RELIABILITY TEST PLAN FOR THE TRANSMUTED NEW WEIBULL-PARETO DISTRIBUTION

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Abstract: **In this paper, a reliability test plan is used for acceptance or rejection of a lot of products submitted for inspection in which the lifetime of the items follow Transmuted New Weibull-Pareto Distribution. Sampling plans in which items that are put to test, to collect the life of the items in order to decide upon accepting or rejecting a submitted lots are called reliability test plans. Here the proposed sampling plan can save the test time in practical situations. Minimum sample size required to accept or reject a submitted lot for a given acceptance number with producer's risk is found. The test plan to determine the termination time of the experiment for a given sample size, producer's risk and termination number is also constructed. Results are illustrated with examples. The comparative study of the proposed reliability test plan with existing sampling plan is explained with respect to time of the experiment.**

Keywords: **Transmuted New Weibull-Pareto Distribution, Reliability Test Plan, Minimum Sample Size, Producer's risk, Experimental time.**

I. INTRODUCTION

Reliability is probability that a component, device, equipment or a system will perform its intended function adequately for a specific period of time under a given set of conditions. Reliability study plays a vital role in the quality control analysis. On the basis of this study, an experimenter can save the time and cost to reach the result which is to accept the submitted lot or to reject it. If the genuine products are rejected on the basis of sample information, this error is called type-I error. On the other hand, if the genuine products are not accepted by the consumer, this error is called type-II error. If a decision to accept or reject the lot are subjected to the risks associated with the two types of errors, this procedure is termed as 'reliability test plan' or 'acceptance sampling based on life test'.

Designing of suitable acceptance sampling plans for various situations is an important exercise in the study of statistical quality control systems. Acceptance sampling plans help us to examine whether the manufactured products meet the pre-specified quality levels. They are primarily used in statistical quality control when it is not possible to perform complete inspection of the manufactured products for various reasons like, the manufactured products being destructive in nature or complete inspection may be a time consuming process. Basically, acceptance sampling plans help us to assess the quality level of the product based on sampled items. Acceptance sampling is a methodology which prescribes the sample size and deals with the procedures for taking decision to accept or reject the manufactured product based on the inspection of samples. It is the purpose of acceptance sampling to sentence lots, not to estimate the lot quality. Most acceptance sampling plans are not designed for estimation purposes.

Characteristics of an acceptance sampling plan are studied mainly with the help of probability distributions which involve certain parametric values. Let us assume that the lifetime of a product follows Transmuted New Weibull-Pareto distribution, whose probability density function and cumulative distribution function are given respectively by

$$f(t;\beta,\sigma,\delta,\lambda) = \frac{\beta\delta}{\sigma}\left({}^{t}/_{\sigma}\right)^{\beta-1} e^{-\delta\left({}^{t}/_{\sigma}\right)^{\beta}}\left(1-\lambda+2\lambda e^{-\delta\left({}^{t}/_{\sigma}\right)^{\beta}}\right); \quad \text{-------------(1)}$$

$$0 < t < \infty, \beta > 0, \sigma > 0, \delta > 0, |\lambda| \le 1$$

$$F(t;\beta,\sigma,\delta,\lambda) = \left(1-e^{-\delta\left({}^{t}/_{\sigma}\right)^{\beta}}\right)\left(1+\lambda e^{-\delta\left({}^{t}/_{\sigma}\right)^{\beta}}\right); \quad \text{----------------(2)}$$

$$0 < t < \infty, \beta > 0, \sigma > 0, \delta > 0, |\lambda| \le 1$$

where σ is the scale parameter and β, δ and λ are shape parameters. Acceptance sampling plans based on truncated life tests for exponential distribution was first discussed by Epstein [2]. The results were extended for the Weibull distribution by Goode and Kao [3], Balakrishnan et al. [1] provide the time truncated acceptance plans for Generalized Birnbaum-Saunders Distribution, Kantam, Rosaiah and Srinivasa Rao [4] developed acceptance sampling based on life tests: log-logistic model. Srinivasa Rao, Ghitany and Kantam [5] developed Reliability Test Plans for Marshall-Olkin Extended Exponential Distribution. Srinivasa Rao [6] developed an Economic Reliability Test Plan Based on Truncated Life Tests for Marshall-Olkin Extended Weibull Distribution. The Reliability Test plan is given in section II. The comparative study is presented in section III. The conclusion is given in section IV.

II. RELIABILITY TEST PLANS

To compare the performance of various acceptance sampling plans, their performance over a range of possible quality levels is studies. In statistical quality control, acceptance sampling plan is concerned with the inspection of a sample of products taken from a lot and the decision whether to accept or not to accept the lot based on the quality of the product. In a life testing experiment, the procedure is to terminate the test by a predetermined time t and note the number of failures. If the number of failures at the end of time t does not exceed a given number c, called acceptance number then we accept the lot with a given probability of at least p. But if the number of failures exceeds c before time t then the test is terminated, and the lot is rejected. For such truncated life test and the associated decision rule, we are interested in obtaining the smallest sample size to arrive at a decision. In the sequel, we assume that the distribution parameters β, δ and λ are known, while σ is unknown. In such a case, the average lifetime of the product depend only on σ, and can be observed that the average lifetime is monotonically increasing in σ. Let σ_0 represent the required minimum average lifetime, then, for given values of β, δ and λ.

The consumer's risk, i.e., the probability of accepting a bad lot should not exceed 1-P^*, where P^* is a lower bound for the probability that a lot of true value of σ below σ_0 is rejected by the sampling plan. For a fixed P^*, sampling plan is characterized by (n, c, t/σ_0).

By sufficiently large lots we can apply binomial distribution to find acceptance probability. The problem is to determine the smallest positive integer n, for given values of P^*, t/σ_0 and c, such that

$$L(P_0) = \sum_{i=0}^{c} \binom{n}{i} P_0^{\,i}(1-P_0)^{n-i} \leq 1 - P^* \text{ ------------(3)}$$

where P_0 = F(t; β, δ, λ, σ_0), obtained from equation (2), indicates the failure probability before time t depends only on the ratio t/σ_0. The function L(P) is the operating characteristic function of the sampling plan, i.e., the acceptance probability of the lot as function of the failure probability P(σ) = F(t;β,δ,λ,σ) is decreasing function in σ which implies that the operating characteristic function is increasing in σ. The minimum values of n satisfying the inequality (3) are obtained and displayed in Table 1 for P^*= 0.90, 0.95, 0.99 and t/σ_0= 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712 for β =2, δ = 2 and λ =1.

Table 1: Minimum sample size required to accept / reject a submitted lot for a given acceptance number (c) with producer's risk (P^*)

P*	c	t/σ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.90	0	2	1	1	1	1	1	1	1
	1	4	2	2	2	2	2	2	2
	2	5	3	3	3	3	3	3	3
	3	7	5	4	4	4	4	4	4
	4	8	6	5	5	5	5	5	5
	5	9	7	6	6	6	6	6	6
	6	11	8	7	7	7	7	7	7
	7	12	9	8	8	8	8	8	8
	8	14	10	9	9	9	9	9	9
	9	15	11	10	10	10	10	10	10
	10	16	12	11	11	11	11	11	11
0.95	0	2	1	1	1	1	1	1	1
	1	4	3	2	2	2	2	2	2
	2	6	4	3	3	3	3	3	3
	3	7	5	4	4	4	4	4	4
	4	9	6	5	5	5	5	5	5
	5	10	7	6	6	6	6	6	6
	6	12	8	7	7	7	7	7	7
	7	13	9	8	8	8	8	8	8
	8	15	10	9	9	9	9	9	9
	9	16	11	10	10	10	10	10	10
	10	17	12	11	11	11	11	11	11

0.99	0	3	2	1	1	1	1	1	1
	1	5	3	2	2	2	2	2	2
	2	7	4	3	3	3	3	3	3
	3	9	5	4	4	4	4	4	4
	4	10	6	5	5	5	5	5	5
	5	12	7	7	6	6	6	6	6
	6	13	9	8	7	7	7	7	7
	7	15	10	9	8	8	8	8	8
	8	16	11	10	9	9	9	9	9
	9	18	12	11	10	10	10	10	10
	10	19	13	12	11	11	11	11	11

Alternatively, we considered another approach for a reliability test plan. We summarize this approach. Let n indicates the number of sampled items to be determined and r stands for a natural number, such that if r failures out of n samples are occurred before the terminated time t the lot would be rejected. In this aspect, r is called as termination number. The sample size is depending upon the cost consideration and the expected time to reach a decision. If the sample size is large it may reduce the expected waiting time but increase the cost of consideration. Let us take sample size as a multiple of the termination number to balance between these two aspects. As we have come to know that the probability of r failures out of n tested items is given as ${}_nC_r p^r(1-p)^{n-r}$, where p=F(t; β, δ, λ, σ) as before. Thus, the probability of accepting the lot is

$$\mathrm{L}(P) = \sum_{i=0}^{r-1} \binom{n}{i} P^i (1-P)^{n-i} \quad \text{------------(4)}$$

If α is producer's risk then equation (5) can be written as:

$$\sum_{i=0}^{r-1} \binom{n}{i} P^i (1-P)^{n-i} = 1\text{-}\alpha \quad \text{-------------(5)}$$

Given the values of n=r.k, equation (5) can be solved for P using cumulative probabilities of binomial distribution. Then the values of P can be used in equation (5) for α = 0.10, 0.05, 0.01 to find the values of t/σ_0. These values for different values of r and n are given in Table 2.

Table 2: Reliability Test Plan for New Weibull-Pareto Distribution for $\beta = 2,\ \delta = 2$ and λ=1 .

$\alpha = 0.10$									
r \ n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.11476	0.09370	0.08115	0.07258	0.06625	0.06134	0.05738	0.05410	0.05132
2	0.19609	0.15586	0.13331	0.11839	0.10758	0.09928	0.09264	0.08718	0.08258
3	0.23679	0.18620	0.15854	0.14043	0.12740	0.11743	0.10949	0.10297	0.09749
4	0.26172	0.20460	0.17379	0.15374	0.13935	0.12837	0.11964	0.11248	0.10647
5	0.27885	0.21720	0.18421	0.16282	0.14750	0.13584	0.12656	0.11897	0.11259
6	0.29151	0.22647	0.19188	0.16951	0.15351	0.14133	0.13166	0.12374	0.11710
7	0.30133	0.23366	0.19782	0.17469	0.15816	0.14559	0.13561	0.12744	0.12051
8	0.30922	0.23943	0.20259	0.17885	0.16189	0.14901	0.13818	0.13041	0.12339
9	0.31573	0.24420	0.20653	0.18228	0.16498	0.15183	0.14140	0.13286	0.12571
10	0.32122	0.24821	0.20985	0.18517	0.16758	0.15421	0.14361	0.13493	0.12766
$\alpha = 0.05$									
r \ n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.08007	0.06538	0.05662	0.05064	0.04623	0.04280	0.04003	0.03774	0.03581
2	0.16024	0.12739	0.10897	0.09678	0.08794	0.08115	0.07573	0.07126	0.06751
3	0.20386	0.16036	0.13655	0.12096	0.10973	0.10115	0.09431	0.08869	0.08398
4	0.23147	0.18102	0.15377	0.13604	0.12331	0.11357	0.10587	0.09953	0.09421
5	0.25080	0.19542	0.16576	0.14652	0.13274	0.12224	0.11390	0.10706	0.10132
6	0.26525	0.20616	0.17469	0.15433	0.13976	0.12868	0.11988	0.11266	0.10662
7	0.27657	0.21455	0.18166	0.16043	0.14525	0.13371	0.12455	0.11704	0.11075
8	0.28573	0.22134	0.18730	0.16536	0.14969	0.13777	0.12832	0.12058	0.11409
9	0.29333	0.22697	0.19198	0.16945	0.15337	0.14115	0.13145	0.12351	0.11686
10	0.29978	0.23174	0.19594	0.17291	0.15648	0.14400	0.13410	0.12600	0.11921
$\alpha = 0.01$									

r \ n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.03545	0.02894	0.02506	0.02242	0.02046	0.01895	0.01772	0.01671	0.01585
2	0.10358	0.08236	0.07046	0.06257	0.05686	0.05247	0.04896	0.04608	0.04365
3	0.14878	0.11708	0.09971	0.08833	0.08013	0.07386	0.06881	0.06477	0.06132
4	0.17953	0.14048	0.11935	0.10559	0.09571	0.08818	0.08218	0.07726	0.07313
5	0.20190	0.15741	0.13354	0.11805	0.10695	0.09849	0.09177	0.08626	0.08164
6	0.21904	0.17035	0.14437	0.12755	0.11552	0.10636	0.09909	0.09313	0.08813
7	0.23269	0.18063	0.15297	0.13510	0.12232	0.11260	0.10489	0.09857	0.09327
8	0.24388	0.18905	0.16001	0.14127	0.12789	0.11771	0.10964	0.10302	0.09748
9	0.25327	0.19610	0.16590	0.14644	0.13255	0.12199	0.11361	0.10675	0.10101
10	0.26129	0.20212	0.17093	0.15085	0.13652	0.12564	0.11700	0.10994	0.10401

As an example of this approach, let us think that we have to derive a life test sampling plan with an acceptance probability of 0.95 for lots with an acceptable mean life of 1000 hours and 10, 5 as sample size, termination number r respectively. From table 2, the entry against r=5 under column 2r is 0.25080. This implies that the termination time t = 250.80 hours. In this test plan, we select 10 items from the submitted lot and put to test. We reject the lot, when the 5th failure is occurred before 250.80 hours, otherwise we accept the lot. In either case terminating the experiment as soon as the 5th failure occurs or the termination time 250.80 hours is reached, or whichever is earlier.

III. COMPARATIVE STUDY

In this study, the proposed reliability test plan based on Transmuted New Weibull-Pareto distribution is compared with the existing reliability test plan based on New Weibull-Pareto distribution. From Table 3, we can be easily observed that the entries reveal that the terminating time of the proposed reliability test plan based on transmuted new weibull-pareto distribution is uniformly smaller than the corresponding time of the existing reliability test plan based on new weibull-pareto distribution and also it would be beneficial in terms of test time and cost.

Table 3: Comparison of proportion of termination time for proposed reliability test plan based on transmuted new weibull-pareto distribution and existing reliability test plan based on new weibull-paretodistribution $\beta = 2, \ \delta = 2$.

Proposed Transmuted New Weibull-Pareto Distribution for $\alpha = 0.05$									
r \ n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.08007	0.06538	0.05662	0.05064	0.04623	0.04280	0.04003	0.03774	0.03581
2	0.16024	0.12739	0.10897	0.09678	0.08794	0.08115	0.07573	0.07126	0.06751
3	0.20386	0.16036	0.13655	0.12096	0.10973	0.10115	0.09431	0.08869	0.08398
4	0.23147	0.18102	0.15377	0.13604	0.12331	0.11357	0.10587	0.09953	0.09421
5	0.25080	0.19542	0.16576	0.14652	0.13274	0.12224	0.11390	0.10706	0.10132
6	0.26525	0.20616	0.17469	0.15433	0.13976	0.12868	0.11988	0.11266	0.10662
7	0.27657	0.21455	0.18166	0.16043	0.14525	0.13371	0.12455	0.11704	0.11075
8	0.28573	0.22134	0.18730	0.16536	0.14969	0.13777	0.12832	0.12058	0.11409
9	0.29333	0.22697	0.19198	0.16945	0.15337	0.14115	0.13145	0.12351	0.11686
10	0.29978	0.23174	0.19594	0.17291	0.15648	0.14400	0.13410	0.12600	0.11921
Existing New Weibull-Pareto Distribution for $\alpha = 0.05$									
r \ n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.11324	0.09246	0.08007	0.07162	0.06538	0.06053	0.05662	0.05338	0.05064
2	0.22662	0.18016	0.15411	0.13686	0.12436	0.11477	0.10710	0.10078	0.09547
3	0.28831	0.22678	0.19311	0.17106	0.15519	0.14305	0.13337	0.12543	0.11876
4	0.32735	0.25601	0.21747	0.19239	0.17438	0.16065	0.14972	0.14076	0.13324
5	0.35468	0.27637	0.23442	0.20721	0.18772	0.17288	0.16108	0.15141	0.14329
6	0.37512	0.29155	0.24704	0.21825	0.19766	0.18198	0.16953	0.15933	0.15078
7	0.39112	0.30342	0.25691	0.22688	0.20542	0.18909	0.17614	0.16552	0.15663
8	0.40408	0.31302	0.26489	0.23385	0.21169	0.19484	0.18147	0.17053	0.16135
9	0.41484	0.32098	0.27151	0.23964	0.21689	0.19961	0.18590	0.17468	0.16527
10	0.42935	0.32773	0.27711	0.24453	0.22130	0.20365	0.18965	0.17819	0.16859

IV. CONCLUSION

In this paper a reliability test plan is developed when the lifetimes of the items follow the Transmuted New Weibull-Pareto distribution. Minimum sample size required to accept or reject a submitted lot for a given acceptance number with producer's risk were obtained. Values of termination time for the given values of sample size were provided. On comparing the two lifetime distributions, we can find that the proposed transmuted new weibull-pareto distribution would result in many savings in the experimental time and cost.

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