Holographic Dark Energy Model With Constant Deceleration Parameter in Scalar Tensor Theory Of Gravitation

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Abstract:- In this paper, we have investigated spatially homogeneous anisotropic Bianchi type-V universe filled with two minimally interacting fields, matter and holographic dark energy components in scalar-tensor theory of gravitation formulated by Saez and Ballester (Phys. Lett.A 113; 467, 1986). Exact solutions of field equations are obtained using the fact that scalar expansion is proportional to the shear scalar and constant deceleration parameter. Some physical and kinematical properties of the model are also discussed.

Keywords:- Bianchi type-V universe. Deceleration parameter. Holographic dark energy. Scalar-tensor theory.

1.Introduction:-

Recent observational data of modern cosmology based on various measurements reveals that our universe is experiencing transition from early inflation to the late time acceleration (Reiss[1], Perlmutter, et al. [2]). The main source responsible for this acceleration is supposed to be 'dark energy'. The concept of dark energy refers to a kind of exotic energy with negative pressure whose origin still remains a mystery. However, it is now believed that the universe consists of 76% dark energy, 20% dark matter and 4% ordinary matter. Two distinct approaches have been suggested to explain cosmic acceleration. The first approach deals with modifying Einstein gravity where in additional energy component is introduced to explain the concept of dark energy. In this approach a number of alternative models have been proposed but so far no suitable candidate is found. The other approach is to modify the Einstein Lagrangian by replacing the scalar curvature by a function of R known as $f(R)$ gravity (Nojini,et al. [3]). Other modified theories are $f(R,T)$ gravity (Harko [4]), Brans – Dicke ([5]) and Saez – Ballester ([6]) scalar – tensor theories of gravity.

Recent observation of the luminosity of type Ia supernovae indicate (Bachall et al. [7]; Perlmutter et al. [8]) an accelerated expansion of the universe and the surveys of clusters of galaxies show that the density of matter is very much less than the critical density. This observation leads to a new type of matter which violate the strong energy condition i.e., $\rho+2p<0$. The matter (fluid) content responsible for such a condition to be satisfied at a certain stage of evaluation of the universe is referred to as dark energy (Sahni and Starobinsky [9]; Peebles and Ratra [10]; Padmanabhan [11]; Copeland et al. [12]). This mysterious fluid is believed to dominate over the matter content of the Universe by 70% and to have enough negative pressure as to drive present day acceleration. Most of the dark energy models involve one or more scalar fields with various actions and with or without a scalar field potential (Maor and Brustein [13]; Cardenas and Campo[14]; Ferreira and Joyce [15]).

In recent years, there has been a considerable interest in holographic dark energy models because of the fact that holographic dark energy is an emerging model as a candidate of dark energy constructed by holographic principle (Guberina et al. [16]). It is argued that this model may solve the cosmological constant problem and some other issues. Using the holographic principle of quantum gravity theory (Susskind [17]) a viable holographic dark energy model was constructed by Li [18]. The holographic dark energy model is successful in explaining the observational data and has been, widely studied by several authors. Cohen et al. [19], Horova and Minic ([20]), Thomas ([21]), Hsu ([22]) are some of the authors who have investigated several aspects of holographic dark energy.

In particular, Setare ([23]) studied holographic dark energy model in Brans – Dicke theory. Sheykhi ([24]) studied interacting holographic dark energy models in Brans-Dicke (1961) theory of gravitation. Setare and Vanegas ([25]) have discussed the cosmological dynamics of interacting holographic dark energy model. Sarkar and Mahanta [26] have investigated holographic dark energy in Bianchi type-I space – time with constant deceleration parameter. Das and Mammon ([27]) studied holographic dark energy models in Brans-Dicke (1961) theory of gravitation. Sarkar ([28]) has discussed the evolution of holographic dark energy model in Bianchi type – I universe with linearly varying deceleration parameter and established a correspondence with generalized Chapligin gas models of the universe. Also holographic scalar field dark energy models are studied by many authors. For instance, Very recently Kiran et al. ([29], [30]) studied minimally interacting holographic dark energy models in Bianchi type-V space time in the scalar-tensor theories of gravitation proposed by Brans and Dicke (1961) and Saez and Ballester (1986).

Motivated by the above discussions and investigations, in this paper, we propose Bianchi type-V cosmological model for minimally interacting holographic dark energy in Seaz-Ballester theory of gravitation. The plan of the paper is follows. In section 2, we established the Seaz-Ballester field equations with the help of Bianchi type-V metric in the presence of matter and holographic dark energy. In section 3, we obtained the solution of the field equations. In section 4, we discuss some important properties of our model for both cases. Some discussions and conclusions are presented in the last section.

2. Metric and field equations:

We consider the Bianchi type-V space time described by the metric

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2x} dy^2 - C^2 e^{-2x} dz^2 \tag{1}$$

where A, B, C are functions of cosmic time t only.

Saez-Ballester (1986) field equations for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2} g_{ij} R - \omega\phi^n \left(\phi,_i \phi,_j - \frac{1}{2} g_{ij} \phi,_k \phi'^k\right) = -\left(T_{ij} + \overline{T}_{ij}\right) \tag{2}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n \phi^i_{;i} + n\phi^{n-1} \phi,_k \phi'^k = 0 \tag{3}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar, ω and n are arbitrary dimensionless constants and $8\pi G = c = 1$ in the relativistic units.

The energy momentum tensor for matter and the holographic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j \tag{4}$$

and

$$\overline{T}_{ij} = \left(\rho_\lambda + p_\lambda\right) u_i u_j - g_{ij} p_\lambda \tag{5}$$

where ρ_m, ρ_λ are the energy densities of matter and the holographic dark energy and p_λ is the pressure of the holographic dark energy.

Also, the energy conservation equation is

$$T^{ij}_{;j} + \overline{T}^{ij}_{;j} = 0 \tag{6}$$

In a commoving coordinate system, the field equations (2) and (3) for the metric (1) with the help of Eqns.(4) and (5) can be, explicitly, written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{1}{A^2} - \frac{\omega}{2}\phi^n \dot{\phi}^2 = -p_\lambda \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{A^2} - \frac{\omega}{2}\phi^n \dot{\phi}^2 = -p_\lambda \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} - \frac{\omega}{2}\phi^n \dot{\phi}^2 = -p_\lambda \tag{9}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{3}{A^2} + \frac{\omega}{2}\phi^n \dot{\phi}^2 = \rho_m + \rho_\lambda \tag{10}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{11}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{12}$$

where an overhead dot indicates differentiation with respect to t.

The energy conservation equation gives

$$\dot{\rho}_m + \rho_m\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \dot{\rho}_\lambda + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\left(1 + \omega_\lambda\right)\rho_\lambda = 0 \tag{13}$$

Here we are considering the minimally interacting matter and holographic dark energy components. Hence both the components conserve separately, so that we have (Sarkar 2014)

$$\dot{\rho}_m + \rho_m\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{14}$$

$$\dot{\rho}_\lambda + \left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)(1+\omega_\lambda)\rho_\lambda = 0 \tag{15}$$

Where $p_\lambda = \omega_\lambda \rho_\lambda$ is barotropic equation of state parameter for holographic dark energy.

The expression for physical and general parameters to be used in solving Saez-Ballester field equations for the metric (1) are as the follows:

The average scale factor $a(t)$ is given by

$$a(t) = (ABC)^{\frac{1}{3}} \tag{16}$$

The spatial volume V is given by

$$V = a^3 = ABC \tag{17}$$

The directional Hubble parameter, respectively, are

$$H_1 = \frac{\dot{A}}{A},\quad H_2 = \frac{\dot{B}}{B},\quad H_3 = \frac{\dot{C}}{C} \tag{18}$$

The average Hubble parameter is

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} \tag{19}$$

The dynamical scalar expansion θ for the space time given by Eq. (1) and the shear scalar σ^2 are

$$\theta = \left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \tag{20}$$

$$\sigma^2 == \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left[\left(\frac{\dot{A}}{A}\right)^2+\left(\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{C}}{C}\right)^2-\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{\dot{B}}{B}\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\frac{\dot{C}}{C}\right] \tag{21}$$

The average anisotropy parameter is

$$A_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{22}$$

where $H_i (i = 1,2,3)$ represent the directional Hubble parameters.

3. Solutions of Field equations:

Integration of Eq. (10) yields

$$A^2 = kBC \tag{23}$$

where k is a constant of integration which can be taken as unity without any loss of generality. So that we have

$$A^2 = BC \tag{24}$$

To find determinate solutions we use the following conditions:

(i) The scalar expansion θ is proportional to shear scalar σ^2 so that we have (Collins et al. 1980).

$$B = C^m \tag{25}$$

where $m \neq 1$ is a positive constant and preserves the anisotropy of the space-time.

(ii) According to Thorne [31], the observations of the velocity red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic within about 30% range approximately (Kantowski and Sachs [32], Kristian and sachs [33]) and red-shift studies place the limit $\frac{\sigma}{H} \le 0.3$, on the ratio of shear σ to Hubble H in the neighborhood of our galaxy today. Collin [34] discussed the physical significance of this condition for perfect fluid and barotropic EoS in a more general case. In many papers (Sarif Zubair [35], Yadav and Yadav [36]), this condition is proposed to find the exact solutions of cosmological models.

Since the line element (1) is completely characterized by Hubble's parameter H. Let us consider that mean Hubble parameter H is related to average scale factor a by following relation

$$H = k_1 a^{-s} \tag{26}$$

Where $k_1 > 0$ and $s \geq 0$ are constant.

The deceleration parameter is defined by as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{27}$$

From equations (26) and (27), we get

$$\dot{a} = k_1 a^{-s+1} \tag{28}$$

$$\ddot{a} = -k_1^{\,2}(s-1)\,a^{-2s+1} \tag{29}$$

Using equations (27), (28) and (29), we get

$$q = s - 1 \tag{30}$$

The signs of q indicates whether the model inflates or not. The positive signs of q corresponds to standard decelerating model where as the negative signs of indicates inflation.

From equation (27), we obtain the law of average scale factor a as

$$a = \begin{cases} (Dt + c_1)^{\frac{1}{s}}, s \neq 0 \\ c_2 e^{k_1 t}, s = 0 \end{cases} \tag{31}$$

Where c_1 and c_2 are the constant of integration.

From equation (30), for $s \neq 0$, it is clear that the condition for expansion of universe is $s > 0$ i.e. $q + 1 > 0$. Therefore for expansion model of universe the deceleration parameter q should be greater than -1.

3.1 Case (i): when $s \neq 0$

Then

$$a = (Dt + c_1)^{\frac{1}{s}} \tag{32}$$

Now, using Eqs. (16), (24), (25) and (32) we get the expressions for the metric coefficients as

$$A = (Dt + c_1)^{\frac{1}{s}} \tag{33}$$

$$B = (Dt + c_1)^{\frac{2m}{s(m+1)}} \tag{34}$$

$$C = (Dt + c_1)^{\frac{2}{s(m+1)}} \tag{35}$$

From equation (12) and (32) we get

$$\phi = \left[\frac{(n+2)s}{2(s-3)} t^{\frac{s-3}{s}} \right]^{\frac{2}{n+2}} \tag{36}$$

Using Eq. (33), (34) and (35) we can write the metric (1) in the form (after suitably choosing the integration constants, i.e. taking $D = 1, c_1 = 0$)

$$ds^2 = dt^2 - t^{\frac{2}{s}} dx^2 - e^{-2x}\left(t^{\frac{4m}{s(m+1)}} dy^2 + t^{\frac{4}{s(m+1)}} dz^2 \right) \tag{37}$$

3.2 Case (ii): when $s = 0$

Then

$$a = c_2 e^{k_1 t}, s = 0 \tag{38}$$

Taking $c_2 = 1$ equation (38) becomes

$$a = e^{k_1 t} \tag{39}$$

Now, using Eqs. (16), (24), (25) and (39) we get the expressions for the metric coefficients as

$$A = e^{k_1 t} \tag{40}$$

$$B = e^{\frac{2mk_1 t}{m+1}} \tag{41}$$

$$C = e^{\frac{2k_1 t}{m+1}} \tag{42}$$

From equation (12) and (38) we get

$$\phi = \left[\frac{-(n+2)}{2k_1 e^{k_1 t}} \right]^{\frac{2}{n+2}} \tag{43}$$

Using Eq. (40), (41) and (42) we can write the metric (1) in the form

$$ds^2 = dt^2 - e^{2k_1 t} dx^2 - e^{-2x}\left(e^{\frac{4mk_1 t}{m+1}} dy^2 + e^{\frac{4k_1 t}{m+1}} dz^2 \right) \tag{44}$$

4. Physical discussion:

4.1. Case (i): when $s \neq 0$

Eq. (37) represents minimally interacting Bianchi type-V holographic dark energy model with constant deceleration parameter in Saez-Ballester theory with the following physical and geometrical parameters .

Spatial volume in the model is

$$V = t^{\frac{3}{s}} \tag{45}$$

The average Hubble parameter is

$$H = \frac{1}{st} \tag{46}$$

The scalar expansion is

$$\theta = \frac{3}{st} \tag{47}$$

The shear scalar is

$$\sigma^2 = \frac{(m-1)^2}{s^2(m+1)^2 t^2} \tag{48}$$

The average anisotropy parameter is

$$A_m = \frac{2(m-1)^2}{3(m+1)^2} \tag{49}$$

From Eqns. (7), (36), and (37) the holographic pressure in the model is

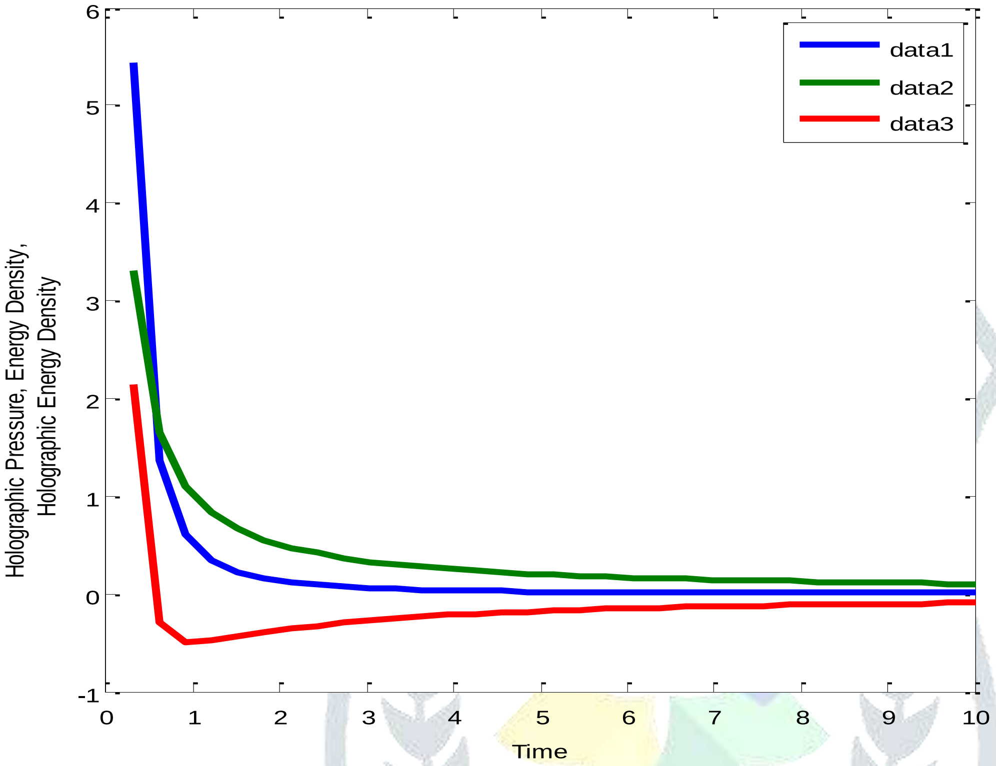
$$p_\lambda = \frac{2\left[s(m+1)^2 - 2(m^2+m+1)\right]}{s^2(m+1)^2 t^2} + \frac{1}{t^{\frac{2}{s}}} + \frac{\omega}{2}\frac{1}{t^{\frac{6}{s}}} \tag{50}$$

The energy density of dark matter is

$$\rho_m = \frac{\rho_0}{t^{\frac{3}{s}}} \tag{51}$$

The holographic energy density in the model is

$$\rho_\lambda = \frac{2(m^2+4m+1)}{s^2(m+1)^2 t^2} - \frac{3}{t^{\frac{2}{s}}} + \frac{\omega}{2}\frac{1}{t^{\frac{6}{s}}} - \frac{\rho_0}{t^{\frac{3}{s}}} \tag{52}$$



Data1- Holographic Pressure Vs Time, Data2- Energy Density Vs Time
Data3- Holographic Energy Density Vs Time

Now by using barotropic equation of state parameter for holographic dark energy, eq[ns] (50) and (51) we get the equation of state of parameter (EoS) as

$$\omega_\lambda = \frac{\dfrac{2\left[s(m+1)^2 - 2(m^2+m+1)\right]}{s^2(m+1)^2 t^2} + \dfrac{1}{t^{\frac{2}{s}}} + \dfrac{\omega}{2}\dfrac{1}{t^{\frac{6}{s}}}}{\dfrac{2(m^2+4m+1)}{s^2(m+1)^2 t^2} - \dfrac{3}{t^{\frac{2}{s}}} + \dfrac{\omega}{2}\dfrac{1}{t^{\frac{6}{s}}} - \dfrac{\rho_0}{t^{\frac{3}{s}}}} \tag{53}$$

which shows that ω_λ is a function of cosmic time t only.

The coincidence parameter is

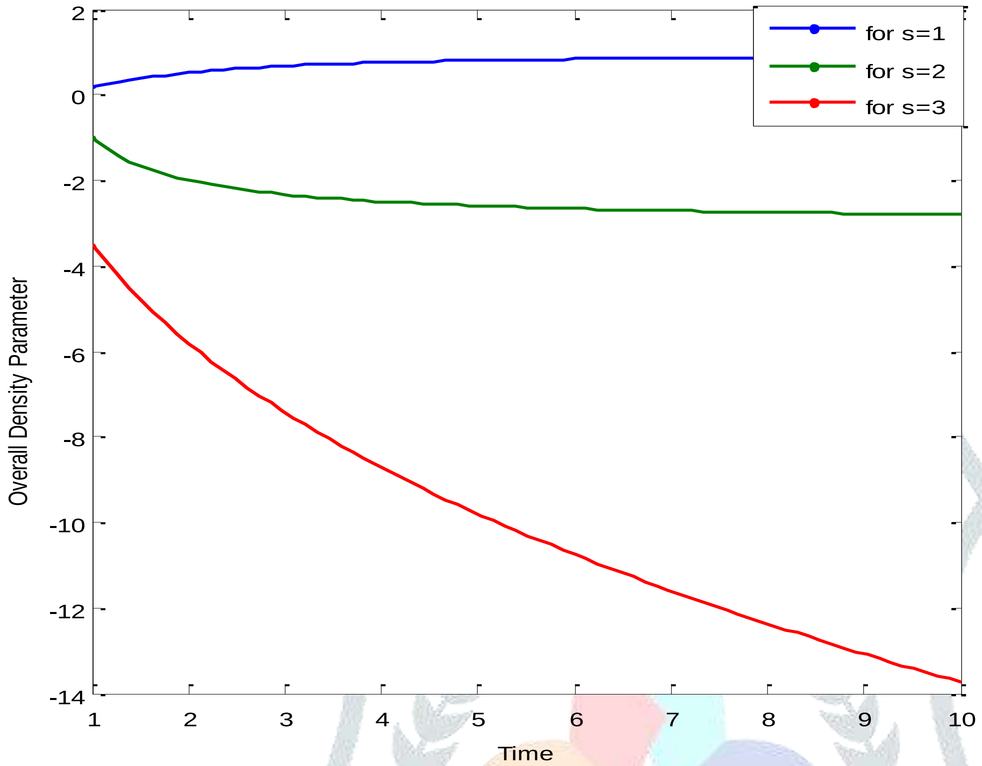
$$r = \frac{1}{\rho_0}\left[\frac{2(m^2+4m+1)}{s^2(m+1)^2 t^{\frac{2s-3}{s}}} - 3t^{\frac{1}{s}} + \frac{\omega}{2}\frac{1}{t^{\frac{3}{s}}} - \rho_0\right] \tag{54}$$

The matter density parameter Ω_m and holographic energy density parameter Ω_λ are given by

$$\Omega_m = \frac{\rho_m}{3H^2} \quad \text{and} \quad \Omega_\lambda = \frac{\rho_\lambda}{3H^2} \tag{55}$$

Using barotropic equation of state parameter for holographic dark energy, (51), (52) and (55) we get the overall density parameter as

$$\Omega = \Omega_m + \Omega_\lambda = \frac{1}{3}\left[\frac{2(m^2+4m+1)}{(m+1)^2} - 3\frac{s^2}{t^{\frac{2-s}{s}}} + \frac{\omega}{2}\frac{s^2}{t^{\frac{6-2s}{s}}}\right] \tag{56}$$



The cosmic jerk parameter is dimension less third order derivative of the scale factor with respect to the cosmic time defined as

$$j(t) = \frac{1}{H^3}\frac{\dddot{a}}{q} = (1-s)(1-2s) \tag{57}$$

4.2. Case (ii): when $s = 0$

Eq. (44) represents minimally interacting Bianchi type-V holographic dark energy model with constant deceleration parameter in Saez-Ballester theory with the following physical and geometrical parameters .
Spatial volume in the model is

$$V = e^{3k_1 t} \tag{58}$$

The average Hubble parameter is

$$H = k_1 \tag{59}$$

The scalar expansion is

$$\theta = 3k_1 \tag{60}$$

The shear scalar is

$$\sigma^2 = k_1^2 \frac{(m-1)^2}{(m+1)^2} \tag{61}$$

The average anisotropy parameter is

$$A_m = \frac{2(m-1)^2}{3(m+1)^2} \tag{62}$$

From Eqns. (7), (43), and (44) the holographic pressure in the model is

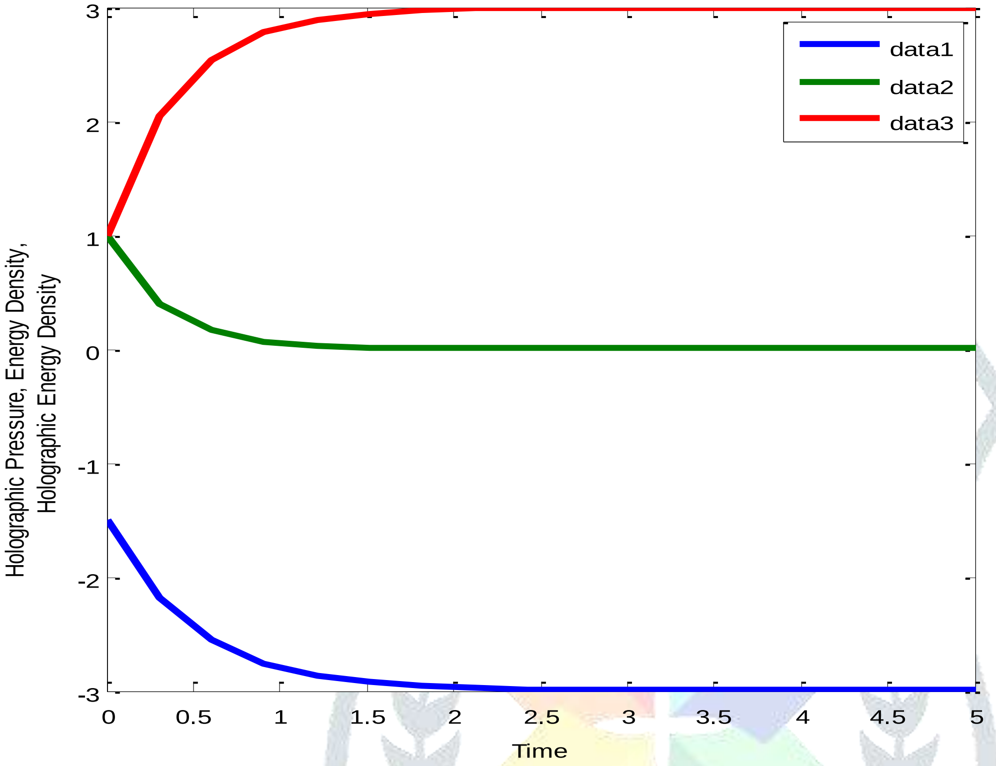
$$p_\lambda = \frac{-4\left(m^2+m+1\right)}{(m+1)^2} + \frac{1}{e^{2k_1 t}} + \frac{\omega}{2}\frac{1}{e^{2k_1 t}} \tag{63}$$

The energy density of dark matter is

$$\rho_m = \frac{\rho_0}{e^{3k_1 t}} \tag{64}$$

The holographic energy density in the model is

$$\rho_\lambda = \frac{2k_1^2(m^2+4m+1)}{(m+1)^2} - \frac{3}{e^{2k_1 t}} + \frac{\omega}{2}\frac{1}{e^{2k_1 t}} - \frac{\rho_0}{e^{3k_1 t}} \tag{65}$$



Data 1- Holographic Pressure Vs Time, Data 2- Energy Density Vs Time
Data 3- Holographic Energy Density Vs Time

Now by using barotropic equation of state parameter for holographic dark energy, eq^{ns} (64) and (65) we get the equation of state of parameter (EoS) as

$$\omega_\lambda = \frac{\dfrac{-4(m^2+m+1)}{(m+1)^2} + \dfrac{1}{e^{2k_1 t}} + \dfrac{\omega}{2}\dfrac{1}{e^{2k_1 t}}}{\dfrac{2k_1^2(m^2+4m+1)}{(m+1)^2} - \dfrac{3}{e^{2k_1 t}} + \dfrac{\omega}{2}\dfrac{1}{e^{2k_1 t}} - \dfrac{\rho_0}{e^{3k_1 t}}} \tag{66}$$

which shows that ω_λ is a function of cosmic time t only.

The coincidence parameter is

$$r = \frac{1}{\rho_0}\left[\frac{2k_1^2(m^2+4m+1)e^{3k_1 t}}{(m+1)^2} - 3e^{k_1 t} + \frac{\omega}{2}e^{k_1 t} - \rho_0\right] \tag{67}$$

Using barotropic equation of state parameter for holographic dark energy, (55), (64) and (65) we get the overall density parameter as

$$\Omega = \Omega_m + \Omega_\lambda = \frac{1}{3}\left[\frac{2(m^2+4m+1)}{(m+1)^2} - \frac{3}{k_1^2 e^{2k_1 t}} + \frac{\omega}{2}\frac{1}{k_1^2 e^{2k_1 t}}\right] \tag{68}$$

The cosmic jerk parameter is dimension less third order derivative of the scale factor with respect to the cosmic time defined as

$$j(t) = \frac{1}{H^3}\frac{\ddot{a}}{q} = 1 \tag{69}$$

5. Conclusions:

In this paper we have investigated spatially homogeneous and anisotropic Bianchi type-V minimally interacting holographic dark energy models in the scalar-tensor theory of gravitation proposed by Saez and Ballester (1986). The present investigation will help for a better understanding of dark energy models in Saez-Ballester theory of gravitation. We have obtained the energy density of matter, holographic dark energy density, holographic pressure, equation of state (EoS) parameter, the scalar field in the model, total energy density in the universe discussed their physical behavior in each of the models. In both cases jerk parameter in the models are constant. It is observed that the average density parameter in the universe is function of t. Average anisotropic parameter $A_m \neq 0$ so our model is anisotropic except $m \neq 1$. Thus the model presented here is anisotropic, shearing except $m \neq 1$. The spatial volume increases with time and the physical parameters decrease and ultimately tend to zero as $T \to \infty$.

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