Root Cube Difference Labeling Of Graphs

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Abstract

In this paper, we contribute some new results for Root Cube Difference Labeling of graphs. We prove that difference labeling of graphs are Root Cube Difference Labeling of Graphs. We use some standard graphs to derive the results for Root Cube Difference Graphs.

Keywords: Mean Labeling of graphs, Root Cube Mean Labeling of graphs and Root Cube Difference Labeling of graphs.

AMS Subject Classification (2010):05C78

1 INTRODUCTION:

If the vertices of the graph are assigned values subject to certain conditions is known as graph labeling. Graph labeling was first introduced in the mid-sixties. A dynamic survey on graph labeling is regularly updated by Gallian [2] and it is published by Electronic Journal of Combinatory. The notion of square difference labeling was introduced by J.Shiama [4]-[6]. Graph labeling can also be applied in areas such as communication network, mobile tele communication, and medical field. The concept of Cube difference labeling was first introduced by J.Shiama and it was proved in [7] that many standard graphs like Path, Cycle, Complete graphs, Ladder, Lattice grids, Wheels, Comb, Star graphs, Crown, dragon, Shell graphs and Coconut trees are cube difference labeling. R.Uma and S. Divya [9] have analysed cube difference labeling of star related graphs. The present work is aimed to discuss one such a labeling namely Root Cube Difference labeling Fan, Crown, Bistar and the Subdivision of the edges of the star graphs $k_{1,n}$.

2 Preliminaries

Definition 2.1 [7]

A crown graph R_n is formed by adding to the n points $v_1, v_2, ..., v_n$ of a cycle C_n ,n more pendent points $u_1, u_2, ..., u_n$ and n more lines $u_i v_i, i = 1,2,...,n$ for $n \geq 3$.

Definition 2.2 [7]

Let $G = (V(G), E(G))$ be a graph. G is said to be cube difference labeling if there exist a injection $f : v(G) \to \{0,1,..., p-1\}$ such that the induced function $f^* : E(G) \to N$ given by $f^*(uv) = \left|[f(u)]^3 - [f(v)]^3\right|$ is injective.

Definition 2.3 [7]

A graph which satisfies the cube difference labeling is called the cube difference graph.

Definition 2.4 [9]

A complete biparitite graph $K_{1,n}$ is called a star and it has n + 1 vertices and n edges.

Definition 2.5 [9]

A bistar graph B(m,n) is a graph obtained by attaching m pendant edges to one end point and n pendant edges to the other end points of K_2.

Definition 2.6 [9]

A merge graph $G_1 * G_2$ can be formed from two graphs G_1 and G_2 by merging a vertex of G_1 with a vertex of G_2.

Definition 2.7 [9]

A subdivision of a graph G is a graph that can be obtained from G by a sequence of edge subdivisions.

3. Root Cube Difference Labeling Of Graphs

Definition 3.1

A graph $G=(V(G),E(G))$ is said to be cube difference labeling if there exists a injection $f:v(G)\rightarrow\{0,1,...,p-1\}$ such that the induced function $f^*:E(G)\rightarrow N$ given by $f^*(uv)=\left\lceil\sqrt{\left|[f(u)]^3-[f(v)]^3\right|}\right\rceil$(or) $\left\lfloor\sqrt{\left|[f(u)]^3-[f(v)]^3\right|}\right\rfloor$ is injective.

Theorem 3.2

F_n is a Root Cube Difference Graph.

Proof:

Let G be a Fan graph F_n and let $|V(F_n)|=2n+1$ and $|E(F_n)|=3n$. Define the function $f:V(G)\rightarrow\{0,1,...n-1\}$ $f(u)=0$, $f(u_i)=i, 1\le i\le 2n$ and the induced edge labeling function $f^*:E(G)\rightarrow N$ is defined by $f^*(uv)=\sqrt{\left|[f(u)]^3-[f(v)]^3\right|}$ are all distinct such that $f(e_i)\neq f(e_j)$ for every $e_i\neq e_j$.

Then the edge sets are,

$E_1=\{uu_i/0\le i\le 2n\}$

$E_2=\{u_{2i+1}u_{2i+2}/0\le i\le n\}$

And the edges labeling are,

In E_1,

$$f^*(uu_i) = \sqrt{|i^3|}, 0 \le i \le 2n$$

$$= \{1,3,5,8,...\}$$

In E_2,

$$f^*(u_{2i+1}u_{2i+2}) = \sqrt{|12i^2 + 18i + 7|}, 0 \le i \le n$$

$$= \{2,6,9,...\}$$

Clearly the labeling of edges of E_1 and that E_2 are all distinct value. Therefore the Fan graph F_n is a Root Cube Difference Graph.

Example 3.3

The following is an example for Fan graph F_5 is Root Cube Difference Graph.

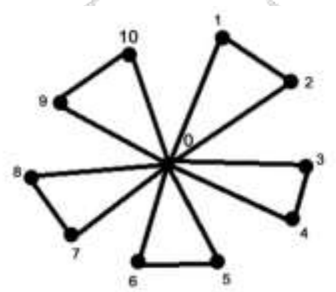

Figure 3.1

Theorem 3.4

Crown graph R_n admits Root Cube Difference Graph.

Proof:

A crown graph R_n is formed by adding to the n points $v_1, v_2, ..., v_n$ of a cycle C_n, n more pendent points $u_1, u_2, ..., u_n$ and n more lines $u_i v_i, i = 1,2,...,n$ for $n \ge 3$. Let $|V(G)| = 2n$ and $|E(G)| = 2n$. Define the function $f : V(G) \to \{0,1,...n-1\}$, $f(u_i) = i, 1 \le i \le 2n-1$, and the induced edge labeling function $f^* : E(G) \to N$ is defined by $f^*(uv) = \sqrt{|[f(u)]^3 - [f(v)]^3|}$ are all distinct such that $f(e_i) \ne f(e_j)$ for every $e_i \ne e_j$. Then the edge sets are,

$$E_1 = \{u_i u_{i+1} / 0 \le i \le n-1\}$$

$$E_2 = \{u_{n-1} u_0\}$$

$$E_3 = \{u_i u_{n+i} / 0 \le n+i \le 2n-1\}$$

In E_1,

$$f^*(u_i u_{i+1}) = \sqrt{|3i^2 + 3i + 1|} \quad 0 \le i \le n-1$$

$$= \{1,2,4,...\}$$

In E_2,

$$f^*(u_{n-1} u_0) = \sqrt{|(n-1)^3|}$$

$$= \{14\}$$

In E_3,

$$f^*(u_i u_{n+i}) = \sqrt{|21i^2 + 147i + 343|}, 0 \le n+i \le 2n-1$$

$$= \{18,22,26,...\}$$

Clearly the edge labels are distinct. Hence the Crown graph R_n admits the Root Cube Difference Labeling.

Example 3.5

Crown graph R_7 is a Root Cube Difference Graph.

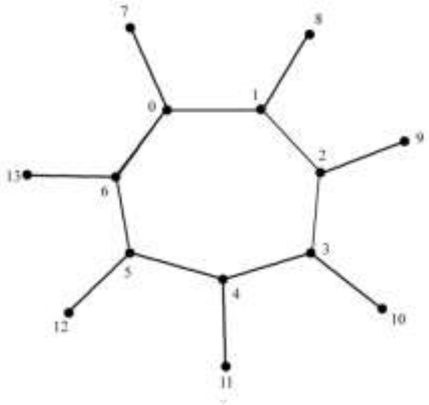

Figure 3.2

Remark 3.6

The following graph (C_3 * $K_{1,n}$) is not satisfied Root Cube Difference Labeling.

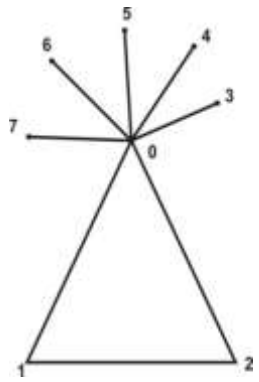

Figure 3.3

Here the vertices (u_0,u_2) and(u_2,u_1) edges value are not distinct. Therefore the graph (C_3 * $K_{1,n}$) is not satisfied Root Cube Difference Labeling.

Theorem 3.7

The subdivision of the edges of the star graph $K_{1,n}$ is a Root Cube Difference graph.

Proof :

Let G be a subdivision of a graph is obtained by the edges of the star graph $K_{1,n}$. Since G being a star graph. Let V(G) = 2n + 1 and E(G) = 2n, where the vertex set V(G) = $\{v, u_0, u_1,..., u_{n-1}, w_0, w_1,..., w_{n-1}\}$, where $u_0, u_1,..., u_{n-1}$ are adjacent vertices of V and $w_0, w_1,..., w_{n-1}$ are adjacent vertices of $u_0, u_1,..., u_{n-1}$.

Define the edge set E(G) = $\{E_1, E_2\}$, where E_1 = (v,u_i) and E_2 = (u_i,w_i) where i = 0,1,…,n-1. Define the function $f: V(G) \to \{0,1,...n-1\}$ as follows, f(v) = 0, f(u_i) = i+1 for $0 \le i \le n$ and f(w_i) = (n+1)+i, for $0 \le i \le n-1$.Then the induced function $f^*: E(G) \to N$ is defined by

$f^*(uv) = \sqrt{\left|[f(u)]^3 - [f(v)]^3\right|}$ are all distinct such that $f(e_i) \ne f(e_j)$ for every $e_i \ne e_j$.

Then the edge sets are,

$E_1 = \{uu_i / 0 \le i \le n-1\}$

$E2 = \{u_i w_i / 0 \le i \le n-1\}$

In E_1,

$f^*(uu_i) = \sqrt{\left|i^3 + 3i^2 + 3i + 1\right|}, 0 \le i \le n-1$

$= \{1,2,5,...\}$

In E_2,

$f^*(u_i w_i) = \sqrt{|15i^2 + 105i + 215|}, 0 \le i \le n-1$

$= \{14, 18, 22, ...\}$

The edge labels are distinct. Hence subdivision of the edges of the star graph $K_{1,n}$ is a Root Cube Difference graph.

Example :3.8

This example shows that subdivision of the edges of the star graph $K_{1,n}$ is a Root Cube Difference graph.

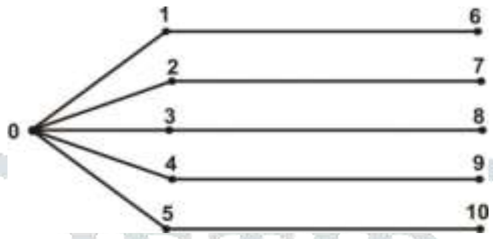

Figure 3.4

Theorem 3.9

The Bistar graph B(m,n) is a Root Cube Difference graph.

Proof:

Let G be a bistar graph B(m,n). Let V(G) = m + n + 2 and E(G) = m + n + 1. The vertex set V(G) = $\{u_0, u_1, ..., u_m, v_0, v_1, ..., v_n\}$, where $u_1, u_2, ..., u_m$ are pendant vertices adjacent to u_0 and $v_1, v_2, ..., v_n$ are n pendant vertices adjacent to v_0. Define the function $f : V(G) \to \{0, 1, ... n-1\}$ as follows, $f(u_0) = 0$, $f(v_0) = 1$, $f(u_i) = i + 1$, for $1 \le i \le m$ and $f(v_i) = m + i + 1$, for $1 \le i \le n$

then the induced function $f^* : E(G) \to N$ is defined by $f^*(uv) = \sqrt{|[f(u)]^3 - [f(v)]^3|}$ are all distinct such that $f(e_i) \ne f(e_j)$ for every $e_i \ne e_j$.

Then the edge sets are,

$E_1 = \{uu_i / 0 \le i \le n-1\}$

$E_2 = \{vv_i / 0 \le i \le n-1\}$

In E_1,

$f^*(uu_i) = \sqrt{|i^3 + 3i^2 + 3i + 1|}, 1 \le i \le n$

$= \{2, 5, 8, ...\}$

In E_2,

$$f^*(vv_i) = \sqrt{|i^3 + 21i^2 + 147i + 342|}, 1 \le i \le n$$

$$= \{22, 26, 31, \ldots\}$$

Here we get all the edges with distinct value. Hence Bistar graphs B(m,n) is a Root Cube Difference graph.

Example 3.10

The graph Bistar B(m,n) is a Root Cube Difference graph. Which shown in Figure 3.5

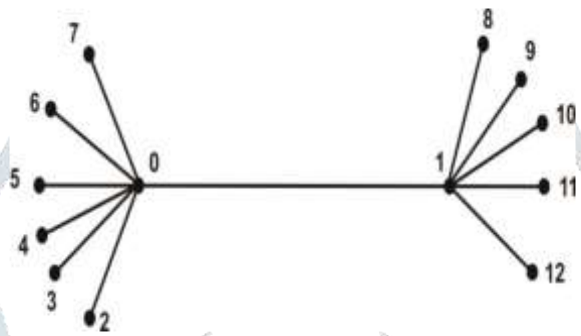

Figure 3.5

Conclusion

It is very interesting to investigate graphs which Root Cube Difference Graph. In this paper we proved that Fan graph, Crown graph, bistar graph, subdivision graph, are Root Cube Difference Graphs. It is possible to investigate similar results for several other graphs.

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