# g-*b-COMPACTNESS, g-b**-COMPACTNESS AND g-**b-COMPACTNESS IN TOPOLOGICAL SPACES

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**Abstract:** In this paper, we introduced and studied new classes of spaces by making use of *b-open cover, b**-open cover and **b-open cover. We have introduced open cover called g-*b-open cover, g-b**-open cover and g-**b-open cover in topological spaces. This paper deals with g-*b-compact spaces, g-b**-compact spaces and g-**b-compact spaces and their properties by using *b-open, b**-open and **b-open sets.

**Mathematics subject classification:** 54D05, 54D30

**Keywords:** g-*b-compact spaces, g-b**-compact spaces and g-**b-compact spaces.

## I. INTRODUCTION

The notions of compactness is useful and fundamental notions of not only general topology but also of other advanced branches of mathematics. Many researchers have investigated the basic properties of compactness. The productivity of these notions of compactness motivated mathematicians to generalize these notions. g-*b-open(g-b**-open, g-**b-open)[8,12,13] sets are introduced by K.Rekha and T.Indira in 2012-13. S. S. Benchalli and Priyanka M. Bansali [3] introduced gb-compactness & gb-connectedness in topological spaces in the year 2011. The aim of this paper is to introduce the concept of g-*b-compactness, g-b**-compactness and g-**b-compactness in topological spaces.

## II. PRELIMINARIES

Throughout this paper $(X,\tau),(Y,\sigma)$ are topological spaces with no separation axioms assumed unless otherwise stated. Let $A \subseteq X$ . The closure of $A$ and the interior of $A$ will be denoted by $cl(A)$ and $\text{int}(A)$ respectively.

**2.1. Definition:** A subset A of a space X is said to be *b-open [7] if $A \subseteq Cl(Int(A)) \cap Int(Cl(A))$

The complement of *b-open set is said to be *b-closed. The family of all *b-open sets (respectively *b-closed sets) of $(X,\tau)$ is denoted by $*bO(X,\tau)$ (respectively $*bcl(X,\tau)$ ).

**2.2. Definition:** A subset A of a space X is said to be b**-open [4] if $A \subseteq Int(Cl(Int(A))) \cup Cl(Int(Cl(A)))$

The complement of b**-open set is said to be b**-closed. The family of all b**-open sets (respectively b**-closed sets) of $(X,\tau)$ is denoted by $b**O(X,\tau)$ (respectively $b**cl(X,\tau)$ ).

**2.3. Definition:** A subset A of a space X is said to be **b-open [7] if $A \subseteq Int(Cl(Int(A))) \cap Cl(Int(Cl(A)))$

The complement of **b-open set is said to be **b-closed. The family of all **b-open sets (respectively **b-closed sets) of $(X,\tau)$ is denoted by $**bO(X,\tau)$ (respectively $**bcl(X,\tau)$ ).

**2.4. Definition:** [8] Let $X$ be a topological space. A subset $A$ of $X$ is said to be generalized *b-closed if $*bcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

**2.5. Definition:** [12] Let $X$ be a topological space. A subset $A$ of $X$ is said to be generalized b**-closed if $b**cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

**2.6. Definition: [13]** Let $X$ be a topological space. A subset $A$ of $X$ is said to be generalized **b-closed i $**bcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

**2.7. Note:**

The complement of generalized *b-closed set is said to be generalized *b-open.

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**2.8. Definition:** [9]A function $f : X \to Y$ is said to be *b-continuous if $f^{-1}(V)$ is *b-closed in $X$ for every closed set $V$ of $Y$.

**2.9. Definition:**[9]A function $f : X \to Y$ is said to be *b-irresolute if $f^{-1}(V)$ is *b-closed in $X$ for every *b-closed set $V$ of $Y$.

**2.10. Definition:**[9]A function $f : X \to Y$ is said to be b**-continuous if $f^{-1}(V)$ is b**-closed in $X$ for every closed set $V$ of $Y$.

**2.11. Definition:**[9]A function $f : X \to Y$ is said to be b**-irresolute if $f^{-1}(V)$ is b**-closed in $X$ for every b**-closed set $V$ of $Y$.

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## III. g-*b-COMPACTNESS

**3.1. Definition:** A collection $\{A_i : i \in \Lambda\}$ of g-*b-open sets in a topological space $X$ is called a g-*b-open cover of a subset $B$ of $X$ if $B \subset \cup\{A_i : i \in \Lambda\}$ holds.

**3.2. Definition:** A topological space $X$ is g-*b-compact if every g-*b-open cover of $X$ has a finite sub-cover.

**3.3. Definition:** A subset $B$ of a topological space $X$ is said to be g-*b-compact relative to $X$ if for every collection $\{A_i : i \in \Lambda\}$ of g-*b-open subsets of $X$ such that there exist a finite subset $\Lambda_0$ of $\Lambda$ such that

$B \subseteq \cup\{A_i : i \in \Lambda_0\}$ .

**3.4. Definition:** A subset $B$ of a topological space $X$ i said to be g-*b-compact if $B$ is g-*b-compact as a subspace of $X$ .

**3.5. Theorem:**

Every g-*b-closed subset of a g-*b-compact space is g-*b-compact relative to $X$ .

**Proof:**

Let $X$ be a g-*b-compact space and $A$ be a g-*b-closed subset of $X$ .Then $X - A = A^C$ is g-*b-open in $X$ .Let $M = \{G_\alpha : \alpha \in \Lambda\}$ be a cover of $A$ by g-*b-open sets in $X$ .Then $M^* = M \cup A^C$ is a g-*b-open cover of $X$ .Since $X$ is g-*b-compact. $\Rightarrow M^*$ is reducible to a finite sub cover of $X$ .That is $X = G_{\alpha_1} \cup G_{\alpha_2} \cup ... \cup G_{\alpha_m} \cup A^C$ , $G_{\alpha_K} \in M$ .But $A$ and $A^C$ are disjoint. Hence

$A \subset G_{\alpha_1} \cup G_{\alpha_2} \cup ... \cup G_{\alpha_m}$ , $G_{\alpha_K} \in M$ .Which implies that any g-*b-open cover $M$ of $A$ contains a finite sub cover. Therefore $A$ is g-*b-compact relative to $X$ .Thus every g-*b-closed subset of a g-*b-compact space $X$ is g-*b-compact.

**3.6. Theorem:**

A *b-continuous image of a *b-compact space is compact.

**Proof:**

Let $f : X \to Y$ be a *b-continuous map from a *b-compact space $X$ onto a topological space $Y$ .

To Prove: $Y$ is compact. Let $\{A_i : i \in \Lambda\}$ be an open cover of $Y$ .$\Rightarrow$ each $A_i$ is an open subset of $Y$ .

Since $f$ is *b-continuous. $\Rightarrow$ $f^{-1}(A_i)$ is *b-open in $X$ . Then $\{f^{-1}(A_i) : i \in \Lambda\}$ is a *b-open cover of $X$ .Since $X$ is *b-compact. It has a finite sub cover say $f^{-1}(A_1), f^{-1}(A_2),..., f^{-1}(A_n)$

Since $f$ is onto. $\Rightarrow$ $A_1, A_2,..., A_n$ is a finite cover of $Y$ .$\Rightarrow$ Every open cover of $Y$ has a finite sub cover.Therefore $Y$ is compact.

**3.7. Theorem:**

If a map $f : X \to Y$ is *b-irresolute and a subset $B$ of $X$ *b-compact relative to $X$ , then the image $f(B)$ is *b-compact relative to $Y$ .

**Proof:**

Let $\{A_\alpha : \alpha \in \Lambda\}$ be any collection of *b-open subsets of $Y$ such that $f(B) \subset \cup\{A_\alpha : \alpha \in \Lambda\}$

$\Rightarrow B \subset \{f^{-1}(\cup A_\alpha) : \alpha \in \Lambda\}$ .Then $B \subset \cup\{f^{-1}(A_\alpha) : \alpha \in \Lambda\}$. By hypothesis $B$ is *b-compact relative to $X$ . Therefore, there exist a finite subset $\Lambda_0$ of $\Lambda$ such that $B \subset \cup\{f^{-1}(A_\alpha) : \alpha \in \Lambda_0\}$

Therefore we have $f(B) \subset \cup\{A_\alpha : \alpha \in \Lambda_0\}$.Which shows that $f(B)$ is *b-compact relative to $Y$ .

**IV. g-b**-COMPACTNESS**

**4.1. Definition:** A collection $\{A_i : i \in \Lambda\}$ of g-b**-open sets in a topological space $X$ is called a g-b**-open cover of a subset $B$ of $X$ if $B \subset \cup\{A_i : i \in \Lambda\}$ holds.

**4.2. Definition:** A topological space $X$ is g-b**-compact if every g-b**-open cover of $X$ has a finite sub-cover.

**4.3. Definition:** A subset $B$ of a topological space $X$ is said to be g-b**-compact relative to $X$ if for every collection $\{A_i : i \in \Lambda\}$ of g-b**-open subsets of $X$ such that there exist a finite subset $\Lambda_0$ of $\Lambda$ such that

$B \subseteq \cup\{A_i : i \in \Lambda_0\}$ .

**4.4. Definition:** A subset $B$ of a topological space $X$ i said to be g-b**-compact if $B$ is g-b**-compact as a subspace of $X$ .

**4.5. Theorem:**

Every g-b**-closed subset of a g-b**-compact space is g-b**-compact relative to $X$ .

**Proof:**

Let $X$ be a g-b**-compact space and $A$ be a g-b**-closed subset of $X$ .Then $X - A = A^C$ is g-b**-open in $X$ .Let $M = \{G_\alpha : \alpha \in \Lambda\}$ be a cover of $A$ by g-b**-open sets in $X$ .Then $M^* = M \cup A^C$ is a g-b**-open cover of $X$ .Since $X$ is g-b**-compact. $\Rightarrow$ $M^*$ is reducible to a finite sub cover of $X$ .

That is $X = G_{\alpha_1} \cup G_{\alpha_2} \cup ... \cup G_{\alpha_m} \cup A^C$ , $G_{\alpha_K} \in M$ But $A$ and $A^C$ are disjoint. Hence $A \subset G_{\alpha_1} \cup G_{\alpha_2} \cup ... \cup G_{\alpha_m}$ , $G_{\alpha_K} \in M$ Which implies that any g-b**-open cover $M$ of $A$ contains a finite sub cover. Therefore $A$ is g-b**-compact relative to $X$ .Thus every g-b**-closed subset of a g-b**-compact space $X$ is g-b**-compact.

**4.6. Theorem:**

A b**-continuous image of a b**-compact space is compact.

**Proof:**

Let $f : X \to Y$ be a b**-continuous map from a b**-compact space $X$ onto a topological space $Y$ .

To Prove: $Y$ is compact. Let $\{A_i : i \in \Lambda\}$ be an open cover of $Y$ . $\Rightarrow$ each $A_i$ is an open subset of $Y$ .

Since $f$ is b**-continuous. $\Rightarrow$ $f^{-1}(A_i)$ is b**-open in $X$ .Then $\{f^{-1}(A_i) : i \in \Lambda\}$ is a b**-open cover of $X$ .Since $X$ is b**-compact. It has a finite sub cover say $f^{-1}(A_1), f^{-1}(A_2),..., f^{-1}(A_n)$

Since $f$ is onto. $\Rightarrow$ $A_1, A_2,..., A_n$ is a finite cover of $Y$ . $\Rightarrow$ Every open cover of $Y$ has a finite sub cover.

Therefore $Y$ is compact.

**4.7. Theorem:**

If a map $f : X \to Y$ is b**-irresolute and a subset $B$ of $X$ b**-compact relative to $X$ , then the image $f(B)$ is b**-compact relative to $Y$ .

**Proof:**

Let $\{A_\alpha : \alpha \in \Lambda\}$ be any collection of b**-open subsets of $Y$ such that $f(B) \subset \cup\{A_\alpha : \alpha \in \Lambda\}$

$\Rightarrow B \subset \{f^{-1}(\cup A_\alpha) : \alpha \in \Lambda\}$. Then $B \subset \cup\{f^{-1}(A_\alpha) : \alpha \in \Lambda\}$. By hypothesis $B$ is b**-compact relative to $X$ .Therefore, there exist a finite subset $\Lambda_0$ of $\Lambda$ such that $B \subset \cup\{f^{-1}(A_\alpha) : \alpha \in \Lambda_0\}$

Therefore we have $f(B) \subset \cup\{A_\alpha : \alpha \in \Lambda_0\}$. Which shows that $f(B)$ is b**-compact relative to $Y$ .

## V. g-**b-COMPACTNESS

**5.1. Definition:** A collection $\{A_i : i \in \Lambda\}$ of g-**b-open sets in a topological space $X$ is called a g-**b-open cover of a subset $B$ of $X$ if $B \subset \cup\{A_i : i \in \Lambda\}$ holds.

**5.2. Definition:** A topological space $X$ is g-**b-compact if every g-**b-open cover of $X$ has a finite sub-cover.

**5.3. Definition:** A subset $B$ of a topological space $X$ is said to be g-**b-compact relative to $X$ if for every collection $\{A_i : i \in \Lambda\}$ of g-**b-open subsets of $X$ such that there exist a finite subset $\Lambda_0$ of $\Lambda$ such that

$B \subseteq \cup\{A_i : i \in \Lambda_0\}$ .

**5.4. Definition:** A subset $B$ of a topological space $X$ i said to be g-**b-compact if $B$ is g-**b-compact as a subspace of $X$ .

**5.5. Theorem:**

Every g-**b-closed subset of a g-**b-compact space is g-**b-compact relative to $X$ .

**Proof:**

Let $X$ be a g-**b-compact space and $A$ be a g-**b-closed subset of $X$ .Then $X - A = A^C$ is g-**b-open in $X$ . Let $M = \{G_\alpha : \alpha \in \Lambda\}$ be a cover of $A$ by g-**b-open sets in $X$ . Then $M^* = M \cup A^C$ is a g-**b-open cover of $X$ . Since $X$ is g-**b-compact. $\Rightarrow$ $M^*$ is reducible to a finite sub cover of $X$ .

That is $X = G_{\alpha_1} \cup G_{\alpha_2} \cup ... \cup G_{\alpha_m} \cup A^C$ , $G_{\alpha_K} \in M$ . But $A$ and $A^C$ are disjoint. Hence $A \subset G_{\alpha_1} \cup G_{\alpha_2} \cup ... \cup G_{\alpha_m}$ , $G_{\alpha_K} \in M$ . Which implies that any g-**b-open cover $M$ of $A$ contains a finite sub cover. Therefore $A$ is g-**b-compact relative to $X$ . Thus every g-**b-closed subset of a g-**b-compact space $X$ is g-**b-compact.

**5.6. Theorem:**

A **b-continuous image of a **b-compact space is compact.

**Proof:**

Let $f : X \to Y$ be a **b-continuous map from a **b-compact space $X$ onto a topological space $Y$ .

To Prove: $Y$ is compact. Let $\{A_i : i \in \Lambda\}$ be an open cover of $Y$ . $\Rightarrow$ each $A_i$ is an open subset of $Y$ .

Since $f$ is **b-continuous. $\Rightarrow$ $f^{-1}(A_i)$ is **b-open in $X$ . Then $\{f^{-1}(A_i) : i \in \Lambda\}$ is a **b-open cover of $X$ . Since $X$ is **b-compact. It has a finite sub cover say $f^{-1}(A_1), f^{-1}(A_2),..., f^{-1}(A_n)$

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Therefore we have $f(B) \subset \cup\{A_\alpha : \alpha \in \Lambda_0\}$. Which shows that $f(B)$ is **b-compact relative to $Y$ .

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