SINGULARITY AND ITS TYPES

Kuljot kaur
Khalsa College for women, civil lines,
Ludhiana(141001)

ABSTRACT: A point or region of infinite mass density at which space and time are infinitely distorted by gravitational forces and which is held to be the final state of matter falling into a black hole and point at which the derivative of a given function of a complex variable does not exist but every neighborhood of which contains points for which the derivative does exist.

INTRODUCTION

Mathematical singularity, a point at which a given mathematical object is not defined or not "well-behaved" for example infinite or not differentiable. In real analysis singularities are either discontinuities or discontinuities of the derivative (sometimes also discontinuities of higher order derivatives). A singularity, as most commonly used, is a point at which expected rules break down. The term comes from mathematics, where a point on a curve that has a sudden break in slope is considered to have a slope of undefined or infinite value; such a point is known as a singularity and A point at which an analytic function f(z) is not analytic, i.e. at which f '(z) fails to exist, is called a singular point or singularity of the function.

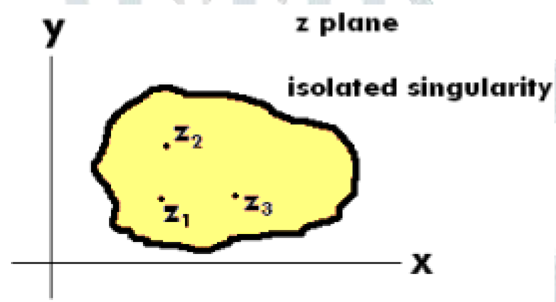

Fig. 1a

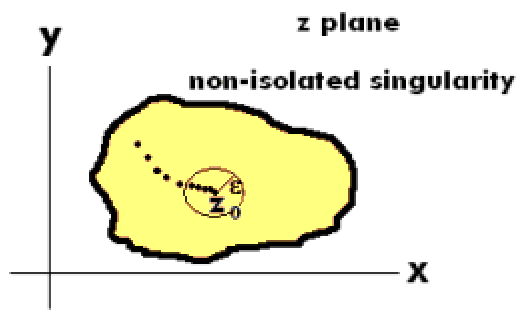

Fig. 1b

There are different types of singular points:

1.Isolated and non-isolated singular points. A singular point z_0 is called an isolated singular point of an analytic function f(z) if there exists a deleted ε-spherical neighborhood of z_0 that contains no singularity. If no such neighborhood can be found, z_0 is called a non-isolated singular point. Thus an isolated singular point is a singular point that stands completely by itself, embedded in regular points. See Fig. 1a where z_1, z_2 and z_3 are isolated singular points. Most singular points are isolated singular points. A non-isolated singular point is a singular point such that every deleted ε-spherical neighborhood of it contains singular points. See Fig. 1b where z_0 is the limit point of a set of singular points. Isolated singular points include poles, removable singularities, essential singularities and branch points.

2. Pole. An isolated singular point z_0 such that f(z) can be represented by an expression that is of the form

$$f(z) = \frac{\phi(z)}{(z - z_0)^n}$$

where n is a positive integer, □(z) is analytic at z_0, and □(z_0) ≠ 0. The integer n is called the order of the pole. If n = 1, z_0 is called a simple pole.

Example. The function

$$f(z) = \frac{5z+1}{(z-2)^3(z+3)(z-2)}$$

has a pole of order 3 at z = 2 and simple poles at z = -3 and z = 2.

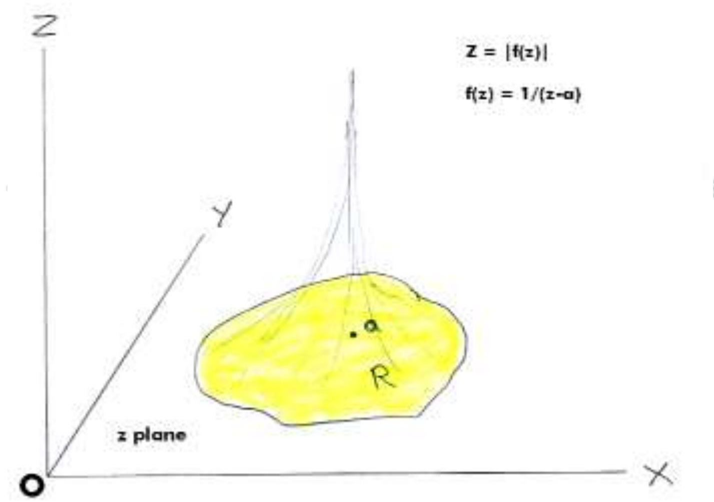

Fig. 2

Shown in Fig. 2 is a modulus surface of the function f(z) = 1/(z-a) defined on a region R. One sees the “pole” arising above point a in the complex plane. Thus the reason for the term “pole”. A modulus surface is obtained by affixing a Z axis to the z plane and plotting Z = |f(z)| [i.e. plotting the modulus of f(z)].

3. Removable singular point. An isolated singular point z_0 such that f can be defined, or redefined, at z_0 in such a way as to be analytic at z_0. A singular point z_0 is removable if $\lim_{z \to z_0} f(z)$ exists.

Example. The singular point z = 0 is a removable singularity of f(z) = (sin z)/z since $\lim_{z \to 0} \frac{\sin z}{z} = 1.$

4. Essential singular point. A singular point that is not a pole or removable singularity is called an essential singular point. Example.f(z) = $e^{1/(z-3)}$ has an essential singularity at z = 3.

5.Singular points at infinity. The type of singularity of f(z) at z = ∞ is the same as that of f(1/w) at w = 0. Consult the following example.

Example. The function f(z) = z^2 has a pole of order 2 at z = ∞, since f(1/w) has a pole of order 2 at w = 0.

Using the transformation $w = 1/z$ the point $z = 0$ (i.e. the origin) is mapped into $w = \infty$, called the point at infinity in the w plane. Similarly, we call $z = \infty$ the point at infinity in the z plane. To consider the behavior of $f(z)$ at $z = \infty$, we let $z = 1/w$ and examine the behavior of $f(1/w)$ at $w = 0$.

CONCLUSION: Singularity functions are a class of discontinuous function that contain singularities, i.e. they are discontinuous at their singular points. Singularity functions have been heavily studied in the field of mathematics under the alternative names of generalized functions and distribution theory.

References

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