A Study on Optimal Reserve Inventory Between Machines Through Order Statistics

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Abstract: In Inventory control one of the interesting and important problems is the determination of optimal reserve inventory size between two machines in a serial system. The output of the machine M_1 is the raw material for machine M_2. The breakdown of the M_1 will force the machine M_2 to go to the breakdown stage. Therefore the reserve inventory in between two machines is maintained. The optimal size of the reserve inventory is determined taking into consideration a random duration of breakdown of M_1, consumption rate of M_2, the holding cost as well as shortage cost of inventory. Several researchers have discussed this problem. In this paper it is assumed that machine M_1 is in the first stage and the two machines M_{21} and M_{22} are in the second stage. The optimal reserve inventory level is derived assuming that the breakdown duration of M_1 follows first and n^{th} order statistics under logistic distribution. Numerical illustration is also provided.

Key words: Optimal Reserve Inventory, Breakdown Duration, Order Statistics, Logistic distribution.

Introduction

In inventory control theory the series system is found to be with greater viability for application in industries, which are productions oriented. The basic model is one in which it is assumed that the two machines namely M_1 and M_2 are in series and the finished product of machine M_1 happens to be a the raw material for machine M_2 will go to the down stage due to none availability of the semi-finished product produced by M_1. Therefore to avoid the shortage cost a reserve inventory in between M_1 and M_2 is suggested taking it consideration the inventory holding cost and also the shortage cost of the semi-finished products. Several researchers have discussed this problem and the basic model is found in Hansmann (1962). Ramachandran and Sathiyamoorthy (1981) have considered a modified version of this model Venkatesan et al.(2010) have discussed such a model, Sachithananantham et al., (2006) have analysed reserve inventory between two machines with SCBZ Property and derived the expression for the optimal reserve inventory level under the condition that the analysis is carriedout before and after the truncation point. Vijayasankar et al., (2014) have designed a production system in which two stages are considered such as (i) recruitment and training stage and (ii) work spot stage. The demand of manpower between two stages is assumed as random variable which follows order statistics through skew logistic distribution. The optimal level of reserve manpower has been obtained empirically and numerically. Govindan et al (2016) have studied the problem under reserve inventory and used the concept of order statistics. Sachithananantham et al (2007) have the optimal size of the reserve inventory is obtained under the assumption that the repair time distribution satisfies the SCBZ property.

In this paper, reserve inventory levels are observed between machines. The process is performed in two stages such that in the first stage machine M_1 is functioning and the machines M_{21} and M_{22} are functioning in the second stage. The semi finished products of M_1 is the input of Machines M_{21} and M_{22}. The breakdown period of M_1 is considered as random variable which follows order statistics under logistic distribution. The expression for optimal reserve inventory levels are obtained in relation with the first and n^{th} order statistics. Numerical values and the corresponding curves are exhibited.

Assumptions and Notations

- M_1 is the machine in the first stage and the output of $M_1$is the raw material for machines $M_{21}$and M_{22} in the second stage.
- If there is a breakdown of M_1 the supply of raw material for M_{21} and M_{22} will be stopped.
- The reserve inventory if maintained between the machine M_1 and the machines M_{21} and M_{22}.
- The breakdown duration of M_1 is a random duration.
- A reserve inventory of the semi-finished product is maintained between the machines in series.

S-Reserve inventory level

μ-mean time interval between the successive breakdowns of machine M_1

τ-A random variable which denotes the duration of breakdown / repair time of M_1 and it has p.d.f. g(.) with c.d.f.G(.)and $\tau \sim \exp(\lambda)$

h-Holding cost per unit of reserve inventory

d_1 – Cost per unit of idle time of machine M_{21}

d_2 – Cost per unit of idle time of machine M_{22}

r_1 – Consumption rate per unit time of machine M_{21}

r_2 – Consumption rate per unit time of machine M_{22}

Case 1

Consider a random sample of n observations which are the time intervals of breakdown of machine M_1. These observations are arranged in increasing order and are denoted as

$$Y_{(1)} \leq Y_{(2)} \leq Y_{(3)} \leq \ldots \leq Y_{(n)}$$

Here $Y_{(1)}$ and $Y_{(n)}$ are the first and n^{th} order statistics and their corresponding pdfs are defined in general as

$$f_{(1)}(t) = n\,[1-G_1(t)]^{n-1}\, g_1(t) \qquad (1)$$

and

$$f_{(n)}(t) = n\,[G_n(t)]^{n-1}\, g_n(t) \qquad (2)$$

The breakdown periods of M_1 are defined as random variables which are distributed according to Logistic distribution.

Now, the p.d.f of the first order statistic for random samples from logistic distribution, [Norman L. Johnson, and Samuel Kotz, (1970)] is given by

$$g_1(\tau) = ne^{-n\tau}(1+e^{-\tau})^{-(n+1)} \qquad (3)$$

and its cumulative distribution function is given by

$$G_1(\tau) = (1+e^{\tau})^{-n} \qquad (4)$$

Based on the first order statistic related to logistic distribution for breakdown periods of M_1, the expected cost is stated as

$$E(c) = \frac{nh}{(r_1+r_2)}\int_0^{\frac{s}{(r_1+r_2)}}\left(\frac{s}{r_1+r_2}-\tau\right)[1-G_1(\tau)]^{n-1}g_1(\tau)d\tau$$

$$+\frac{n(d_1+d_2)}{\mu}\int_{\frac{s}{r_1+r_2}}^{\infty}\left(\tau-\frac{s}{r_1+r_2}\right)[1-G_1(\tau)]^{n-1}g_1(\tau)d\tau$$

$$E(c) = \frac{nh}{(r_1+r_2)}I_1 + \frac{n(d_1+d_2)}{\mu}I_2 \qquad (5)$$

Where, $I_1 = \int_0^{\frac{s}{(r_1+r_2)}}\left(\frac{s}{r_1+r_2}-\tau\right)[1-G_1(\tau)]^{n-1}g_1(\tau)d\tau$

Apply the expressions of $g_1(\tau)$ and $G_1(\tau)$ from (3) and (4) respectively and get,

$$I_1 = \int_0^{\frac{s}{r_1+r_2}}\left(\frac{s}{r_1+r_2}-\tau\right)[1-(1+e^{\tau})^{-n}]^{n-1}e^{-n\tau}(1+e^{-\tau})^{-(n+1)}\,d\tau$$

For solving this integral, differentiating I_1 by using Leibnitz rule of differentiation of an integral, we get

$$I_1 = \frac{1}{n^2(r_1+r_2)}\left[\left\{1-\left(1+e^{\frac{s}{r_1+r_2}}\right)^{-n}\right\}^n-(1-2^{-n})^n\right]$$

Again,

$$I_2 = \int_{\frac{s}{(r_1+r_2)}}^{\infty}\left(\tau - \frac{s}{r_1+r_2}\right)[1-G_1(\tau)]^{n-1} g_1(\tau)d\tau$$

similarly, use the expressions (3) and (4) in the above equation and then differentiate I_2 by using Leibnitz rule of differentiation of an integral, we get

$$I_2 = \frac{1}{n^2(r_1+r_2)}\left[-1+\left\{1-\left(1+e^{\frac{s}{r_1+r_2}}\right)^{-n}\right\}^{n}\right]$$

Now, substitute the expression of I_1 and I_2 in the equation (5) which becomes

$$\frac{\partial}{\partial s}E(c) = \frac{h}{n(r_1+r_2)^2}\left[\left\{1-\left(1+e^{\frac{s}{r_1+r_2}}\right)^{-n}\right\}^{n} - (1-2^{-n})^{n}\right]$$

$$+\frac{(d_1+d_2)}{n\mu(r_1+r_2)}\left[-1+\left\{1-\left(1+e^{\frac{s}{r_1+r_2}}\right)^{-n}\right\}^{n}\right] \qquad (6)$$

Apply some simple mathematics in the equation (6), the optimal reserve inventory level $\hat{S}$ is obtained as

$$\hat{S} = \frac{(r_1+r_2)}{loge}\log\left[-1+\left\{1-\left(\frac{(d_1+d_2)(r_1+r_2)+\mu h(1-2^{-n})^{n}}{\mu h+(d_1+d_2)(r_1+r_2)}\right)^{\frac{1}{n}}\right\}^{-1/n}\right] \qquad (7)$$

Case 2

In this case, the model studied in case (1) is extended to the n^{th} order statistic. Here also, the random variable τ follows Logistic distribution.

The general p.d.f. of the n^{th} order statistic is already given in the equation (2). Also, the p.d.f. of the n^{th} order statistic based on Logistic distribution is stated as

$$g_n(\tau) = ne^{-\tau}(1+e^{-\tau})^{-(n+1)} \qquad (8)$$

Its cumulative distribution function is defined as

$$G_n(\tau) = (1+e^{-\tau})^{-n} \qquad (9)$$

Using the above stated p.d.f. and cumulative distribution function along with relavent costs, the expected cost is framed as

$$E(c) = \frac{nh}{(r_1+r_2)}\int_0^{\frac{s}{(r_1+r_2)}}\left(\frac{s}{r_1+r_2}-\tau\right)[G_n(\tau)]^{n-1}g_n(\tau)d\tau$$

$$+\frac{n(d_1+d_2)}{\mu}\int_{\frac{s}{(r_1+r_2)}}^{\infty}\left(\tau-\frac{s}{r_1+r_2}\right)[G_n(\tau)]^{n-1}g_n(\tau)d\tau$$

$$E(c) = \frac{nh}{(r_1+r_2)}I_3 + \frac{n(d_1+d_2)}{\mu}I_4 \qquad (10)$$

Here, $I_3 = \int_0^{\frac{s}{(r_1+r_2)}}\left(\frac{s}{r_1+r_2}-\tau\right)[G_n(\tau)]^{n-1}g_n(\tau)d\tau$

Using the expressions of $g_n(\tau)$ and $G_n(\tau)$ from the expressions (8) and (9) respectively, and get

$$I_3 = \int_0^{\frac{s}{(r_1+r_2)}} \left(\frac{s}{r_1+r_2} - \tau\right) [1 + e^{-\tau})^{-n}]^{n-1}\{ne^{-\tau}(1+e^{-\tau})^{-(n+1)}\}d\tau$$

Now, differentiate I_3 by using Leibnitz rule of differentiation of an integral, we get,

$$I_3 = \frac{1}{n^2(r_1+r_2)}\left\{\left[1 + e^{-\frac{s}{r_1+r_2}}\right]^{-n^2} - (2)^{-n^2}\right\}$$

Again,

$$I_4 = \int_{\frac{s}{(r_1+r_2)}}^{\infty} \left(\tau - \frac{s}{r_1+r_2}\right) [G_n(\tau)]^{n-1} g_n(\tau) d\tau$$

As usual, use the expressions of (8) and (9) in the above equation and differentiate I_4 by using Leibnitz rule of differentiation of an integral. We have

$$I_4 = \frac{1}{n^2(r_1+r_2)}\left[-1 + \left\{1 + e^{-\frac{s}{r_1+r_2}}\right\}^{-n^2}\right]$$

Now, substitute the expressions of I_3 and I_4 in the equation (10) which becomes

$$\frac{\partial}{\partial S}E(c) = \frac{h}{n(r_1+r_2)}\left[\left\{1 + e^{-\frac{s}{r_1+r_2}}\right\}^{-n^2} - 2^{-n^2}\right]$$

$$+\frac{(d_1+d_2)}{n\mu(r_1+r_2)}\left[-1 + \left\{1 + e^{-\frac{s}{r_1+r_2}}\right\}^{-n^2}\right] \qquad (11)$$

Apply some simple mathematics in equation (11), the optimal reserve inventory level $\hat{S}$ is obtained as

$$\hat{S} = \frac{(r_1+r_2)}{-loge}\log\left[-1 + \left\{\frac{(d_1+d_2)(r_1+r_2)+\mu h 2^{-n^2}}{\mu h+(d_1+d_2)(r_1+r_2)}\right\}^{-1/n^2}\right] \qquad (12)$$

Numerical Illustration

Case 1: The variations in $\hat{S}$ consequent to the changes in d_1, d_2, μ and h have been studied by assuming other parameters as r_1=14, r_2 =14.5, d_1 = 40, d_2=60, μ=7.5, h=30 and n =10. The computed values and their curves are presented here.

Table 1.1: Optimal reserve inventory for varying d_1

d_1	30	35	40	45	50
$\hat{S}$	12.9215	13.1534	13.3736	13.5832	13.7838

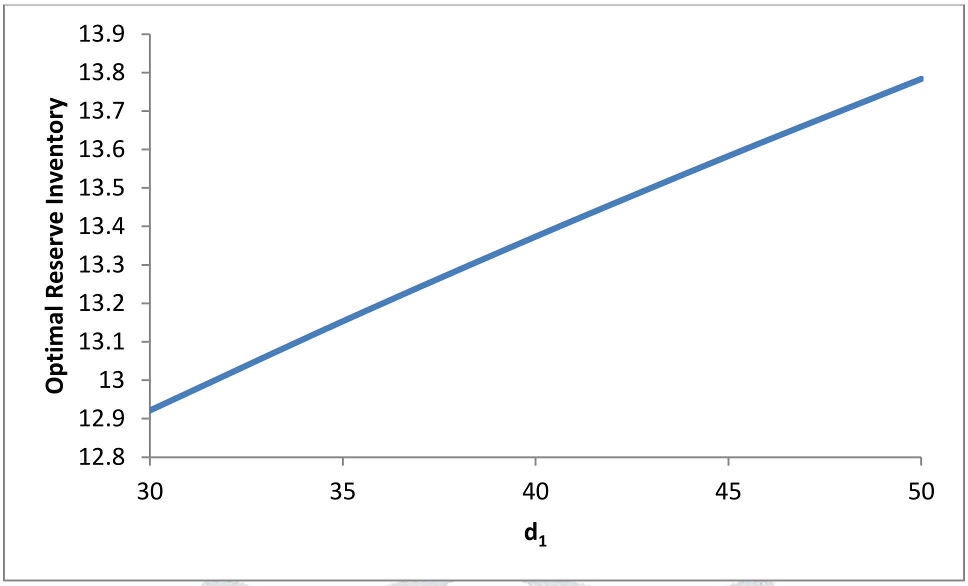

Fig. 1.1: Optimal reserve inventory for varying d_1

Table 1.2: Optimal reserve inventory for varying d_2

d_2	40	50	60	70	80
$\hat{S}$	12.4173	12.9213	13.3736	13.7832	14.1574

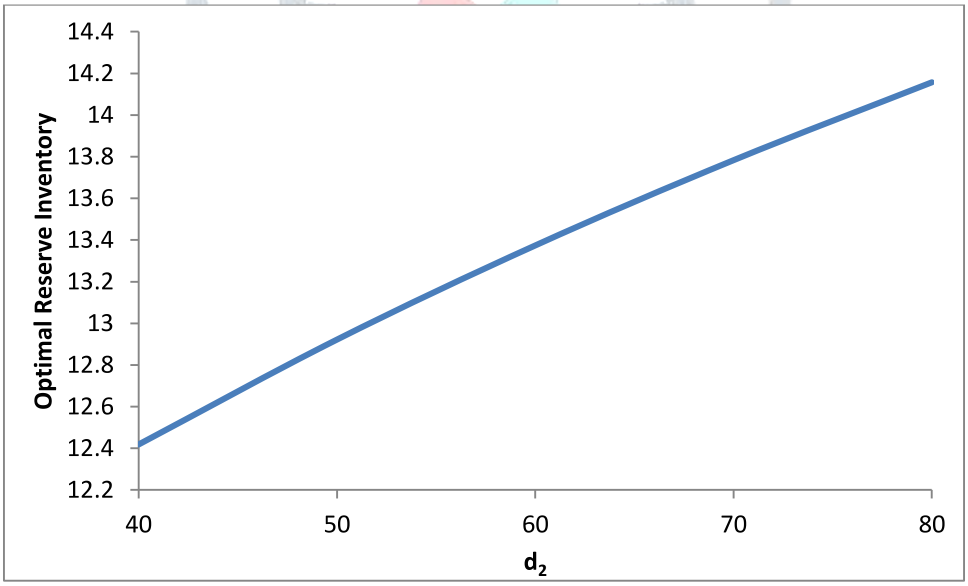

Fig. 1.2: Optimal reserve inventory for varying d_2

Table 1.3: Optimal reserve inventory for varying μ

μ	2.5	5.0	7.5	10.0	12.5
$\hat{S}$	18.0844	15.1176	13.3736	12.1418	11.1947

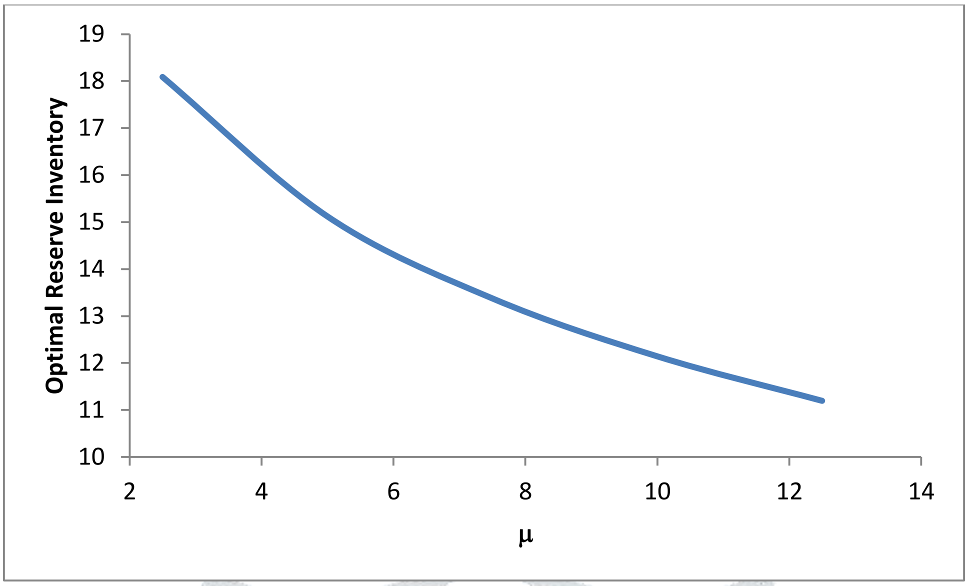

Fig. 1.3: Optimal reserve inventory for varying μ

Table 1.4: Optimal reserve inventory for varying h

h	20	25	30	35	40
$\hat{S}$	15.1176	14.1574	13.3736	12.7124	12.1418

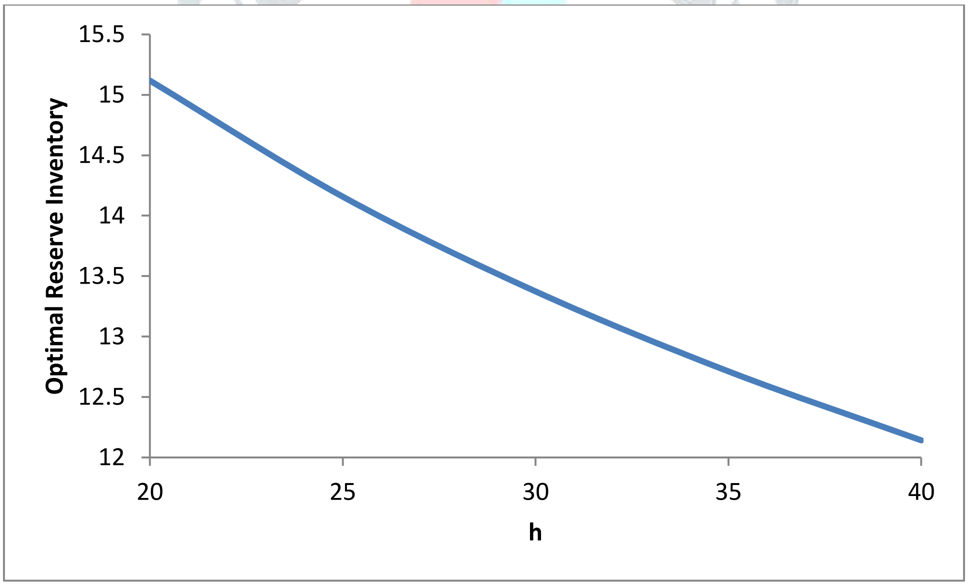

Fig. 1.4: Optimal reserve inventory for varying h

Case 2: In this case, the variations in $\hat{S}$ are computed for the changes in d_1, d_2, μ and h when other parameters are fixed as r_1=2, r_2 =2.5, d_1 =5, d_2=10, μ=12.5, h=45 and n =10.

Table 2.1: Optimal reserve inventory for varying d_1

d_1	5	10	15	20	25
$\hat{S}$	17.0564	17.6022	18.0583	18.4538	18.8051

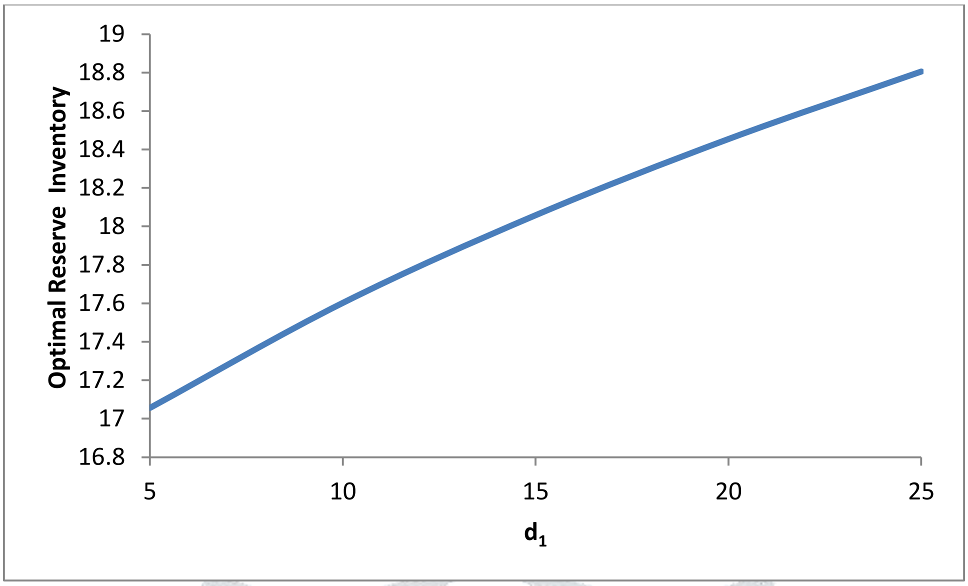

Fig. 2.1: Optimal reserve inventory for varying d_1

Table 2.2: Optimal reserve inventory for varying d_2

d_2	10	20	30	40	50
$\hat{S}$	17.0564	18.0583	18.8051	19.4127	19.9301

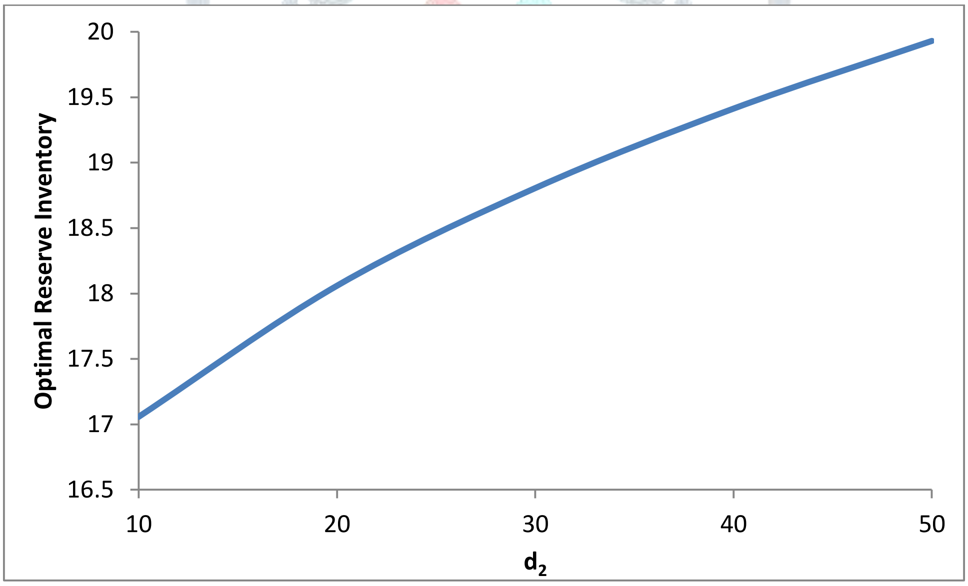

Fig. 2.2: Optimal reserve inventory for varying d_2

Table 2.3: Optimal reserve inventory for varying μ

μ	10.0	12.5	15.0	17.5	20.0
$\hat{S}$	17.4757	17.0564	16.7333	16.4729	16.2562

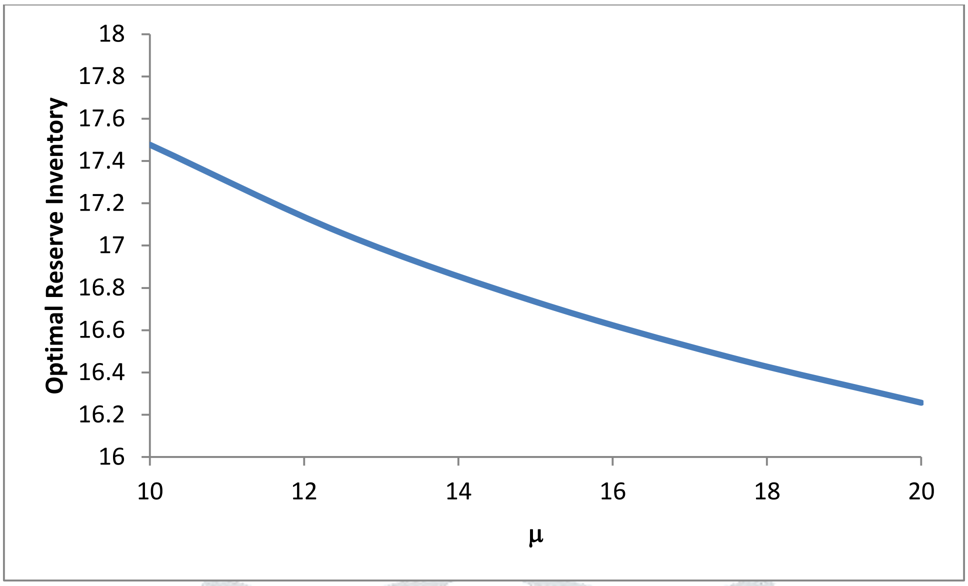

Fig. 2.3: Optimal reserve inventory for varying µ

Table 2.4: Optimal reserve inventory for varying h

h	40	45	50	55	60
$\hat{S}$	17.2744	17.0564	16.8677	16.7017	16.5541

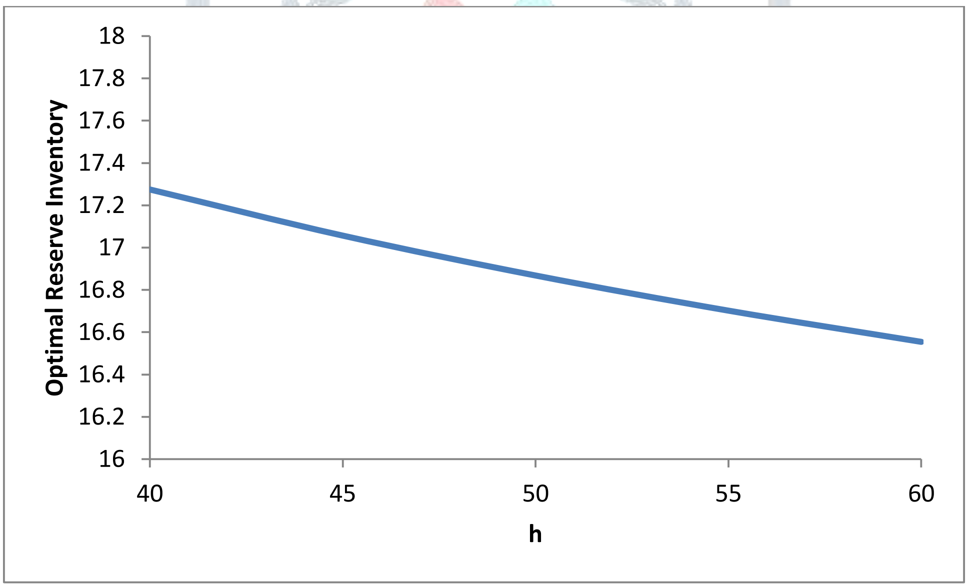

Fig. 2.4: Optimal reserve inventory for varying h

Conclusion

The expressions for the optimum reserve inventory ($\hat{S}$) are derived based on the first and n^{th} order statistics from Logistic distribution. The nature of the optimum reserve inventory has been analysed through numerical results by assuming shortage costs (d_1 and d_2), holding cost (h) and average time duration (µ) between successive breakdowns of initial machine of the working process are variables and other parameters are fixed.

The figures (1.1), (1.2), (2.1) and (2.2) revealed that, in both cases, the optimum reserve inventory increases when the shortage costs increase due to insufficient supply to machines M_{21} and M_{22}. Also, the figures (1.3), (1.4), (2.3) and (2.4) showed that the optimum reserve inventory decreases for increasing both holding cost and average time duration between successive breakdowns of initial machine of the working process.

It is remarked that the minimum reserve inventory requires higher breakdown period and higher holding cost. It is also concluded that smaller shortage cost provides minimum reserve inventory.

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