Design of MIMO ball and beam system using PID controller in Automation

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Abstract : This paper employing the method of design a MIMO Non-Linear system using fuzzy controller, the BALL and BEAM System has to be taken as a MIMO Non Linear system. The ball and beam system can usually be found in most university control labs, model and control theoretically. Controller design for the ball and beam is one of the most challenging tasks in control design as the system is open loop unstable. It is very difficult to achieve high-level performance in such systems. The system includes a ball, a beam, a motor and several sensors. The basic idea is to use the torque generated from motor to the control the position of the ball on the beam. The ball rolls on the beam freely. By employing linear sensing techniques, the information from the sensor can be taken and compared with desired positions values. The difference can be fed back into the controller, and then in to the motor in order to gain the desired position. The mathematical model for this system is inherently nonlinear but may be linearized around the horizontal region. This simplified linearised model, however, still represents many typical real systems, such as horizontally stabilizing an airplane during landing and in turbulent airflow. By considering real plant problems such as the sensor noise and actuator saturation, the controllers of the system become more efficient and robust.

The mathematical model of the system is formulated. Initially the system is simulated with classical controller in MATLAB. Regulation and tracking of the system are tested by giving a step input and sine input respectively. The control algorithm used in the present study was found to be effective in controlling the open loop, non-linear unstable system.

***Keywords* - MIMO , PID Controller, Non-Linear System ,Stepper motor, MATLAB.**

I. INTRODUCTION

The aim is to design a PID controller for ball and beam system to track the ball to a commanded position by varying the beam angle. The ball and beam system is also called ‘balancing a ball on a beam’. It can usually be found in most university control labs. It is generally linked to real control problems such as horizontally stabilizing an airplane during landing and in turbulent airflow. There are two degrees of freedom in this system. One is the ball rolling up and down the beam, and the other is beam rotating through its central axis. The aim of the system is to control the position of the ball to a desired reference point. The control signal can be derived by feeding back the position information of the ball. The control voltage signal goes to the DC motor via a power amplifier， then the torque generated from the motor drives the beam to rotate to the desired angle. Thus, the ball can be located at the desired position.

It is important to point out that the open loop of the system is unstable and nonlinear. The problem of ‘instability’ can be overcome by closing the open loop with a feedback controller. The modern state-space method can be employed to stabilize the system. The nonlinear property is not significant when the beam only deflects a small angle from the horizontal position. In this case, it is possible to linearize the system. However, the non-linearities become significant when the angle of the beam from the horizontal is larger than 30 degrees, or smaller than -30 degrees. In this case, a simple linear approximation is not accurate. Thus a more advanced control technique such as nonlinear control should work better.

II. Related Works

Arroyo (2005) built the system named the 'Ball on Balancing Beam' in 2005, A system employed the resistive wire sensor to measure the position of the ball. The resistive position sensor acted as a wiper similar to a potentiometer resulting in the position of the ball. The signal from the sensor was processed in a DSP. A DC motor with reducing gear was used. The system was controlled by PD controller. This system was easy to build, and the simple PD controller was easy to design. The negative aspects of the 'Ball on Balancing Beam' system includes that the beam was made of acrylic, which may be too brittle for a sudden impact. Additionally, although the position of the ball was controlled by the PD controller, the tilt angle of the beam was not measured and controlled. Therefore, the system may be not very robust.

Quanser (2006) present its commercial product named 'ball and beam module', The ball and beam module consisted of the position sensor made by resistive wires and a DC servo motor with reducing gearbox. The system could be controlled by a PID controller or a state space controller. A relative small motor could be used for the system due to the leverage effect. The configuration of the 'ball and beam module' is more complicated than 'Ball and Beam Balancer'.

Hirsch (1999) built his 'Ball on Beam System' in 1999. The system employed an ultrasonic sensor to measure the position of the ball. The angle of the beam was measured though the use of a potentiometer. The motor with a gearbox was driven with a high power op-amp circuit. The system is controlled by a PD controller. Hirsch's system was easy to build due to the simple mechanical configuration. However, the shaft, which supported the weight the beam, was too long for the motor bearing. Therefore the motor bearing would experience a large moment from the beam.

Rosales (2004) built the ball and beam system, which was similar with the 'Ball and Beam Balancer' system (Ambalavanar, Moinuddin & Malyshev 2006). Rosales's system was made of acrylic, but the 'Ball and Beam Balancer' system was made of aluminum.

Lieberman (2004) build a system named 'A Robotic Ball Balancing Beam', The system is similar with 'Ball on Beam System' (Hirsch 1999). The difference between the two systems is that Lieberman's system used a resistive wire position sensor, and the Hirsch's system used an ultrasonic position sensor.

III. Ball and beam system

The ball and beam system is one of the basic examples of nonlinear and unstable control system. This system commonly used for control theory verification or control system design and implementation practice. This system getting popular and become an important laboratory models for teaching control system engineering due to very simple to understand as a system and the control technique that can be studied and it cover many important classical and modern design methods. The system also used as a control training tool in many industrial processes and their application. The ball and beam mechanism consists of a beam and solid ball on it. The ball is rolling free along the beam according to the changing angle of the beam. The beam rotates in the vertical plane driven by a torque usually using servo motor at the side. The required ball position is being control by applying an electrical control signal to the motor amplifier. The position of ball on the beam can measured using special sensor. The control task is difficult because the ball does not stay in one place on the beam but moves with an acceleration that is proportional to the tilt of beam. In control technology the system is open loop unstable because the system output (ball position) increases without limit for a fixed input (beam angle). Feedback control must be used to keep the ball in a desired position on the beam. There are so many method and type in designing the model of ball and beam system hardware [4]. Most the ball and beam system using stainless steel material because it looks clean and it using gears to connect to the motor. Fig.1 Shows the ball on balancing beam build by Berkeley Robotics Laboratory.

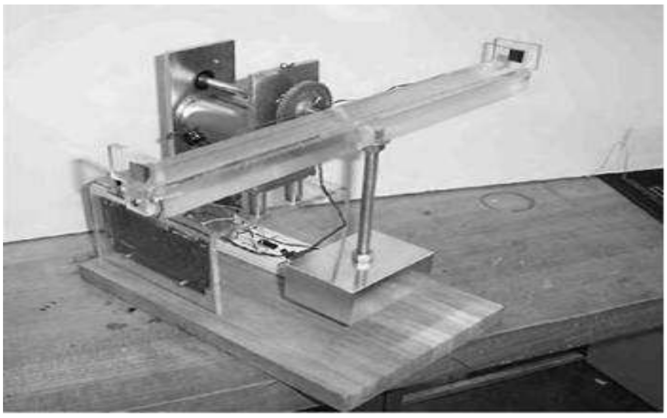

Figure.1. Ball on balancing beam build by Berkeley Robotics Laboratory.

The ball on beam balancer system is one of the most enduringly popular and important laboratory models for teaching control systems engineering. Control Job: Automatically regulating the position of the ball on the beam by changing the angle of the beam. The open loop is unstable.

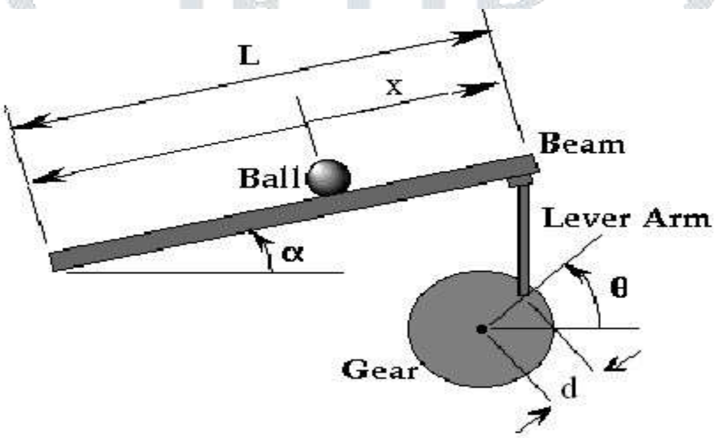

Figure.2 System identification of ball and beam system.

L - Length of the beam

x - Ball position coordinate

α -Beam angle

Θ -gear angle

d - lever radius

Ball is placed on the beam where it is allowed to roll with one degree of freedom along the length of the beam. Ball is positioned by controlling the beam angle using DC stepper motor [8]. To control the position of the ball two variables have to be measured: ball position and beam angle.

IV. Rotary servo plant with encoder

The DC servo motor plant is used as an actuator for the ball and beam system. A high quality DC servo motor is mounted in a solid aluminum frame. The motor drives a built-in Swiss-made 14:1 gearbox whose output drives an external gear. The motor gear drives a gear attached to an independent output shaft that rotates in a precisely machined aluminum ball bearing block. The output shaft is equipped with an encoder. This second gear on the output shaft drives an anti-backlash gear connected to a precision potentiometer. The potentiometer is used to measure the output angle.

The external gear ratio can be changed from 1:1 to 5:1 using various gears. Two inertial loads are supplied with the system in order to examine the effect of changing inertia on closed loop performance. In the high gear ratio configuration, rotary motion modules attach to the output shaft using two 8-32 thumbscrews. The square frame allows for installations resulting in rotations about a vertical or a horizontal axis.

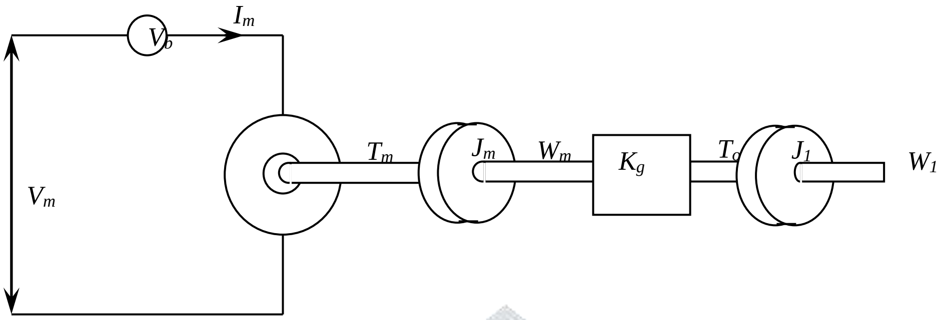

Figure.3 Block diagram of the servo motor system.

V. Mathematical model of the DC motor

This is derived from the basic equations of a DC motor

$$V_m = I_m R_m + V_b \tag{1}$$

Where V_m - Input voltage given to motor

I_m - Current

R_m - Armature resistance and

V_b – Back emf

$$V_b = K_m W_m \tag{2}$$

Where K_m – Back emf constant

W_m – Angular velocity of motor shaft and is given by

$$W_m = K_g W_1 \tag{3}$$

Where K_g – motor gear ratio and

W_l – Angular velocity of output shaft

Torque generated by motor is

$$T_m = K_m I_m \tag{4}$$

Torque at the output after gearbox can be obtained using Lagrange equation:

$$KE_{motor} = \frac{1}{2} J_m \theta^2$$

$$KE_{motor} = \frac{1}{2} J_m K_g{}^2 \theta_1{}^2 \qquad \text{(From 3)}$$

$$KE_{load} = \frac{1}{2} J_1 \theta_1$$

Total $KE = \frac{1}{2}\theta_1\left(J_m K_g^{\,2} + J_1\right)\theta^2$

$PE = 0$

Lagrangian function L = KE - PE = $\frac{1}{2}\theta_1\left(J_m K_g^{\,2} + J_1\right)\theta^2$

Lagrange Equation is $\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \left(\frac{\partial L}{\partial \theta_1}\right) = T_o$

Where $\frac{\partial L}{\partial \theta_1} = 0; \frac{\partial L}{\partial \dot{\theta}_1} = \theta_1\left(J_m K_g^{\,2} + J_1\right)$

Therefore $\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = W_1\left(J_m K_g^{\,2} + J_1\right) = W_1 J_{eq} = T_o$ (5)

From (4) $\mathrm{I_m} = T_m / K_m$

Substituting T_m from (5) we get

$$\mathrm{I_m} = \frac{\dot{W}_1 J_{eq}}{K_m K_g} \quad (6)$$

From (2) $V_b = K_m \mathrm{W_m}$

Substituting W_m from (3) we get

$$V_b = K_m K_g W_1 \quad (7)$$

On substituting (6) and (7) in (1) we get

$$V_m = \frac{W_1 J_{eq} R_m}{K_m K_g} + K_m K_g W_1$$

$$ie;\ V_m = \frac{\ddot{\theta} J_{eq} R_m}{K_m K_g} + K_m K_g \dot{\theta}$$

Taking Laplace transform we get

$$\frac{\theta(s)}{V_m(s)} = \frac{1}{s\left(\frac{sJ_{eq}R_m}{K_g K_m} + K_g K_m\right)}$$

From rotary servo motor parameters:

R_m=2.6Ω

K_m=0.00767Nm/amp

K_g=70

$J_{eq} = J_m K_g^2 + J_1$

J_m=3.87×10^{-7} Kgm^2

J_1=0.00003 Kgm^2

Therefore J_{eq}=1.9263×10^{-3} Kgm^2

K_m×K_g=0.5369

$$\frac{J_{eq}R_m}{K_mK_g} = 9.328\times10^{-3}$$

Therefore $$\frac{\theta(s)}{V_m(s)} = \frac{1}{s\left(0.009328s + 0.5369\right)} \quad (8)$$

5.1 CONTROL SYSTEM DESIGN OF MOTOR

The open loop position response of the Dc motor is unstable due to the pole at the origin. A PD and PID controller can be used for the output to track the desired angle.

5.1.1 PD Control

$$V_m = K_p\ (\theta_d - \theta) - K_d\theta$$

$$V_m(s) = N_p\theta_d(s) - \left(K_p + sK_d\right)\theta(s)$$

Substituting in (8) we get

$$\frac{\theta(s)}{V_m(s)} = \frac{K_p}{s\left(0.009328s^2 + (0.5369 + K_d)s + K_p\right)}$$

Characteristic equation is

$$\frac{s^2}{0.009328} + \frac{(0.5369 + K_d)s}{0.009328} + K_p \quad (9)$$

Constraints are:

- T_p=100ms=0.1s
- ζ (z)= 0.707

Since $T_p = \dfrac{\pi}{W_n\sqrt{1-\varsigma^2}}$; $W_n = 44.42$

General 2nd order equation is $s^2 + 2\varsigma W_n s + W_n{}^2$

Substituting the values of W_n and ζ we get:

$$s^2 + 62.809\ s + 1973.1364 \quad (10)$$

Comparing 9 and 10 we get:

K_p= 18.40
K_d= 0.0489

5.1.2 PID Control

$$V_m(s) = Kp\left(\theta_d(s) - \theta(s)\right) + sK_d\theta(s) + \frac{Ki\left(\theta_d(s) - \theta(s)\right)}{s}$$

Substituting in 8 we get

$$\frac{\theta(s)}{\theta_d(s)} = \frac{\left(K_p + K_i/s\right)}{0.00932s^3 + (0.5369 + K_d)s^2 + K_p s + K_i}$$

Characteristic equation is

$$s^3 + \frac{(0.5369 + K_d)s^2}{0.009328} + \frac{K_p s}{0.009328} + \frac{K_i}{0.009328} \qquad (11)$$

Constraints are: Constraints are:

- T_p=100ms=0.1s
- ζ (z)= 0.707

Since $T_p = \frac{\pi}{W_n\sqrt{1-\varsigma^2}}$; $W_n = 44.42$

General 3rd order equation is $s^3 + 1.75W_n s^2 + 2.15W_n^2 s + W_n^3$

Substituting the values of W_n and ζ we get:

$$s^3 + 77.735\ s^2 + 4242.24326s + 87646.7189 \qquad (12)$$

Comparing (11) and (12) we get:

Kp = 0.1882
Kd = 39.5716
Ki = 817.5686

5.2 MATHEMATICAL MODELLING OF THE PLANT

A ball is placed on a beam, where it is allowed to roll with one degree of freedom along the length of the beam. The position of the ball is changed by changing the angle of the beam. When the angle is changed from the vertical position, gravity changes the ball to roll along the beam. Forces acting on the system are shown below:

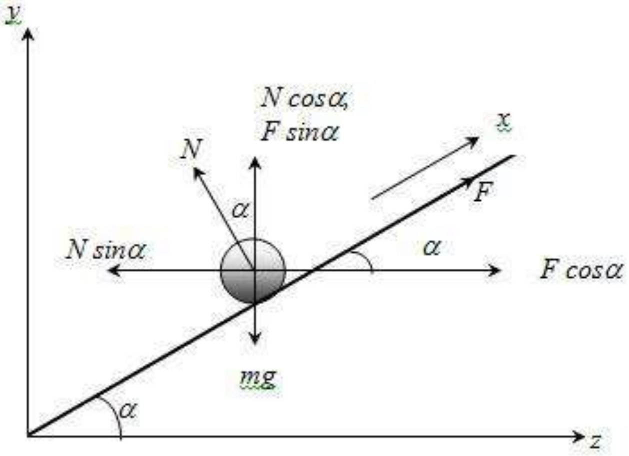

Figure.4 Forces act on the system

Forces acting along y-axis are

$$m\ddot{y} = -mg + N\cos\alpha + F\sin\alpha \qquad (1)$$

Forces acting along z-axis are

$$m\ddot{z} = -N\sin\alpha + F\cos\alpha \qquad (2)$$

(1)×sinα + (2)×cosα gives

$$m(\ddot{y}\sin\alpha + \ddot{z}\cos\alpha) = -mg\sin\alpha + F$$

But $y = x\sin\alpha$ and $z = x\cos\alpha$, therefore

$$\ddot{z} = \ddot{x}\cos\alpha - 2\dot{x}\dot{\alpha}\sin\alpha - x\dot{\alpha}^2\cos\alpha - x\ddot{\alpha}\sin\alpha \qquad (4)$$

$$\ddot{y} = \ddot{x}\sin\alpha - 2\dot{x}\dot{\alpha}\cos\alpha - x\dot{\alpha}^2\sin\alpha - x\ddot{\alpha}\cos\alpha \qquad (5)$$

(3) $(4)\times\cos\alpha + (5)\times\sin\alpha$ gives

$$\ddot{y}\sin\alpha + \ddot{z}\cos\alpha = \ddot{x} - x\dot{\alpha}^2$$

Therefore (3) becomes

$$\mathrm{m}\left(\ddot{x} - x\dot{\alpha}^2\right) = -mg\sin\alpha + F \qquad (6)$$

The condition for a rolling ball without friction is

$$F_r = J\dot{w} = -J\ddot{x}/r^2, \text{ hence}$$

$$\mathrm{F} = -J\ddot{x}/r^2$$

Therefore (6) becomes

$$\mathrm{m}\left(\ddot{x} - x\dot{\alpha}^2\right) = -mg\sin\alpha - J\ddot{x}/r^2$$

$$\left(\mathrm{m} + J/r^2\right)\ddot{x} + mg\sin\alpha - mx\dot{\alpha}^2 = 0$$

Thus the non-linear mathematical model of Ball and beam is

$$\left(\mathrm{m} + J/r^2\right)\ddot{x} + mg\sin\alpha - mx\dot{\alpha}^2 = 0$$

Linearization of the plant can be done at the operating point α, by putting α=0. Thus the linear mathematical model of the plant is:

$$\ddot{x}\left(\frac{J}{r^2} + m\right) + mg\alpha = 0$$

5.2.1 Control system design of the plant

5.2.1.1 PD Control

The mathematical model of ball and beam is

$$\frac{X(s)}{\theta(s)} = -\frac{mgd}{L\left(\frac{J}{r^2} + m\right)}\frac{1}{s^2}$$

Substituting the parameters:

m=0.06Kg

r=0.0027m

L=0.425m

$J=9.99\times10^{-6}Kgm^2$

d=0.0254m

we get $\frac{X(s)}{\theta(s)} = -\frac{-0.0246}{s^2}$ (7)

$$\theta = \left(L/d\right)\alpha = 17\alpha$$

$$\alpha = K_{p_1}(X_d - X) + K_{d_1}\frac{d(X_d - X)}{dt}$$

Therefore $\theta(s) = 17\ (K_{p_1} +\ sK_{d_1})\ X_d(s)\text{-}17\ (K_{p_1} + sK_{d_1})\ X(s)$

Substituting in 7 we get

$$\frac{X(s)}{X_d(s)} = -\frac{0.416(K_{p_1} + sK_{d_1})}{s^2 - 0.416sK_{d_1} - 0.416K_{p_1}}$$

Characteristics equation is

$s^2\text{-}\ 0.416\ s\ K_{d1}\text{-}\ 0.416\ K_p$ (8)

Constraints are:

- T_p=3s
- $\zeta(z)$=0.707

Therefore W_n=1.481

Substituting the values in general 2nd order equation we get

$s^2 + 2.094\ s + 2.193$ (9)

Comparing (8) and (9) we get:

K_{p1}= -5.272
K_{d1}= -5.034

5.2.2 PID Control

$$\theta = \left(L/d\right)\alpha = 17\alpha$$

$$\alpha = K_{p_1}(X_d - X) + K_{d_1}s(X_d - X) + K_{i_1}\frac{(X_d - X)}{s}$$

Therefore $\theta(s) = 17\left(K_{p_1} +\ sK_{d_1}\ + K_{i_1}/s\right)\ Xd(s)\ \text{-}17\left(K_{p_1} +\ sK_{d_1}\ + K_{i_1}/s\right)\ X(s)$

Substituting in (7) we get

$$\frac{X(s)}{X_d(s)} = -\frac{0.416(K_{p_1} + s^2 K_{d_1} + K_{i_1})}{s^3 - 0.416 s^2 K_{d_1} - 0.416 s K_{p_1} - 0.416 K_{i_1}} \quad (10)$$

Characteristics equation is

$s^3 - 0.416\, K_{d1} s^2 - 0.416\, K_{p1} s - 0.416\, K_{i1}$

Constraints are:

- Tp=3s
- ζ=0.707

Therefore W_n=1.481

Substituting the values in general 3rd order equation we get

$$s^3 + 2.592 s^2 + 4.716 s + 3.248 \quad (11)$$

Comparing (10) and (11) we get:

Kp_1= -11.497
Kd_1= -6.3195
Ki_1 = -7.918

5.3 RELATION BETWEEN BEAM ANGLE AND GEAR ANGLE

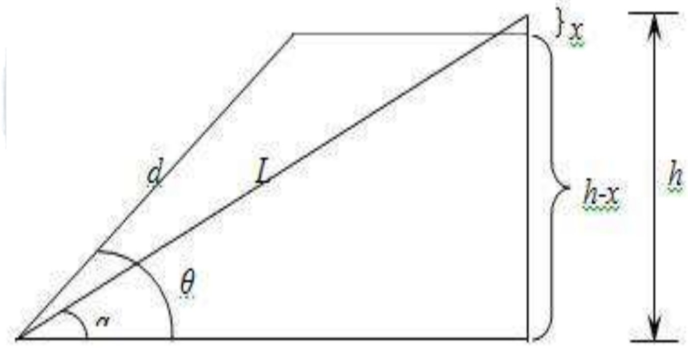

Figure.5 Relation between beam angle and gear angle.

$$Sin\alpha = \frac{h}{L} \ and \ \ Sin\theta = (h - x)/d$$

Therefore $h = LSin\alpha = dSin\theta + x$

Since $x <<$ $dSin\alpha$, neglect x.

Therefore $LSin\alpha = dsin\alpha$,

Since α and θ are small, Sinα □ α and Sinθ =θ,

Therefore $L\alpha = d\theta$,

Therefore $\alpha = d/L\theta$

Taking Laplace transform

$$\frac{X(s)}{\theta(s)} = -\frac{mgd}{L\left(\frac{J}{r^2}+m\right)}\frac{1}{s^2}$$

VI. RESULTS AND DISCUSSION

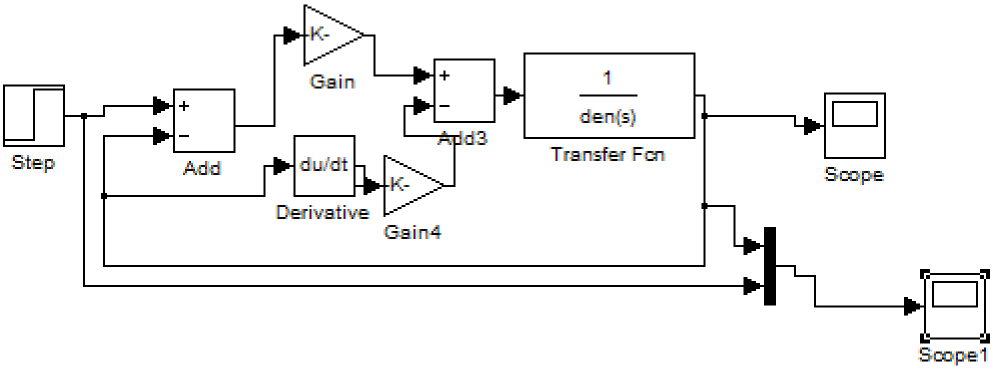

Figure.6 PD controller for simulink model of motor

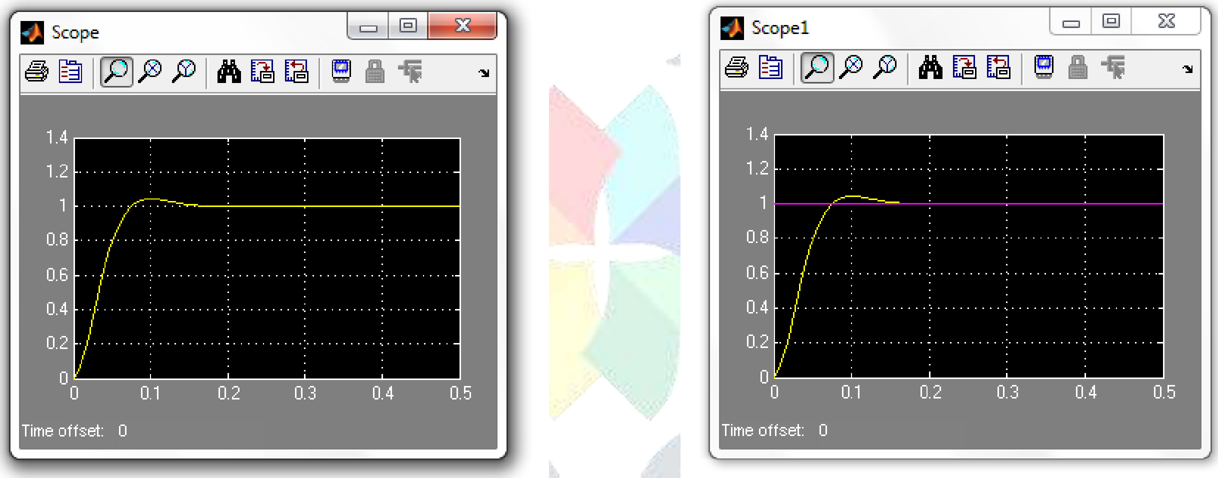

Figure.7 Response of the motor

The above Simulink model of PD motor was implemented for a step input of 1 unit and step time zero, with the designed values of K_p and K_d. It is observed that the response of the system has a peak time of 0.1sec, as per the design. At steady state the response settles at the desired value of 1 unit.

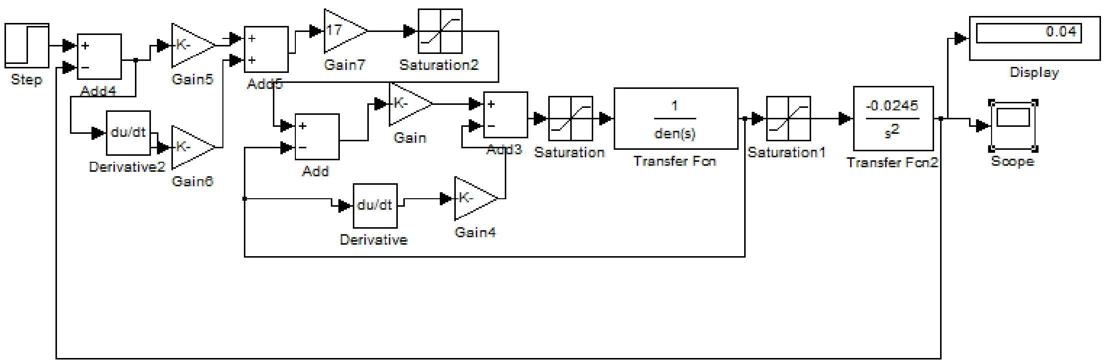

Figure.8 Simulink model of plant for tracking step input

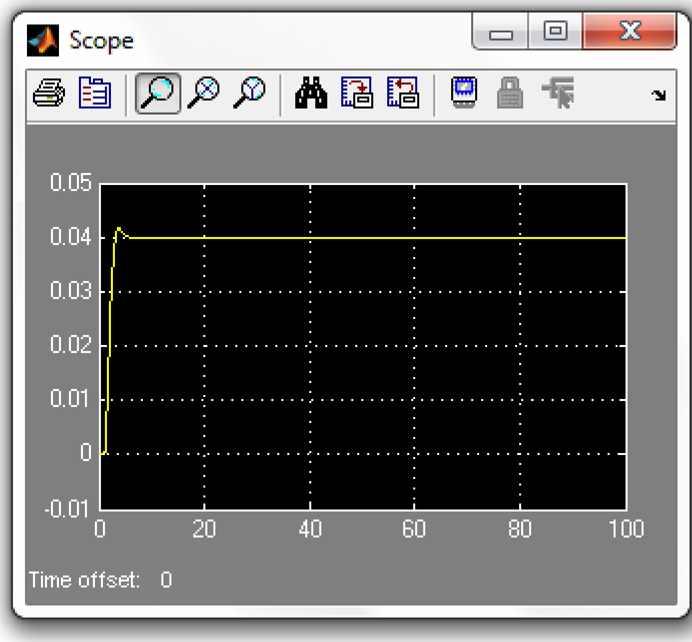

Figure.9 Response for step input

The above Simulink model of PD control of plant was implemented for a step input of 0.04 units and step time one, with the designed values of Kp and Kd. It is observed that the response of the system has a peak time of 3sec (4sec-1sec). At steady state the response settles at the desired value of 0.04 units.

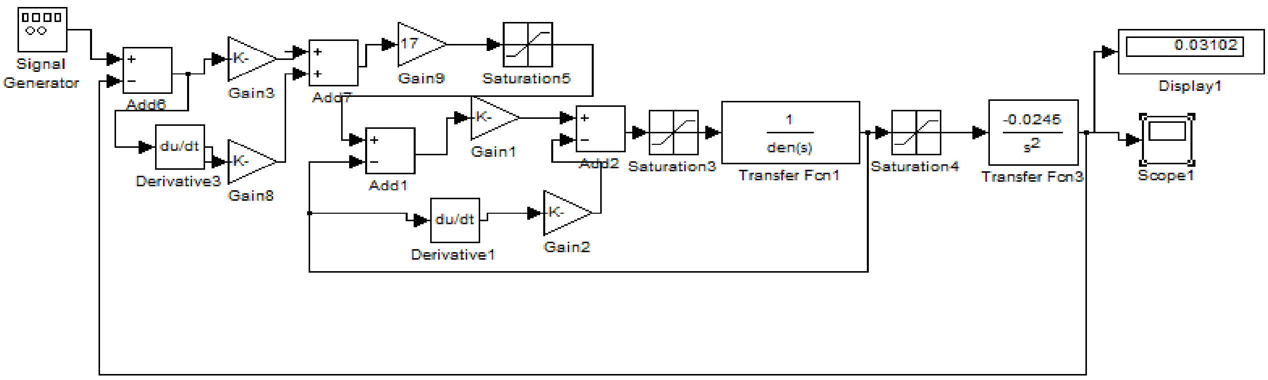

Figure.10 Simulink model of plant for tracking sine input

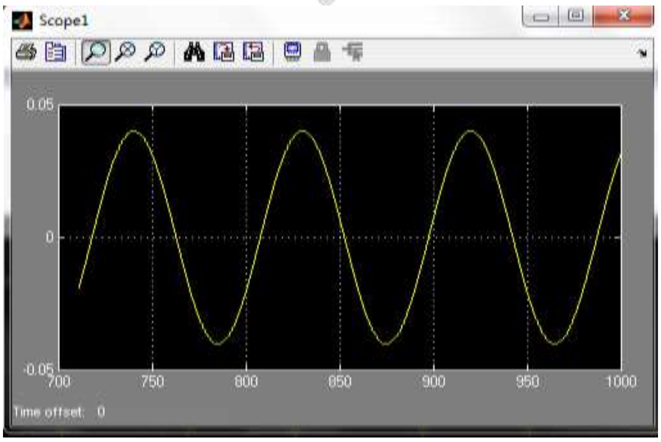

Figure.11 Response for sine input

The above Simulink model of PD control of plant was implemented for a sine input of 0.04 units and frequency of 0.7 rad/sec, with the designed values of K_p and K_d. At steady state the response settles at the desired value of 0.04 units.

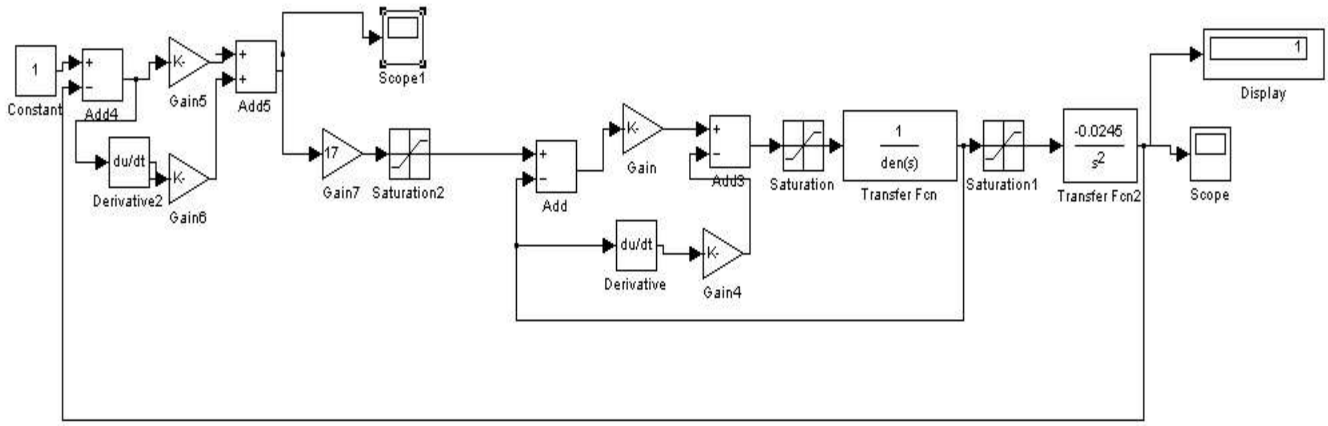

Figure.12 Simulink model of the plant

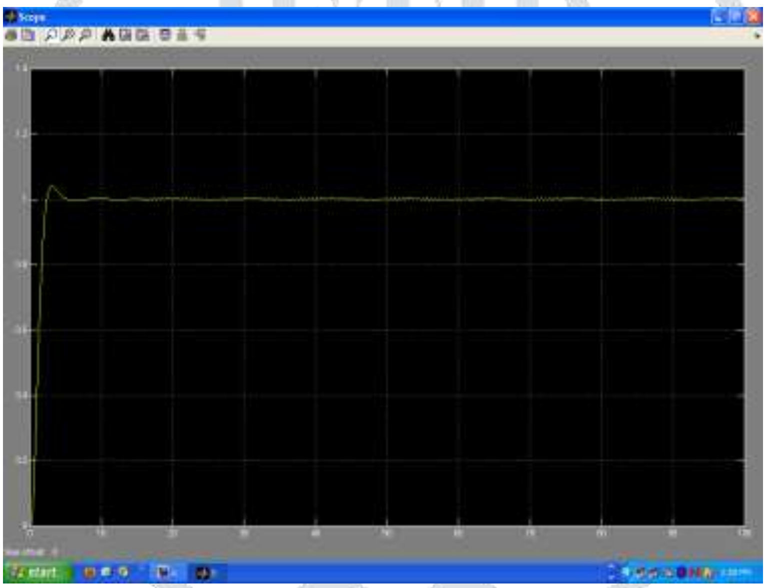

Figure.13 Response of the plant

The above Simulink model of PD control of plant was implemented for a step input of 1 units and step time one, with the designed values of K_p and K_d. It is observed that the response of the system has a peak time of 3sec (4sec-1sec). At steady state the response settles at the desired value of 1 units.

VII. CONCLUSION

To Conclude, we have shown the mathematical model of the DC motor has been developed and the mathematical model of the plant has been developed. The open loop unstable system will be controlled by the classical and PID controller. The PID controller is to be designed for control the system and the motor position is controlled by the controller. And also we have shown the simulation results for PD and PID controller.

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