# ON #$\alpha$ Regular Generalized Continuous Functions in Topological Spaces

S. Thilaga Leelavathi [1] and M. Mariasingam[2]

[1]Assistant Professor,Department of Mathematics, Pope's College, (Affilated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli- 627012), Sawyerpuram, Thoothukudi, Tamilnadu – 628 251, India.

[2]Head and Associate Professor (Rtd), Post Graduate and Research Department of Mathematics, V.O.Chidambaram College, (Affilated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli- 627012), Thoothukudi, Tamilnadu – 628 008, India.

**Abstract :** In this paper we introduce a new type of functions called the #$\alpha$ - regular generalized continuous functions. Also we study some characterizations and basic properties of #$\alpha$ - regular generalized continuous functions. Moreover we study #$\alpha rg$ $-$irresuolute functions by using #$\alpha rg$ - closed sets.

**Keywords :** #$\alpha$rg-closed, #$\alpha$rg-open, #$\alpha$rg-continuous, #$\alpha$rg- irresolute

AMS subject Classification : 54A05, 54C08

**1.Introduction** :-The concept of regular continuous function was first introduced by Arya S.P and Gupta R, Later Palaniappan N and Rao, K.C{13} studied the concept of regular generalized continuous function. Syed Ali Fathima and Maria Singam {19} studied the concept of # regular generalized continuous function. Thilaga and Maria Singam {21} introduced and studied the properties of #$\alpha rg$-closed sets. The purpose of this paper is to introduce the concept of #$\alpha rg$-continuos and #$\alpha rg$-irresolute functions and we study the relation among them.

**2. Preliminaries :-** Throughout this paper (X,$\tau$) represents a topological space on which no separation axiom is assumed. Unless otherwise mentioned. For a subset A of a topological space X, cl (A) and int (A) denote the closure of A and the interior of A respectively. X\A (or) $A^c$ denotes the complement of A in X. We recall the following definition and results.

Definition : 2.1 A subset A of a space X is called.

1) a pre open set [11 ] of A⊆intcl(A) and preclosed set if clint(A) ⊆A.
2) a semi open set [8] if A⊆clint(A) and semi closed set if int cl (A) ⊆ A.
3) a $\alpha$-open set [21]if A⊆ int (cl (intA)) and an $\alpha$-closed set if cl (int(cl(A)) ⊆ A.
4) a regular open set [18] if A=int cl (A) and a regular closed set if A = cl(int(cl(A)) .
5) a $\pi$-open set[19] if A is finite union of regular open sets.
6) regular semi open [4] if there is a regular open U such U⊆A⊆cl(U).

**Definition :2.2 A** subset A of (X,$\tau$) is called.

1) an $\alpha$-generalized closed set [10] (briefly $\alpha g$-closed ) if $\alpha$cl(A) ⊆U whenever A⊆U and U is open in (X,$\tau$).
2) a generalized pre-closed set [21] (briefly gp-closed) if pcl(A) ⊆ U whenever A⊆U and U is open in (X,$\tau$).
3) a generalized semi pre-closed set [21](briefly gsp-closed) if spcl (A)⊆U whenever A⊆U and U is open in (X,$\tau$).
4) a generalized $\alpha$-closed (g$\alpha$-closed)[10] set if $\alpha$cl(A)⊆U whenever A⊆U and U is open in (X,$\tau$).
5) rw-closed [2] if cl(A)⊆U whenever A⊆U and U is regular semi open.
6) #rg-closed [19] if cl(A) ⊆U whenever A⊆U and U is rw-open.
7) #$\alpha$rg-closed [21] if $\alpha$cl(A)⊆U whenever A⊆U and U is rw-open.

The complements of the above mentioned closed sets are their respective open sets.

**Definition:-2.3** A function f: X→Y from a topological space X into a topological space Y is called.

1) A $\alpha$-generalized continuous [8] (briefly $\alpha g$ - continuous) if $f^{-1}$(v) is $\alpha g$-closed in X for every closed set V in Y.
2) A generalized pre continuous [1] (briefly gp-continuous) if $f^{-1}$(V) is gp-closed in X for every closed set V in Y.
3) A generalized semi pre-continuous [10] (briefly gsp-continuous) if $f^{-1}$(V) is gsp-closed in X for every closed set in V in Y.
4) A generalized $\alpha$ $-$continuous[12] (briefly g$\alpha$ $-$continuous) if $f^{-1}$(V) is g$\alpha$-closed in X for every closed set in V in Y.
5. A α -generalized pre continuous[12] (briefly αgp-continuous ) if $f^{-1}$(V) is αgp-called in X for every closed set V in Y.

**Definition :2.4** For a subset A of a space X #αrg-cl(A)=∩{F:A⊆F, F is #αrg-closed in X} is called the #αrg-closure of A.

**Definition :2.5** Let (X,$\tau$ ) be a topological space and $\tau_{\#\alpha rg}$ ={V⊆X, #αrg-cl(X\V)=X\V}

**Lemma :2.6** For any x∈X,x∈# αrg-cl(A) if and only if V∩A≠ ∅ for every #αrg-open set V containing X.

**Lemma :2.7** Let A and B be subsets of (X,$\tau$ ) Then

1. # αrg-cl(∅) = ∅ and #αrg-cl(X)=X
2. If A⊆B, then #αrg-cl(A) ⊆ # αrg-cl(B)
3. A⊆#αrg-cl(A)
4. if A is #αrg-closed then #αrg-cl(A)=A
5. #αrg -closure of a set A is not always #αrg -closed.

**Remark :2.8** Suppose $\tau_{\#\alpha rg}$ is a topology, If A is #αrg-closed in (X,$\tau$)

**Lemma:2.9** A set A⊆X is #αrg-open if and only if F⊆intA whenever F⊆A, F is rw-closed.

**3. #αrg continuous Functions:**

In this section we introduce and study # αrg-continuous functions.

**Definition :3.1.1** A function f: (X,$\tau$ ) → $(y, \sigma)$ is called #αrg-continuous if $f^{-1}$(V) is #αrg-closed in (X,$\tau$ ) for every closed subset V of $(y, \sigma)$.

**Theorem: 3.1.2** Every continuous map is # αrg-continuous .

**Proof:** Let f: (X,$\tau$ ) → $(y, \sigma)$ be a continuous map then for every closed set A in y, $f^{-1}$(A) is closed in X.Since every closed set is #αrg-closed, $f^{-1}$(A) is #αrg -closed in X. Hence f is #αrg -continuous map.

**Theorem :3.1.3** Every #rg-continuous map is #αrg -continuous map .

**Proof:** Let f: (X,$\tau$ ) → $(y, \sigma)$ is #rg-continuous map then for every closed set A in y, $f^{-1}$(A) is #rg-closed in X. Since every #rg closed set is #αrg-closed, $f^{-1}$(A) is #αrg -closed in X.Hence f is #αrg-continuous map.

The converse of the theorem 3.1.2 and 3.1.3 is not necessarily true as seen from the following example.

**Example :**3.1.4 Let X {a,b,c}=y, $\tau$=({∅, X,{c},{b,c}} $\sigma = \{y, \emptyset, \{b\}\}$

**Define :** f: (X,$\tau$ ) ) → $(y, \sigma)$ by f(a)=a f(b)=c, f(c)=b clearly

i) f is #αrg - continuous but it is not continuous.
ii) f is #αrg - continuous but it is not #rg-continuous .

**Corollary:** 3.1.5 Every regular continuous map is #αrg-continuous but converse is not true.

**Proof :** Follows from Theorem 3.1.2and the fact that every regular continuous map is #rg-continuous.

**Theorem :**3.1.6 In a topological space(X,$\tau$) ,

(a)Every #αrg-continuous map is gp-continuous map .
(b) Every #αrg-continuous map is αg-continuous map.
(c) Every #αrg-continuous map is gsp-continuous map.

**Proof (a):** Suppose f: (X,$\tau$) → $(y, \sigma)$ is #αrg - continuous. Let V be a closed set in $(y, \sigma)$. Since f is #αrg - continuous then $f^{-1}$(V) is #αrg - closed set in (X,$\tau$). Since every #αrg-closed set is gp-closed set, then $f^{-1}$(V) is also gp-closed set in X. Thus f is gp-continuous.

**Proof (b)**: Suppose f: (X,$\tau$) → $(y, \sigma)$ is #αrg-continuous. Let V be a closed set in $(y, \sigma)$ since f is #αrg-continuous then $f^{-1}$(V) is #αrg-closed set in (X,$\tau$). Since every #αrg-closed set is αg-closed set then $f^{-1}$(V) is also αg-closed set in X. Thus f is αg-continuous.

**Proof (c):** Suppose f: (X,$\tau$) → $(y, \sigma)$ is #αrg-continuous let V be a closed set in $(y, \sigma)$ . Since f is #αrg-continuous then $f^{-1}$(V) is #αrg-closed set in (X,$\tau$). Since every #αrg-closed set is gsp closed set then $f^{-1}$(V) is also gsp-closed set in X. Thus f is gsp-continuous.

**Remark3.1.7:** The following example shows that converses of Theorem3.1.6 (a),(b)and (c) are not true.

**Example 3.1.8:**Let X={a,b,c}=Y $\tau$=({∅, X,{a}}$\sigma = \{y, \emptyset, \{a\}$ {a, b}}

**Define :** f: (X,$\tau$ ) ) $\rightarrow (y,\sigma)$ by f(a)=b, f(b)=c, f(c)=a,clearly

i) f is gp- continuous but it is not #αrg ontinuous .
ii) f is αg - continuous but it is not #αrg continuous.
ii) f is gsp - continuous but it is not #αrg continuous .

**Theorem3.1.9:** Let f: (X,$\tau$) $\rightarrow (y,\sigma)$ be a function then the following are equivalent.

(i). f is #αrg-continuous
(ii). The inverse image of each set in $(y,\sigma)$ is #αrg-open in (X,$\tau$)
(iii). The inverse image of each closed set in $(y,\sigma)$ is #αrg - closed in (X,$\tau$).

**Proof :** Suppose (i) holds. Let G be open in Y. Then Y\G is closed in Y. By (i) $f^{-1}$(Y\G) is #αrg-closed in X . But $f^{-1}$(Y\G) = X\ $f^{-1}$(G) which is #αrg-closed in X. Therefore $f^{-1}$(G) is #αrg-open in X. The proves (i)⟹(ii).

Suppose (ii) holds. Let V be any closed set in $(y,\sigma)$. Then Y\V is open set in Y. By (ii) $f^{-1}$(Y\V) is #αrg-open. But $f^{-1}$(Y\V)=X\$f^{-1}$(V) which is #αrg-open Therefore $f^{-1}$(V) is #αrg-closed. This prove (ii) ⟹(iii).

The implication (iii) ⟹ (i) follows from definition.

**Theorem3.1.10:** If a function f: (X,$\tau$) $\rightarrow (y,\sigma)$ is #αrg-continuous then f (#αrg-cl(A))⊆cl(f(A)) for every subset A of X.

**Proof :** Left: (X,$\tau$) $\rightarrow (y,\sigma)$ be #αrg - continuous. Let A⊆X then cl (f(A)) is closed in Y. Since f is #αrg-continuous, $f^{-1}$(cl(f(A)) is #αrg-closed in X and A⊆ $f^{-1}$(f(A)) ⊆ $f^{-1}$(cl(f(A)) implies #αrg-cl(A)⊆ $f^{-1}$(cl f(A)) hence f(#αrg-cl (A)) ⊆ cl (f(A).

**Theorem 3.1.11**: Let X be a space in which every singleton set is rw-closed. Then f: (X,$\tau$) $\rightarrow (y,\sigma)$ is #αrg-continuous iff x∈int($f^{-1}$(V)) for every open subset V of Ycontains f (x).

**Proof :** Suppose f: (X,$\tau$) $\rightarrow (y,\sigma)$ is #αrg-continuous. Fix x∈X and an open set V in Y such that f(x)∈V. Then $f^{-1}$(V) is #αrg-open. Since x∈ $f^{-1}$(V) and {x} is rw-closed, x∈int ($f^{-1}$(V)).

Conversely, assume that x∈int($f^{-1}$(V)) for every open subset V of Y containing f(x). Let V be an open set in Y. Suppose F⊆ $f^{-1}$(V) and F is rw-closed. Let x∈F then f(X)∈V so that x∈int($f^{-1}$(V)). That implies F⊆ x∈int($f^{-1}$(V)). Therefore $f^{-1}$(V) is #αrg-open . This proves f is #αrg-continuous.

**Theorem 3.1.12:** Let f: (X,$\tau$) $\rightarrow (y,\sigma)$ be a function .Let (X,$\tau$) and $(y,\sigma)$ be any two spaces such that $\tau_{\#\alpha rg}$ is a topology on X. Then the following statement are equivalent.

(i) For every subset A of X, f(#αrg-cl(A)) ⊆cl(f(A)) holds.
ii) f: (X,$\tau_{\#\alpha rg}$) $\rightarrow (y,\sigma)$ is continuous.

**Proof:** Suppose (i) holds. Let A be closed in Y. By hypothesis f (#αrg-cl ($f^{-1}$(A)) ⊆cl(f($f^{-1}$(A)) ⊆cl (A) = A.(ie) #αrg-cl($f^{-1}$(A)) ⊆ $f^{-1}$(A). Also $f^{-1}$(A) ⊆#αrg-cl($f^{-1}$(A) Hence #αrg-cl ($f^{-1}$(A)) =$f^{-1}$(A). This implies ($f^{-1}$(A))$^c$∈ $\tau_{\#\alpha rg}$. Thus $f^{-1}$(A) is closed in (X,$\tau_{\#\alpha rg}$) and so f is continuous.This Proves (ii).

Suppose (ii) holds. For every subset A of X, cl(f(A)) is closed in Y. Since f :(X,$\tau_{\#\alpha rg}$) $\rightarrow (y,\sigma)$ is continuous, $f^{-1}$(cl(f(A)) is closed in (X, $\tau_{\#\alpha rg}$) that implies #αrg-cl( $f^{-1}$(cl(f(A)) )= $f^{-1}$(cl(f(A) ).Now we have A⊆ $f^{-1}$((f(A)) ⊆ $f^{-1}$(cl(f(A)) and #αrg-cl(A) ⊆#αrg-cl($f^{-1}$(cl(f(A)) = $f^{-1}$(cl(f(A)) ). Therefore f(#αrg-cl(A) ⊆cl(f(A)).

**Remark :3.1.13** The composition of two #αrg-continuous maps need not be # αrg -continuous shown by an example.

**Example:3.1.14** Let X=Y= Z={a,b,c},$\tau$={$\phi$, X, {b},{b,c}} , $\sigma$={$\phi$, X, {a}} , $\mu$= {$\phi$, X, {b}}.Define a map by f(a)=b, f(b)=a and f(c)=c

**Theorem:3.1.15** Let (X,$\tau$) $(y,\sigma)$ and (z,$\mu$) be topological space such that $\sigma_{\#\alpha rg} = \sigma$ . Let f:(X,$\tau$) $\rightarrow (y,\sigma)$ and g: $(y,\sigma)$ →(z,$\mu$) be #αrg-continuous functions. Then the composition gof: (X,$\tau$) →(z,$\mu$) is #αrg-continuous.

**Proff :** Let V be closed in (z,$\mu$) .Since g is #αrg-continuous, $g^{-1}$(V) is #αrg-closed in $(y,\sigma)$ .Since $\sigma_{\#\alpha rg}$ =$\sigma$, $g^{-1}$(V) is closed in $(y,\sigma)$. Since f is #αrg - continuous, $f^{-1}$($g^{-1}$(V)) is #αrg-closed. (ie) $(gof)^{-1}$ (v) is #αrg - closed in (X,$\tau$).Therefore gof is #αrg - continuous.

**3.2.#αrg - irresolute functions.**

In this section #αrg - irresolute function is introduced and their basic properties are discussed.

**Definition :3.2.1** A function f: (X,$\tau$) $\rightarrow (y,\sigma)$ is called #αrg-irresolute if $f^{-1}(v)$ is # αrg - closed in (X,$\tau$) for every #αrg-closed subset Vof $(y,\sigma)$.

**Theorem : 3.2.2** Every #αrg - irresolute function is #αrg - continuous but converse is not necessarily true.

**Proof :** Suppose f: (X,$\tau$) $\rightarrow (y,\sigma)$ is called #αrg-irresolute. Let V be any closed subset of Y. Then V is #αrg-closed set in Y. Since f is #αrg-irresolute, $f^{-1}(v)$ is #αrg-closed in X. Hence f is #αrg-continuous .

The Converse of the theorem need not be true as seen from the following example.

**Example.3.2.3** Let X={a,b,c}=Y $\tau$=({∅, X,{b},{a,c}}$\sigma = \{y,\emptyset,\{a\}\{a,b\}\}$

**Define :** f: (X,$\tau$ ) ) → $(y,\sigma)$ by f(a)=a, f(b)=b, f(c)=c, #αrg – continuous but not #αrg – irresolute.

**Theorem :3.2.4** If a map f: (X,$\tau$) $\rightarrow (y,\sigma)$ is #αrg - continuous map Y is $\tau_{\#\alpha rg}$ - space then f is #αrg - irresolute.

**Proof :** Let f: (X,$\tau$) $\rightarrow (y,\sigma)$ is #αrg - continuous map then inverse image of every closed set in Y is #αrg-closed set is X. Since Y is $\tau_{\#\alpha rg}$ - space, inverse image of every #αrg-closed set in Y is #αrg-closed set in X. (ie) f is #αrg - irresolute.

**Theorem :3.2.5** Let f: (X,$\tau$) $\rightarrow (y,\sigma)$ be rw-irresolute and closed. Then f maps a #αrg-closed set in (X,$\tau$) into a #αrg - closed set in $(y,\sigma)$.

**Proof :** Let A be #αrg - closed in (X,$\tau$). Let f(A)⊆U where U is rw-open. Then A⊆$f^{-1}(U)$. since f is rw-irresolute, $f^{-1}(U)$ is rw-open in X. Since A is # αrg - closed , αcl(A) ⊆$f^{-1}(U)$ ,that implies f(αcl(A)) ⊆U since f is closed, f(αcl(A)) is closed that implies αcl (f(A))⊆ αclf(αcl(A))= f(αcl(A)) ⊆U. Hence f (A) is #αrg-closed in $(y,\sigma)$.

**Theorem :** 3.2.6Let f: X→Y and g: Y→ Z be any two function. Let h =gof. Then

(i) h is #αrg - continuous if f is #αrg − irresolute and g is #αrg-continuous.
(ii) h is #αrg-irresolute. If both f and g are #αrg - irresolute and
(iii) h is #αrg - continuous if g is continuous and f is #αrg - continuous.

**Proof :** Let V be closed in Z

(i) Suppose f is #αrg - irresolute and g is #αrg – continuous. since g is #αrg - continuous, $g^{-1}(V)$ is #αrg - closed in Y. Since f is #αrg - irresolute, using the definition 3.2.1 $f^{-1}(g^{-1}(V))$ is #αrg-closed in X. This prove (i)

(ii) Let f and g be #αrg - irresolute. Then $g^{-1}(V)$ is #αrg-closed in Y. Since f is #αrg - irresolute using the definition 3.2.1 $f^{-1}(g^{-1}(v))$ is #αrg - closed in X. This proves (ii)

(iii) Let g be continuous and f be #αrg - continuous. Then $g^{-1}(v)$ is closed in Y. Since f is #αrg - continuous using definition _3.1.1 $f^{-1}(g^{-1}(v))$ is #αrg-closed in X. This Proves (iii).

Theorem :3.2.7 A function f: (X,$\tau$) $\rightarrow (y,\sigma)$ is #αrg - irresolute if and only if the inverse image of every #αrg - open set in y is #αrg − open in X.

**Proof :** If follows easily as a direct consequence of definition.

**Theorem :** 3.2.8 If a map f: X→Y is #αrg - irresolute then for every subset A of X, f(#αrg - cl(A)) ⊆cl(F(A)).

**Proof :** For every subset A of X, cl (f(A)) is closed in Y. Thus cl(f(A) is # αrg - closed in Y. By hypothesis, $f^{-1}$(cl(f(A)) is #αrg - closed in X, As A⊆$f^{-1}$(f(A) ⊆ $f^{-1}$(cl(f(A)). We have #αrg-cl(A)⊆#αrg-cl($f^{-1}$(cl(f(A))=$f^{-1}$(cl(f(A)). Hence f (#αrg-cl(A) ⊆ cl (f(A).

**Theorem :3.2.9** If a map f: X→Y is #αrg - irresolute then for every A⊆Y, #αrg-cl ($f^{-1}$(A) ⊆ $f^{-1}$(cl(f(A))

**Proof :** For every subset A of Y, cl (A) is closed set in Y. Thus cl (A) is #αrg-closed in Y. By hypothesis, $f^{-1}$(cl(A)) is #αrg - closed in X, since A⊆cl (A), $f^{-1}$(A) ⊆$f^{-1}$cl(A)) which implies that # αrg-cl($f^{-1}$(A) ⊆#αrg-cl($f^{-1}$ (cl(A)) = $f^{-1}$(cl(f(A))

## REFERENCES

[1] K. Balachandran, P. Sundram, and H. Maki. On generalized continuous maps in topological spaces. Mem.fac.Sci.kochi Univ (Math) 12:5:13:1991
[2] Benchalli, S.S., and Wali. R.S., on RW-Closed sets in topological spaces, Bull, Malays, Math.Sci.Soc(2) 30(2)(2007), 99-110.
[3] Buswas, N., On Characterization of semi-continuous functions, Atti Accad, Naz. Lincei Rend,Cl.Sci.Fis.Mat. Natur. 48(8) (1970), 399-402.
[4] Cameron, D.E., Properties of S-closed spaces, Proc.Amer Math.Soc.72(1978), 581-586.
[5] Dontchev. J and Noiri. T Quasi-normal spaces and πg-closed sets, Acta Math. Hungar, 89(3)(2000), 211-219.
[6] Gnanmbal Y., On generalized pre-regular closed sets in topological spaces, Indian J. Pure App. Math. 28 (1997), 351-360.
[7] Gnanambal. Y. And Balachandran. K., On gpr-continuous functions in topological spaces, Indian J. Pure Appl. Math. 30(6)(1999), 581-593.
[8] Levine. N., Semi-open sets and semi-continuity in topological spaces, Amer, Math. Monthly, 70(1973), 36-
[9] Levine. N., Generalized closed sets in topology, Rend Circ. Mat. Palermo 19(1970), 89-96.
[10] Maki. H. Devi. R and Balachandran, K., Associated topologies of generalized α-cloed and α- generalized closed sets, Mem.Sci. Kochi Univ. Ser. A. Math 15(1994), 51-63.
[11] Mashhour. A.S., ABD. El-Monsef. M.e. and El-Deeb S.N., On pre continuous mappings and weak pre- continuous mappings, Proc Math, Phys. Soc. Egypt 53(1982), 47-53.
[12] Nagaveni. N., Studies on Generalizations of Homeomorphisms in Topological Spaces., Ph.D. Thesis, Bharathiar University, Coimbatore, 1999.
[13] Palaniappan N., and Rao. K.c., Regular generalized closed sets, Kyungpook Math. J. 33(1993),211-219.
[14] Palaniappan N, Pious Missier S and Antony Rex Rodrigo αgp -closed sets in Topological Spaces , Acta Ciencia Indica Vol.XXXI M, No.1,243 (2006).
[15] Park. J.K. and Park, J.H., Mildly generalized closed sets, almost normal and mildly normal spaces, Chaos Solitions and Fractals 20(2004), 1103-1111.
[16] Pushpalatha. A., Studies on Generalizations of Mappings in Topological Spaces, Ph.D. Thesis, Bharathiar University, Coimbatore, 2000.
[17] Sundaram. P and Shiek John. M., On w-closed sets in topology, Acta Ciencia Indica 4(2000), 389-392.
[18] Stone. M., Application of the theory of Boolean rings to general topology, Trans. Amer . Math. Soc.41(1937), 374-481.
[19] Syed Ali Fathima S and Mariasingam M on #regular generalized closed sets in Topological Spaces, International Journal of Mathematical Archive -2(11),2011 2497-2502.
[20] Syed Ali Fathima S and Mariasingam M on #regular generalized open sets in Topological Spaces, International Journal of Computer Applications (0795-8887)Volume 42-No.7 March 2012.
[21] Thilaga Leelavathi S and Mariasingam M on #$\alpha$-regular generalised closed in Topological Spaces, International Journal of Mathematical Trends and Technology (IJMTT) Volume 56 Number 8 – April 2018.
[22] Tong.J., Weak almost continuous mapping and weak nearly compact spaces, Boll. Un.Mat.Ital. (1982),385-391.
[23] Thakur C.K. Raman, Vidyottamma Kumari and Sharma M.K α--Generalized & α * - Separation Axioma for Topological Spaces, IOSR Journal of Mathematics Volume 10, Issue 3 Ver.VI(May-Jun 2014) PP 32-36.