2-VARIABLE SIMPLE DISCRETE HEAT EQUATION

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INTRODUCTION

Partial difference and differential equation take vital rule in heat equation. To get discrete partial difference equation. We need to introduce an operator called generalized 2 dimensional difference operator,

The two dimensional difference operator as $\underset{(l_1,l_2)}{\Delta}$ is defined as

$\underset{(l_1,l_2)}{\Delta} v(k_1,k_2)=v(k_1+l_1,k_2+l_2)-v(k_1,k_2)$

For real valued function $v(k_1,k_2)$

The equation involving $\underset{(l_1,l_2)}{\Delta}, \underset{(l_1,0)}{\Delta}$ and $\underset{(0,l_2)}{\Delta}$ is called generalized second order partial difference equation

(i.e)

$f(\underset{(l_1,l_2)}{\Delta}, \underset{(l_1,0)}{\Delta}, \underset{(0,l_2)}{\Delta}, v(k_1,k_2))$ =0 is called generalized second order partial difference operator

By taking $(l_1=1), (l_2=0)$ (or) $(l_1=0), (l_2=1)$

We get second order partial difference equation

By taking $(l_1 \to 0)$ and $(l_2 \to 0)$

We can make equation (1.1) as a partial difference equation using $\underset{(l_1,l_2)}{\overset{-1}{\Delta}} v(k_1,k_2)$ we can get solution (1.1)

1.1 FORMATION OF 2-VARIABLE SIMPLE DHE

Consider the temperature distribution of a very long rod

Assume that the rod is so long, that it can be laid on the set R of real numbers. Let $v(k_1,k_2)$ be the temperature at the real time (k_1) and real position (k_2) of a rod at time (k_1)

If the temperature $v(k_1,k_2-l_2), l_2>0$ is higher than $v(k_1,k_2)$ heat will flow from the point k_2-l_2 to k_2

The amount of increase is $v(k_1+l_1,k_2)-v(k_1,k_2)$ and

It is reasonable to postulate that the increase is proportional to the difference $v(k_1,k_2-l_2)-v, \alpha>0$

(I,e)

$$\underset{(l_1,0)}{\Delta} v(k_1,k_2)=\alpha \underset{(0,-l_2)}{\Delta} v(k_1,k_2) \qquad (1.1)$$

$$v(k_1+l_1,k_2)-v(k_1,k_2)=\alpha\big(v(k_1,k_2-l_2)-\ v(k_1,k_2)\big)$$

$$v(k_1+l_1,k_2)-v(k_1,k_2)=\alpha\ v(k_1,k_2-l_2)-\alpha\ v(k_1,k_2)$$

$$v(k_1,k_2)-\alpha\ v(k_1,k_2)=v(k_1+l_1,k_2)-\alpha\ v(k_1,k_2-l_2)$$

$$(1-\alpha)v(k_1,k_2)=v(k_1+l_1,k_2)-\alpha\ v(k_1,k_2-l_2)$$

$$v(k_1,k_2)=\frac{1}{1-\alpha}v(k_1+l_1,k_2)-\frac{\alpha}{1-\alpha}v(k_1,k_2-l_2)$$

If $k_1=k_2$ and α=$\frac{l_1}{-l_2}$s

Then $v(k_1,k_2)=(k_1.k_2)$ is a solution of discrete heat equation

Here k_1,k_2 are variable and l_1,l_2 are parameters.

1.2: SOLUTION OF 2-V SIMPLE HEAT EQUATION

In this section we derive a solution of equation (1.1), here we have to obtain a function $v(k_1,k_2)$ satisfying the equation (1.1)

THEOREM 1.2.1

Let $u(k_1,k_2)$=$\underset{l_1,0}{\Delta}$ $v(k_1,k_2)$ and integer m such that $u(k_1,k_2-rl_2)$ and are known for r=1, 2 ...m then 1-dimensional of the form

$v(k_1,k_2)-v(k_1-ml_1,k_2)=\alpha\sum_{r=1}^{\left[\frac{k_1}{l_1}\right]}u(k_1-r_1l_1,k_2)$(1.2)

Proof

From the (1.1), since α is constant

We have

$v(k_1,k_2)=\ \alpha\underset{(l_1,0)}{\overset{-1}{\Delta}}\left(\underset{(0,-l_2)}{\Delta}v(k_1,k_2)\right)$(1.3)

From the given condition, we express (1.2) as

$$v(k_1,k_2)=\ \alpha\underset{(l_1,0)}{\overset{-1}{\Delta}}u(k_1,k_2)$$

$$v(k_1,k_2)=u(k_1-l_1,k_2)+v(k_1-l_1,k_2)$$

Replace $k_1$by k_1-l_1 and $k_2by\ k_2$

$$v(k_1-l_1,k_2)=u(k_1-2l_1,k_2)+v(k_1-2l_1,k_2)$$

$$v(k_1,k_2)=u(k_1-l_1,k_2)+u(k_1-2l_1,k_2)+v(k_1-2l_1,k_2)$$

Now (1.2) follows by applying inverse principle with respect to $\underset{(l_1,0)}{\Delta}$

EXAMPLE 1.2.2

Taking $v(k_1,k_2) = \frac{k_2}{2l_1}(k_1)_{l_1}^{(2)}$ we get

$\frac{k_2}{2l_1}(k_1)_{l_1}^{(2)} - \frac{k_2}{2l_1}(k_1 - \frac{k_1}{l_1}l_1)_{l_1}^{(2)} = \sum_{r=1}^{\left[\frac{k_1}{l_1}\right]}(k_1 - r_1 l_1, k_2)$

Again taking $k_1 = 20, k_2 = 15, l_1 = 4$, we get

$\frac{15}{8}[20(20-4)] - \frac{15}{8}(20-20)_4^{(2)} = \sum_{r=1}^{5}(20 - r(4))(15) = 600$

THEOREM 1.2.3

If $1-\alpha \neq 0$, then the second type solution of simple heat equation (1.1) is

$$v(k_1,k_2) = \frac{1}{(1-\alpha)^m}\, v(k_1+l_1,k_2) - \sum_{r=1}^{m}\frac{\alpha}{(1-\alpha)^r}[v(k_1+(r-1)l_1,k_2-l_2)] \qquad (1.5)$$

Proof

From (1.1), we have

$$v(k_1+l_1,k_2) - v(k_1,k_2) = \alpha\big(v(k_1,k_2-l_2) - v(k_1,k_2)\big)$$

$$v(k_1,k_2) = \frac{1}{1-\alpha}v(k_1+l_1,k_2) - \frac{\alpha}{1-\alpha}v(k_1,k_2-l_2) \qquad (1.6)$$

Replace k_1 *by* $k_1 + l_1$ *in* (1.6) and substituting the corresponding values, we get

$$v(k_1+l_1,k_2) = \frac{1}{1-\alpha}v(k_1+2l_1,k_2) - \frac{\alpha}{1-\alpha}v(k_1+l_1,k_2-l_2)$$

$$v(k_1,k_2) = \frac{1}{1-\alpha}[\frac{1}{1-\alpha}v(k_1+2l_1,k_2) - \frac{\alpha}{1-\alpha}v(k_1+l_1,k_2-l_2)] - \frac{\alpha}{1-\alpha}v(k_1,k_2-l_2)$$

$$v(k_1,k_2) = \frac{1}{(1-\alpha)^2}v(k_1+2l_1,k_2) - \frac{\alpha}{(1-\alpha)^2}v(k_1+l_1,k_2-l_2)] - \frac{\alpha}{1-\alpha}v(k_1,k_2-l_2) \qquad (1.7)$$

Replace k_1 *by* $k_1 + 2l_1$ *in* (1.6)

$$v(k_1+2l_1,k_2) = \frac{1}{1-\alpha}v(k_1+3l_1,k_2) - \frac{\alpha}{1-\alpha}v(k_1+2l_1,k_2-l_2)$$

Substitute in Equation (1.7),

$$v(k_1,k_2) = \frac{1}{(1-\alpha)^2}[\frac{1}{1-\alpha}v(k_1+3l_1,k_2) - \frac{\alpha}{1-\alpha}v(k_1+2l_1,k_2-l_2)] - \frac{\alpha}{(1-\alpha)^2}v(k_1+l_1,k_2-l_2)] - \frac{\alpha}{1-\alpha}v(k_1,k_2-l_2)$$

$$v(k_1,k_2)=\frac{1}{(1-\alpha)^3}v(k_1+3l_1,k_2)-\frac{\alpha}{(1-\alpha)^3}v(k_1+2l_1,k_2-l_2)-\frac{\alpha}{(1-\alpha)^2}v(k_1+l_1,k_2-l_2)-\frac{\alpha}{1-\alpha}v(k_1,k_2-l_2)$$

In general

$$v(k_1,k_2)=\frac{1}{(1-\alpha)^m}\,v(k_1+l_1,k_2)-\sum_{r=1}^{m}\frac{\alpha}{(1-\alpha)^r}[v(k_1+(r-1)l_1,k_2-l_2)]$$

THEOREM 1.2.4

If $1-\alpha\neq 0$, then the third type solution of heat equation (1.9) is

$v(k_1,k_2)=\frac{1}{1-\alpha}v(k_1+l_1,k_2)-\frac{\alpha}{(1+\alpha)^{m+1}}v(k_1+ml_1,k_2)+\sum_{r=1}^{m}\frac{\alpha^2}{(1-\alpha)^{r+1}}[v(k_1+(r-1)l_1,k_2-2l_2)]$ (1.8)

Proof

From (1.1), we have

$$v(k_1+l_1,k_2)-\ v(k_1,k_2)=\alpha[v(k_1,k_2-l_2)-v(k_1,k_2)$$

Replace $k_1\ by\ k_1\ and\ k_2\ by\ k_2-l_2$ in (1.6) and substituting corresponding values, we get

$$v(k_1,k_2-l_2)=\frac{1}{1-\alpha}v(k_1+l_1,k_2-l_2)-\frac{\alpha}{1-\alpha}v(k_1,k_2-2l_2)$$

$$v(k_1,k_2)=\frac{1}{1-\alpha}v(k_1+l_1,k_2)-\frac{\alpha}{1-\alpha}[\frac{1}{1-\alpha}v(k_1+l_1,k_2-l_2)-\frac{\alpha}{1-\alpha}v(k_1,k_2-2l_2)]$$

$$v(k_1,k_2)=\frac{1}{1-\alpha}v(k_1+l_1,k_2)-\frac{\alpha}{(1-\alpha)^2}v(k_1+l_1,k_2-l_2)+\frac{\alpha^2}{(1-\alpha)^2}v(k_1,k_2-2l_2)$$

Replace $k_1\ by\ k_1+l_1\ and\ k_2\ by\ k_2-l_2$ in (1.6)

$v(k_1+l_1,k_2-l_2)=\frac{1}{1-\alpha}v(k_1+2l_1,k_2-l_2)-\frac{\alpha}{1-\alpha}v(k_1+l_1,k_2-2l_2)$(1.9)

Substitute in equation (1.9)

$$v(k_1,k_2)=\frac{1}{1-\alpha}v(k_1+l_1,k_2)-\frac{\alpha}{(1-\alpha)^2}[\frac{1}{1-\alpha}v(k_1+2l_1,k_2-l_2)-\frac{\alpha}{1-\alpha}v(k_1+l_1,k_2-2l_2)]+\frac{\alpha^2}{(1-\alpha)^2}v(k_1,k_2-2l_2)$$

$$v(k_1,k_2)=\frac{1}{1-\alpha}v(k_1+l_1,k_2)-\frac{\alpha}{(1-\alpha)^3}v(k_1+2l_1,k_2-l_2)+\frac{\alpha^2}{(1-\alpha)^3}v(k_1+l_1,k_2-2l_2)+\frac{\alpha^2}{(1-\alpha)^2}v(k_1,k_2-2l_2)$$

In general

$$v(k_1,k_2)=\frac{1}{1-\alpha}v(k_1+l_1,k_2)-\frac{\alpha}{(1+\alpha)^{m+1}}v(k_1+ml_1,k_2)+\sum_{r=1}^{m}\frac{\alpha^2}{(1-\alpha)^{r+1}}[v(k_1+(r-1)l_1,k_2-2l_2)]$$

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