gadain. (2009), An application of Double Laplace transform and Sumudu transform, Lobachevskii J.Math.30(3),pp.214-223. http://dx.doi.org/10.1134/s1995080209030044.

[7] J. Zhang, Asumudu based algorithm m for solving differential equations, Comp.Sci.Moldova 15(3) (2007), pp – 303-313.

[8] Hassan Eltayeb and AdemKilicman, (2010), A Note on the Sumudu Transforms and differential Equations, Applied Mathematical Sciences, VOL, 4, no. 22, 1089-1098.

[9] Raisinghania, M.D., Integral equations and boundary value problems, S.Chand & Company, New- Delhi, 2017

[10] Polyanin, A.D. and Manzhirov, A.V., Handbook of integral equations, Chapman & Hall/CRC, 2008.