# A Study for **Zakharov** system by Laplace - Modified Decomposition Method

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**Abstract:-** Analytical and numerical solutions are obtained for coupled Klein-Gordon Zakharov Equation by the well-known Laplace - Modified decomposition method [LMDM]. We combined Laplace transform and Modified decomposition method and present a new approach for solving Coupled Klein-Gordon Zakharov equation. The method does not need linearization, weak nonlinearity assumptions, or perturbation theory. We compared the numerical solutions with corresponding analytical solutions.

**Keywords** Laplace - Modified Decomposition Method**,** Klein-Gordon Zakharov Equation.

## Introduction

Wave-wave interaction is an important problem for both physical and mathematical reasons. Physically, the wave-wave interaction or the wave collisions are common phenomena in science and engineering for both solitary and non-solitary waves. Mathematically solitary wave collision is a major branch of nonlinear wave interaction in ionic media. We consider the coupled nonlinear Klein-Gorden- Zakharov Equation (KGZ) in the form of

$$E_{tt} - E_{xx} + E_{xx} + E + \eta E = 0$$
$$\eta_{tt} - \eta_{xx} - \left(|E|^2\right)_{xx} = 0 \qquad (1)$$

Here $E(x,t)$ is a complex function and $\eta(x,t)$ is a real function. The Coupled nonlinear Klein-Gorden- Zakharov Equation (KGZ) describe the interaction of Langmuir waves and ion-acoustic waves in plasmas [1-3]. Due to their potential application in plasma physics, the nonlinear Klein-Gordon-Zakharov system has been paid attention by many researchers. Some exact solutions for the Zakharov equations are obtained by different methods [4-19].

In this paper, we use the Laplace - Modified decomposition method (in short LMDM) to obtain the solutions of the equation (1). Large classes of linear and nonlinear differential equations, both ordinary as well as partial, can be solved by the Adomian decomposition method [20-33]. A reliable modification of Adomian decomposition method has been done by Wazwaz [22]. The decomposition method provides an effective procedure for analytical solution of a wide and general class of dynamical systems representing real physical problems [20-25, 32]. This method efficiently works for initial-value or boundary-value problems and for linear or nonlinear, ordinary or partial differential equations and even for stochastic systems. Recently, the solution of fractional differential equation has been obtained through Adomian decomposition method by the researchers [31-33]. The method has features in common with many other methods, but it is distinctly different on close examinations, and one should not be mislead by apparent simplicity into superficial conclusions [20-21].

## Analysis of the Method

The method consists of first taking the Laplace transform of both sides of equations in equation (1) and (2) with differentiation property of Laplace transform and taking inverse Laplace transform. We get

$$E(x,t) = E(x,0) + tE_t(x,0) + L^{-1}\left[\frac{1}{p^2} L\{(E_{xx} - E - N); p\}\right] \qquad (3)$$

$$\eta(x,t) = \eta(x,0) + t\eta_t(x,0) + L^{-1}\left[\frac{1}{p^2} L\{(\eta_{xx} + M); p\}\right] \qquad (4)$$

Where $N(E,\eta) = \eta E, M(E,\eta) = |E|^2_{xx} = \left(E\overline{E}\right)_{xx}$ are symbolize nonlinear operators, The Adomain decomposition method assumes infinite series solutions for unknown function $E(x, t)$, $\eta(x, t)$ and by the infinite series given by

$$E(x,t) = \sum_{n=0}^{\infty} E_n , \qquad \eta(x,t) = \sum_{n=0}^{\infty} \eta_n \qquad (5)$$

The nonlinear terms $N(E,\eta) = \eta E, M(E,\eta) = |E|^2_{xx} = \left(E\overline{E}\right)_{xx}$ are usually represented by an infinite series of the so-called Adomian polynomials

$$N(E,\eta) = \sum_{n=0}^{\infty} A_n\left(E_0, E_1, \ldots\ldots, E_n, \eta_0, \eta_1 \ldots\ldots \eta_\eta\right) \qquad (6)$$

$$M(E,\eta) = \sum_{n=0}^{\infty} B_n\left(E_0, E_1, \ldots\ldots, E_n, \eta_0, \eta_1 \ldots\ldots \eta_\eta\right) \qquad (7)$$

The Adomian polynomials can be generated for all forms of nonlinearity. They are determined by the following relations:

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} N\left(\sum_{n=0}^{\infty}\lambda^i E_i, \sum_{n=0}^{\infty}\lambda^i \eta_i,\right)\right], n \geq 0 \tag{8}$$

$$B_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} M\left(\sum_{n=0}^{\infty}\lambda^i E_i, \sum_{n=0}^{\infty}\lambda^i \eta_i,\right)\right], n \geq 0 \tag{9}$$

This formula is easy to set computer code to get as many polynomials as we need in calculation of the numerical as well as explicit solutions. For the sake of convenience of the readers, we can give the first few Adomian polynomials for $N(E,\eta) = \eta E, M(E,\eta) = |E|^2_{xx} = \left(E\overline{E}\right)_{xx}$ of the nonlinearity as

$$\begin{aligned} A_0 &= \eta_0 E_0 \\ A_1 &= \eta_1 E_0 + \eta_0 E_1 + \eta_1 E_1 \\ A_2 &= \eta_2 E_0 + \eta_0 E_2 + \eta_2 E_1 + \eta_1 E_2 + \eta_2 E_2 \end{aligned} \quad ..(10)$$

$$\begin{aligned} B_0 &= \left(E_0 \overline{E_0}\right)_{xx} \\ B_1 &= \left(E_0 \overline{E_1} + E_1 \overline{E_0} + E_1 \overline{E_1}\right)_{xx} \\ B_2 &= \left(E_1 \overline{E_2} + E_0 \overline{E_2} + E_2 \overline{E_0} + E_2 \overline{E_1} + E_2 \overline{E_2}\right)_{xx} \end{aligned} \quad ..(11)$$

and so on, the rest of the polynomials can be constructed in a similar manner. Substituting the initial conditions into (2) identifying the zeroth components $E_0$ and $\eta_0$ then we obtain the subsequent components by the following recursive equations by using the standard ADM

$$E_{n+1}(x,t) = L^{-1}\left[\frac{1}{p^2} L\{(E_{xx} - E - A_n); p\}\right] \tag{12}$$

$$\eta_{n+1}(x,t) = L^{-1}\left[\frac{1}{p^2} L\{(\eta_{xx} + B_n); p\}\right] \tag{13}$$

General term $E_{n+1}(x,t)$ in a form of the modified recursive scheme as follows:

$$\begin{aligned} E_0 &= E(x.0) \\ E_1 &= tE_t(x,0) + L^{-1}\left[\frac{1}{p^2} L\{(E_{0xx} - E_0 - A_0); p\}\right] \\ &\ldots\ldots\ldots\ldots \\ E_{n+1} &= L^{-1}\left[\frac{1}{p^2} L\{(E_{nxx} - E_n - A_n); p\}\right] \end{aligned} \tag{14}$$

Similarly $\eta_{n+1}(x,t)$ in a form of the modified recursive scheme as follows:

$$\begin{aligned} \eta_0 &= \eta(x.0) \\ \eta_1 &= t\eta_t(x,0) + L^{-1}\left[\frac{1}{p^2} L\{(\eta_{0xx} + B_0); p\}\right] \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \eta_{n+1} &= L^{-1}\left[\frac{1}{p^2} L\{(\eta_{nxx} + B_n); p\}\right] \end{aligned} \tag{15}$$

The practical solutions will be the $n$ -term approximations $\chi_n$ and $\mu_n$

$$\chi_n = \sum_{n=0}^{\infty} E_n, n \geq 0 \quad \text{with} \quad \lim_{n \to \infty} \chi_n = E(x,t) \tag{16}$$

$$\mu_n = \sum_{n=0}^{\infty} \eta_n, n \geq 0 \quad \text{with} \quad \lim_{n \to \infty} \mu_n = \eta(x,t) \tag{17}$$

# Implementation of the method

We consider the application of Klein-Gordon-Zakharov Equations with initial condition are given as

$$E_0 = E(x,0) = \mp e^{ix} \tanh(x) \tag{18}$$

$$\eta_0 = \eta(x,0) = \pm \frac{\omega^2}{1-\omega^2} e^{2ix} \tanh^2(x) - 2 \tag{19}$$

$$E_t(x,0) = E_1 = \mp\omega\left(i \tanh(x) + \mathrm{sech}^2(x)\right)e^{ix} \tag{20}$$

$$\eta_t(x,0) = \pm\frac{2\omega^2}{1-\omega^2}\left(i\tanh^2(x) + \tanh x\,\mathrm{sec}h^2(x)\right)e^{2ix} \tag{21}$$

We get solution from eqs. (14) and (15)

$$\begin{aligned} E_1 &= \pm t\omega\left[i\tanh(x) + \mathrm{sec}h^2(x)\right]e^{ix} \\ &\mp \frac{t^2}{2}\left[\left\{2i\,\mathrm{sec}h^2(x) + 2\,\mathrm{sec}h^2(x)\tanh(x)\right\}e^{ix} - \frac{\omega^2}{1-\omega^2}\tanh^3(x)e^{3ix}\right] \end{aligned} \tag{22}$$

$$\begin{aligned} \eta_1 &= \pm\frac{2t\omega^2}{1-\omega^2}\left[i\tanh^2(x) + \tanh x\,\mathrm{sec}h^2(x)\right]e^{2ix} \pm \frac{\omega^2 t^2}{\left(1-\omega^2\right)}\left[\left\{2\left(\sqrt{2}+1\right)i\tanh(x)\,\mathrm{sec}h^2(x)\right.\right. \\ &\left.\left.+\,\mathrm{sec}h^4(x) - \tanh^2(x)\right\}e^{2ix} - 4\,\mathrm{sec}h^4(x) - 8\,\mathrm{sec}h^2(x)\tanh^2(x)\right] \end{aligned} \tag{23}$$

$$
\begin{aligned}
E_2 = {} & \pm \frac{t^2\omega}{2}\Big[\{(6i-1)\operatorname{sech}^2(x)\tanh(x)+4\operatorname{sech}^2(x)\tanh^2(x)-\tanh(x)\}e^{ix} \\
& \pm \frac{\omega(2+\omega)}{2(1-\omega^2)}\{i\tanh^3(x)+\operatorname{sech}^2(x)\tanh^2(x)\}\{e^{2ix}+e^{3ix}\Big] \\
& \mp \frac{t^4}{12}\Big[\{(2i-2)\operatorname{sech}^4(x)-i\operatorname{sech}^2(x) \\
& +4(2i-1)\operatorname{sech}^2(x)\tanh^2(x)+\frac{4}{1-\omega^2}\operatorname{sech}^2(x)\tanh^3(x) \\
& +8\operatorname{sech}^4(x)\tanh(x)-\left(\frac{5-7\omega^2}{1-\omega^2}+i\right)\operatorname{sech}^2(x)\tanh(x)\Big\}e^{ix} \\
& -\frac{\omega^2}{1-\omega^2}\Big\{\left(\frac{13}{2}-\frac{\omega^3}{1-\omega^2}\right)\operatorname{sech}^4(x)\tanh(x) \\
& +6\operatorname{sech}^2(x)\tanh^3(x)+\left(\sqrt{2}+19-\frac{2\omega^3}{1-\omega^2}\right)i\operatorname{sech}^2(x)\tanh^2(x) \\
& +\left(-4+\frac{\omega^3}{1-\omega^2}\right)\tanh^3(x)-\frac{\omega^2}{1-\omega^2}\tanh^3(x)\Big\}e^{3ix} \\
& -\frac{\omega^2}{2(1-\omega^2)}\{\operatorname{sech}^2(x)\tanh^3(x)+i\operatorname{sech}^2(x)\tanh^2(x)\}e^{2ix} \\
& +\frac{\omega^4}{4(1-\omega^2)}\tanh^5(x)e^{4ix}\Big]\mp\frac{t^5\omega^2}{20(1-\omega^2)}\Big[\{-(2\sqrt{2}\omega+3\omega+2)\operatorname{sech}^2(x)\tanh^2(x) \\
& +(2\sqrt{2}\omega+3\omega+2)i\operatorname{sech}^4(x)\tanh(x)-\omega\tanh^3(x) \\
& +2i\tanh^3(x)\operatorname{sech}^2(x)\}+2\operatorname{sech}^4(x)\tanh^3(x)\}e^{3ix}-4i\omega\operatorname{sech}^2(x)\tanh(x) \\
& -8i\omega\operatorname{sech}^2(x)\tanh^3(x)-4\omega\operatorname{sech}^4(x)-8\omega\operatorname{sech}^4(x)\tanh^2(x) \\
& -\frac{\omega^2e^{5ix}\Big]}{(1-\omega^2)}\{i\tanh^5(x)+\tanh^4(x)\operatorname{sech}^2(x)\} \\
& -\frac{t^6\omega^2}{(1-\omega^2)}\Big[\{-4(\sqrt{2}+1)\operatorname{sech}^4(x)\tanh(x)+2i\operatorname{sech}^6(x) \\
& -2i\operatorname{sech}^2(x)\tanh^2(x)+4(\sqrt{2}+1)i\tanh^2(x)+2\operatorname{sech}^6(x)\tanh(x) \\
& +\left(\frac{-2+6\omega^2}{1-\omega^2}\right)\operatorname{sech}^2(x)\tanh^3(x) \\
& +\frac{8\omega^2}{(1-\omega^2)}\operatorname{sech}^2(x)\tanh^5(x)\Big\}e^{3ix}-8\{i\operatorname{sech}^4(x) \\
& +2i\operatorname{sech}^4(x)\tanh^2(x)+\operatorname{sech}^4(x)\tanh(x)+2\operatorname{sech}^4(x)\tanh^3(x)\}e^{ix} \\
& -\frac{\omega^2}{(1-\omega^2)}\{2(\sqrt{2}+1)i\tanh^4(x)\operatorname{sech}^2(x)+\tanh^3(x)\operatorname{sech}^4(x)-\tanh^3(x)\}e^{5ix}\Big]
\end{aligned} \tag{24}
$$

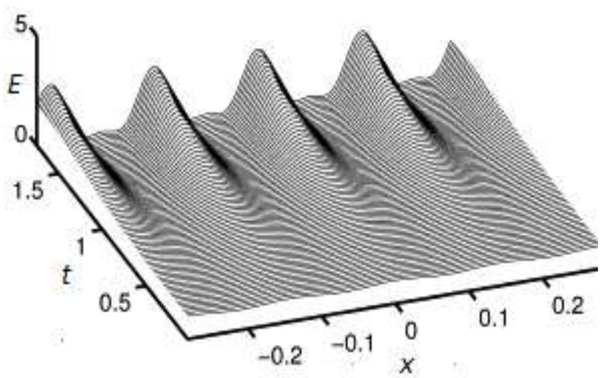
$$\begin{aligned}
\eta_2 &= \pm\frac{\omega^2 t^2}{(1-\omega^2)}\Big[6i\,\mathrm{sech}^4(x)+8i\,\mathrm{sech}^2(x)\tanh^2(x)+4\,\mathrm{sech}^4(x)\tanh(x)\\
&+8\,\mathrm{sech}^3(x)\tanh(x)+2\,\mathrm{sech}(x)\tanh^3(x)-12\,\mathrm{sech}^2(x)\tanh(x)\\
&+4i\,\mathrm{sech}(x)\tanh^2(x)-4i\tanh^2(x)-\frac{(1-\omega^2)}{\omega^2}\mathrm{sech}^2(x)\tanh(x)\Big]\\
&\pm\frac{t^4}{12}\Big[\mathrm{sech}^2(x)\tanh^2(x)-\frac{\omega^2}{(1-\omega^2)}\tanh^4(x)\cos 2(x)+\{\tanh^2 x+\mathrm{sech}^4 x\}\Big]\\
&\pm\frac{\omega^2 t^5}{20(1-\omega^2)}\Big[\{8i(\sqrt{2}+3)\mathrm{sech}^4(x)\tanh(x)+4(\sqrt{2}+1)i\,\mathrm{sech}(x)\tanh^3(x)\\
&+8i(\sqrt{2}+1)\tanh(x)\mathrm{sech}^3(x)+16\,\mathrm{sech}^4(x)\tanh^2(x)+4\,\mathrm{sech}^6(x)\\
&-2(4\sqrt{2}+7)\mathrm{sech}^4(x)-4(4\sqrt{2}+5)\tanh^2(x)\mathrm{sech}(x)\\
&-8i(\sqrt{2}+2)\tanh(x)\mathrm{sech}^2(x)+4\tanh^2(x)\}e^{2ix}-16\,\mathrm{sech}^6(x)\\
&-112\,\mathrm{sech}^4(x)\tanh^2(x)-32\,\mathrm{sech}^2(x)\tanh^4(x)-32\,\mathrm{sech}^6(x)\tanh(x)\\
&-64\tanh^3(x)\mathrm{sech}^4(x)-\frac{(1-\omega^2)}{\omega^2}\{2\,\mathrm{sech}^2(x)\tanh(x)+2\,\mathrm{sech}^4(x)\tanh(x)\\
&-\frac{\omega^2}{(1-\omega^2)}\tanh^3(x)\mathrm{sech}^2(x)\cos 2(x)\}\Big]\\
&\pm\frac{t^6}{24}\Big[4\,\mathrm{sech}^4(x)+4i\,\mathrm{sech}^2(x)\tanh(x)\}e^{2i(x+1)}-\frac{2\omega^2 i}{(1-\omega^2)}\tanh^3(x)\mathrm{sech}^2(x)\cos 2(x)\\
&-4i\,\mathrm{sech}^4(x)\tanh(x)+4\,\mathrm{sech}^4(x)\tanh^2(x)-\frac{4\omega^2}{(1-\omega^2)}\mathrm{sech}^2(x)\tanh^4(x)\cos 2(x)\Big]
\end{aligned} \tag{25}$$

The other components of the decomposition series can also be determined in a similar way, substituting these values into equations (16) and (17); we can obtain the expression of $E(x,t)$ and $\eta(x,t)$ which is in a Taylor series, then the closed form solutions yield as Ahmet Bekir et al. (2015).

$$E(x,t)=\mp e^{i(x+\omega t)}\tanh(x+\omega t) \tag{26}$$

$$\eta(x,t)=\mp\frac{\omega^2}{(1-\omega^2)}e^{2i(x+\omega t)}\tanh^2[(x+\omega t)] \tag{27}$$

We take for $E_2(x,t)$ the time step $\Delta t$ = 0.02 and a space step $\Delta x$ = 0.02, $-0.3 \le x \le 0.3$, the computation is done for $0 \le t \le 2$ $\omega = 0.05$




It's closed [19]

## Conclusion

The Laplace Modified decomposition method [LMDM] is a powerful method which has provided an efficient potential for Klein-Gordon-Zakharov Equation with initial condition. The approximate solitary wave solutions to the equations have been calculated by using the method [LMDM]. Additionally, it does not need any discretization method to get numerical solutions. This method thus eliminates the difficulties and massive computation work. The algorithm can be used without any need to complex calculations except for simple and elementary operations. The LMDM provides the solution in successive components that will be added to get the series solution which are expeditiously convergent.

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