ZERO DIVISOR LATTICE AND COMPRESS FORM OF ZERO DIVISOR GRAPH OF FINITE COMMUTATIVE RING

Varta Teotia[1] and Sapna Shrimali[2]
[1]Research Scholar, Department of Mathematics, Pacific Academy of Higher Education & Research University, Udaipur (Rajasthan), India
[2]Department of Mathematics, Pacific Academy of Higher Education & Research University, Udaipur (Rajasthan), India

Abstract: In this paper we consider on some zero divisor graph and its compressed form of zero divisor graph Γ_C (R)and zero divisor lattice Γ_L (R)of commutative ring R. we are shows when the two commutative ring S and R then $\Gamma(S) \cong \Gamma(R)$, Γ_C (S) ≅ Γ_C (R) and $\Gamma_L(S) \cong \Gamma_L$(R) and defined by structure.

Key words: commutative ring, zero divisor graph, connectivity, lattice.

1. INTRODUCTION:

Great researcher I Beck [3] introduced the graph of zero divisor and this topic was studied further by D. D. Anderson and M. Naseer [9]. The definition of zero divisor graph of ring used by Anderson and Livingston in 1999 [1] from the last ten years so many researcher did research on zero divisor graph properties. Introducing Co zero divisor graph of ring by Afkhami and Khashyarmanesh [2]. We are focused on structure of compressed zero divisor graph of ring and lattice of zero divisor graph. we are using the idea from mathematica note book [7] to creat these graphs. It is very useful to creating these graphs.

We try to give another definition of compressed zero divisor graph and lattice of zero divisor graph. We give some examples and explain them. Similar definition of compressed zero divisor graph is firstly given in [10] and its studied continued by [11].

We provided some definition and examples and we shows the connection between compressed zero divisor and zero divisor graphs.

2. DEFINITION:

In this paper the infinite commutative ring with unity is denotes by R and any element x is zero divisor there exist non zero element r∈R s.t x r = 0.

The set of zero divisor is denotes by Z (R). If the ring has one unique ideal it is called a local ring. The annihilator of R element x is ann(x) = { a | ax = 0 }

In a graph G vertices and edges are denotes by V (G) and E (G). The path between the two vertices x1 and x2 ∈ V (G) to be an ordered sequence of distinct vertices and edges { a_1 e_1 a_2 e_2…………. e_{n-1} a_n } of G.

The minimum length of the path x to y is called distance from x to y.distance of x to y is denoted by d (x,y) = 0. If d(x,y)=∞ that means no path from x to y. The diameter of a graph between x to y is diam (G) = { sup d (x,y) | x,y ∈V (G) }. In the graph neighborhood of any vertex is the set nbd (x) = z ∈ V (G) | x→z }. If two vertices are connected by an edge it is called a connected graph.

The two graphs are said to be isomorphic if there exist one to one and on to Ø V (G) → V (H) x and y two elements x,y E V (G) x→y if Ø (x) → d (y).

For a commutative ring R define a relation of equivalence is $x \equiv y$ iff ann(x)=ann(y). the equivalence classes of x is denoted by $\bar{x}$.

ann(0)=R and $\bar{x} = \bar{y}$ of all x,y belongs to R \ Z(R) .Compressed form of zero divisor graph of R whose vertices are equivalence of non zero divisors. $\bar{x}$ and $\bar{y}$ are connected by an edge iff xy=0.

Lemma: Let the two graph G and H are congruent. If Ø(x) = y then the neighborhood is Ø (nbd (x)) = nbd (y).

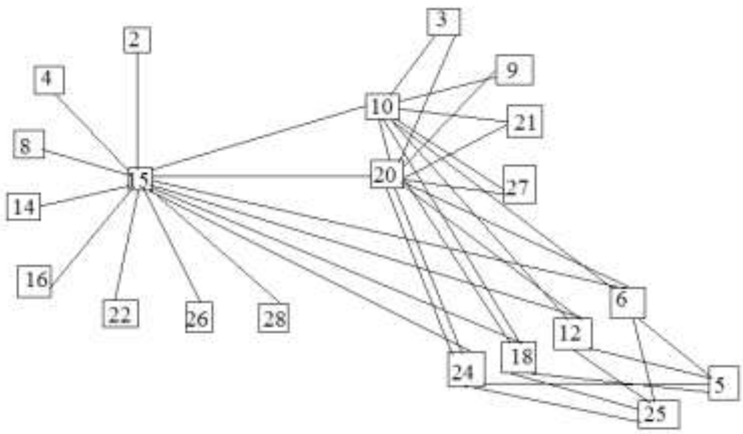
Proof: Suppose two graph G and H are isomorphic Ø: G → H. Suppose two element x and y which x ∈ V(G) and Ø (x) = y ∈ V (H) the the neighborhood is

Ø (nbd (x)) = {Ø (z) | x −z} = { Ø (z) | Ø (x) −Ø (z) } = { Ø (z) | y − Ø(z) } = nbd (y).

Ґ(R) is the zero divisor graph of a ring R with V (Ґ(R)) = Z(R) \ {0} and E (Ґ(R)) = {a −b / a b = 0 }. Z(R) is the set of zero divisor. In [1] Anderson and Livingston is showed the zero divisor graph is always connected and its

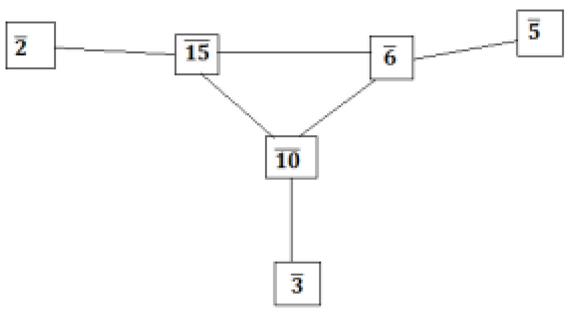
diam z (R) ≤ 3 for any ring R.

Example: In the figure we shows Zero divisor graph and compressed zero divisor graph of Z_{30}.



Zero Divisor graph of Z_{30}

(Ґ Z_{30})



Compress structure of Zero Divisor graph of Z_{30}

($Ґ_C$ Z_{30})

In figure zero divisor graph of Z_{30} there are three cut sets {10,20},{15},{6,12,18,24}and the cut set of compressed zero divisor graph Z_{30} is $\{\overline{15}\},\{\overline{6}\},\{\overline{10}\}$.

Axtell, Stickles and Trampbachls was started the study on cut set of zero divisor graph and this study continued by Cote B. ,weber, huhn.

The graph theory was given in [5]. The algebraic definition and concepts are given by David, Richard and Foote.

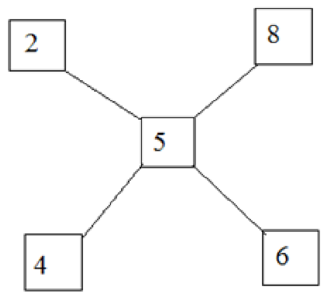
Theorem: Two commutative S and R. If $\Gamma(S) \cong \Gamma(R)$ then Γ_C (S) $\cong$ Γ_C (R).

Proof: Let two commutative S and R then vertices of S and R is $V(\Gamma(S))=\{s_1,s_2,s_3,----------s_n\}$and $V(\Gamma(R))=\{r_1,r_2,r_3----------r_n\}$ such that isomorphism $\emptyset$ $\Gamma(S)\rightarrow \Gamma(R)$ satisfies $\emptyset(s_i)= r_i$ for each i={1,2,3-------------n}. The mapping of edges $\emptyset$: $E(\Gamma_C(S))\rightarrow E(\Gamma_C(R))$ which sends the edges $\bar{s}_i\rightarrow \bar{s}_j$ to $\bar{r}_i\rightarrow \bar{r}_j$.

now we see the compressed form of zero divisor graphs are congruent but zero divisor graphs are not congruent.

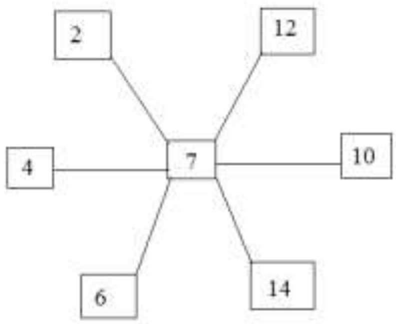
Example: Zero divisor graph and compressed form of zero divisor graph of Z_{10} and Z_{14}.

Compressed structure Zero Divisor graph Γ_C Z_{10}



Zero Divisor graph of Z_{10}

Compressed structure Zero Divisor graph Γ_C Z_{14}



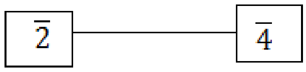
Zero Divisor graph of Z_{14}

3. ZERO DIVISOR LATTICE:

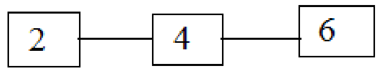
The root of zero divisor lattice one vertex a to other vertex b, either b<a or a and b are incomparable. The zero divisor lattice has multiple roots.

Theorem: Two commutative S and R. If Γ_C (S) $\cong$ Γ_C (R) then $\Gamma_L(S) \cong \Gamma_L(R)$.

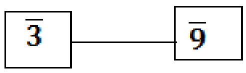
Proof: Suppose the vertices of Γ_C S and Γ_C R is $V(\Gamma_C(S))=\{s_1,s_2,s_3,----------s_n\}$and $V(\Gamma_C(R))=\{r_1,r_2,r_3----------r_n\}$ such that isomorphism $\emptyset$ $\Gamma(S)\rightarrow \Gamma(R)$ satisfies $\emptyset(s_i)= r_i$ for each i={1,2,3-------------n}.we showed in the lemma $\emptyset(ann(s_i))=ann(r_j)$ for each i, if any $ann(s_i)=ann(s_j)$ for any $1\leq i, j\leq n$ then i=j. the mapping of edges $\emptyset$: $E\Gamma_L(S) \rightarrow E\, \Gamma_L(R)$ which sends the edges $\bar{s}_i\rightarrow \bar{s}_j$ to $\bar{r}_i\rightarrow \bar{r}_j$. Thus $\Gamma_L(S) \cong \Gamma_L(R)$.



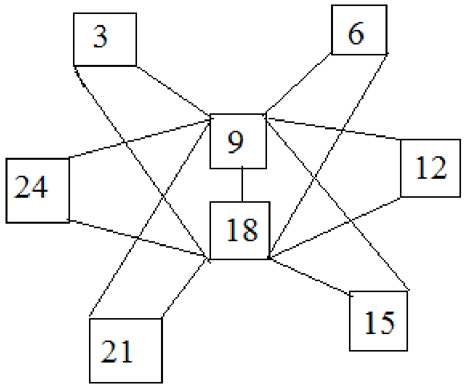
Zero Divisor lattice graph of Z_8



Zero Divisor graph of Z_8



Zero Divisor lattice graph of Z_{27}



Zero Divisor graph of Z_{27}

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