Some New Results On Helm Graphs

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Abstract: In a graph G two vertices of a graph are said to be radial to each other if the distance between them is equal to the radius of the graph. The radial graph of a graph G, denoted by R(G), has the vertex set as in G and then two vertices are adjacent in R(G) if and only if they are radial to each other in G.

The radial graph of helm graphs are obtained. The geodetic polynomials and detour geodetic polynomials of helm graphs are derived and some important results are proved.

Keywords: Distance, Detour geodetic polynomial, Geodetic polynomial, Helm graph, Radial graph.

AMS Classification : 05C12, 05C60, 05C75

I. Introduction

In this paper we discuss only finite simple and connected graph. For basic graph theoretical terminology we refer [1]. In [5] the concept of radial graph R(G) is introduced and the characterization for R(G) is proved . The concept of Geodetic polynomials of a graph using Geodetic sets of a graph are introduced in [8]. Geodetic polynomial, Detour geodetic polynomial of some radial graphs are discussed in [7]. Here we have derived some results, on radial graph of helm graphs and geodetic polynomial, detour geodetic polynomial of helm graphs .

1.1. Preliminaries

For a graph G, the distance d(u,v) between a pair of vertices u and v is the length of a shortest path joining them. The eccentricity e(u) of a vertex u is the distance to a vertex farthest from u. The radius r(G) of G is defined as the minimum eccentricity of all the vertices of G and the diameter d(G) of G is defined as the maximum eccentricity of all the vertices of G.

A graph G for which r(G) = d(G) is called a self centred graph. Two vertices of a graph are said to be radial to each other if the distance between them is equal to the radius of the graph. The radial graph of a graph G, denoted by R(G), has the vertex set as in G and then two vertices are adjacent in R(G) if and only if they are radial to each other in G.

II. Radial graph of Helm Graph

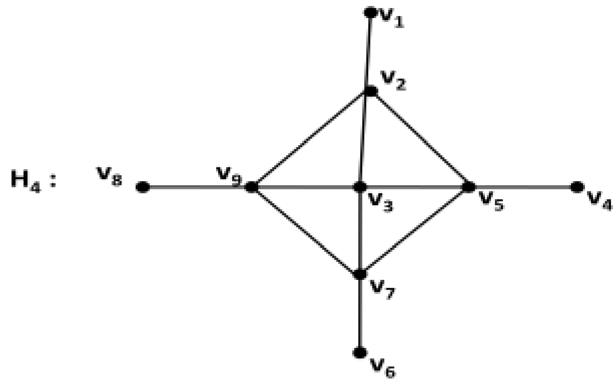
In this section we discuss radial graph of helm graphs and proved some theorems for finding the radial graphs of helm graphs.

Definition 2.1

The Helm graph H_n is the graph with 2n+1 vertices obtained from an n-wheel graph by adjoining a pendant edge at each node of the cycle.

Example 2.2

The following is the example for helm graph H_4 with 9 vertices.



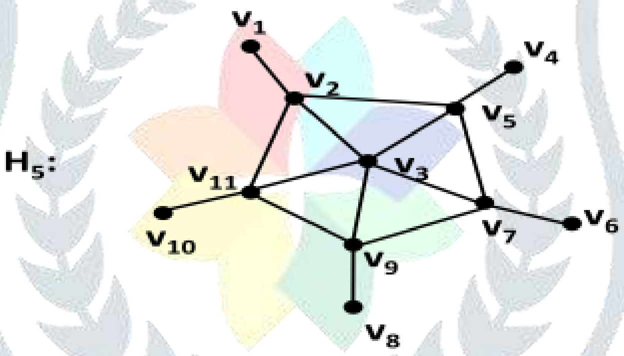
Theorem 2.3

The radial graph of helm graph with 2n+1 vertices $n \geq 4$ has n vertices of degree 3, n vertices has degree 4 and one vertex has degree n.

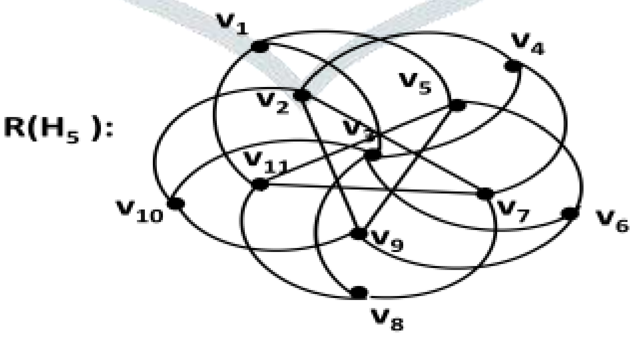
Proof:

Let us prove the theorem by induction on the number of vertices.

Let n=5 then H_5 is a helm graph with 11 vertices and it will be of the form,

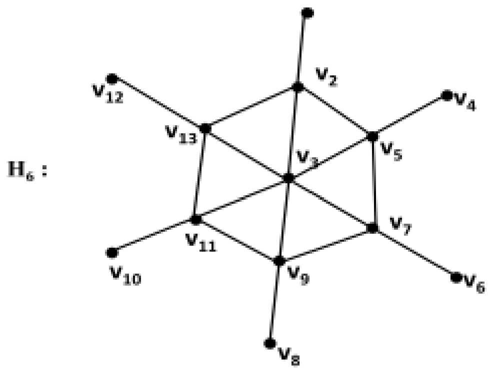


The radial graph of the helm graph H_5 is

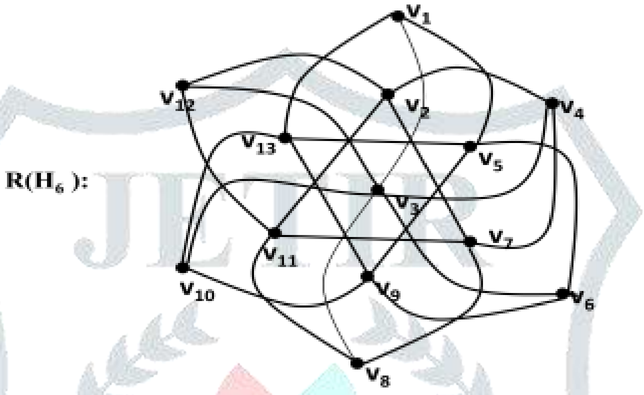


From the radial graph of helm graph we observe that the vertices v_1, v_4, v_6,v_8 and v_{10} has degree 3. The vertices v_2,v_5,v_7,v_9 and v_{11} has degree 4.The cut vertex v_3 has degree 5.

If n = 6, Let H_6 is a helm graph with 13 vertices and it is of the form,



The radial graph of the above helm graph is,



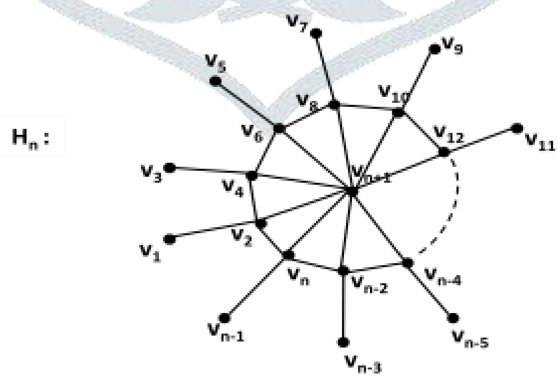
Here the vertices v_1,v_4,v_6,v_8,v_{10} and v_{12} is of degree 3, the vertices v_2,v_5,v_7,v_9,v_{11} and v_{13} has degree 4. The centre vertex v_3 has degree n.

The theorem is true for n = 5 and n = 6.

Let us assume that the theorem is true for all the helm graph H_{n-1} with 2n-1 vertices (i. e) the radial graph of helm graph H_{n-1} has n-1 vertices of degree 3 and n-1 vertices of degree 4 . one vertex has degree n.

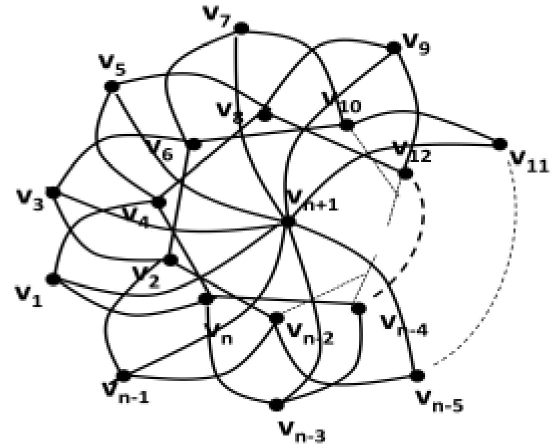
Now we prove the theorem for helm graphs H_n with 2n+1 vertices.

Let H_n is the helm graph with 2n+1 vertices



The radial graph of the above helm graph is

R(H_n):



From the above radial graph of helm graph H_n , we observe that the degree of all inner vertices $v_2, v_4, v_6, v_8, v_{10}, v_{12}, \ldots, v_{n-4}, v_{n-2}, v_n$ is greater than the degree of the corner vertices $v_1, v_3, v_5, v_7, v_9, v_{11}, \ldots, v_{n-5}, v_{n-3}, v_{n-1}$ .In the radial graph of the helm graph ,all the corner vertices has 3, all the inner vertices except the centre vertex has degree 3. The centre vertex has degree n.

Hence the radial graph of the helm graph has n vertices of degree 3, and n vertices of degree 4. One vertex has degree n.

III. Geodetic Polynomial of Helm Graphs

In this section we discuss geodetic polynomial of helm graphs

Definiton 3.1

Let $\mathcal{G}$ (G,i) be the family of geodetic sets of the graph G with cardinality i and let

$g_e\ (G,i) = \ |\mathcal{G}\ (G,i)|$. Then the geodetic polynomial $\mathcal{G}\ (G,x)$ of G is defined as

$\mathcal{G}\ (G,x) = \sum_{i=\ g(G)}^{|V(G)|}\ g_e\ (G,i)\ x^i$ where $g(G)$ is the geodetic number of G.

Theorem 3.2

The geodetic polynomial of H_n, if $n \geq 3$ is $\mathcal{G}$ (H_n ,x) $= x^n(1+x)^{1+n}$.

Proof:

Let H_n be a helm graph with 2n+1 vertices, without loss of generality we choose $n \geq 3$.

Let X={ $v_1, v_2, v_3, \ldots \ldots \ldots . v_n$ }. Helm graph has n pendant vertices. The only geodetic set with minimum cardinality is n in X. Therefore $g_e\ (H_n, n) = 1$.The geodetic set with cardinality n+1 are { $v_1, v_2, v_3, \ldots \ldots \ldots . v_{n+1}$ }.

$g_e\ (H, n+1) = \ (n+1)C\ _1.$

$g_e\ (H, n+2) = \ (n+2)C\ _2.$

Hence

$\mathcal{G}\ (H_n, x) \ = x^n + \ (n+1)C_1\, x^{n+1} + (n+1)C_2 x^{n+2} + \ldots\ldots\ldots\ldots\ldots + (n+1)\ C_{n+1}\, x^{2n+1}$

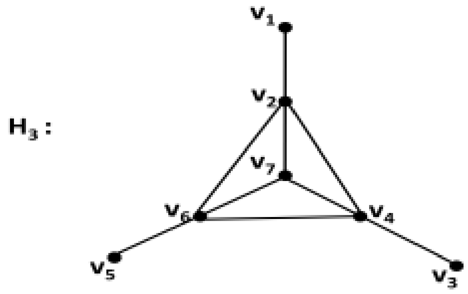
$= x^n\{\ 1 + \ (n+1)C_1\, x^1 + (n+1)C_2 x^2 + \ldots\ldots\ldots\ldots\ldots + (n+1)\ C_{n+1}\, x^{n+1}\ \}$

$\mathcal{G}$ (H_n ,x) $= x^n(1+x)^{1+n}$.

Hence The geodetic polynomial of H_n, if $n \geq 3$ is $\mathcal{G}$ (H_n ,x) $= x^n(1+x)^{1+n}$.

Example 3.3

Let H_3 is the Helm graph with 7 vertices**.**



$g_e\ (H_3, 3) = \{\ (v_1\ v_5\ v_7)\ \}$

$|g_e\ (H_3, 3)| = 1$

$g_e\ (H_3, 4) = \{\ (v_1\ v_2\ v_5\ v_7), (v_1\ v_3\ v_5\ v_7), (v_1\ v_4\ v_5\ v_7), (v_1\ v_5\ v_6\ v_7)\ \}$

$|g_e\ (H_3, 4)| = 4$

$g_e\ (H_3, 5) = \{(\ v_1\ v_2\ v_3\ v_5\ v_7\ ), (\ v_1\ v_2\ v_4\ v_5\ v_7\ ), (\ v_1\ v_2\ v_6\ v_5\ v_7\ ), (\ v_1\ v_3\ v_4\ v_5\ v_7\ ),$

$(\ v_1\ v_3\ v_6\ v_5\ v_7)\ , (\ v_1\ v_4\ v_6\ v_5\ v_7)\ \}$

$|g_e\ (H_3, 5)| = 6$

$g_e\ (H_3, 6) = \{\ (\ v_1\ v_2\ v_3\ v_4\ v_5\ v_7), (\ v_1\ v_2\ v_3\ v_5\ v_6\ v_7\ ), (\ v_1\ v_2\ v_4\ v_6\ v_5\ v_7), (\ v_1\ v_3\ v_4\ v_5\ v_6\ v_7)\ \}$

$|g_e\ (H_3, 6)| = 4$

$g_e\ (H_3, 7) = \{\ (v_1\ v_2\ v_3\ v_4\ v_5\ v_6\ v_7)\ \}$

$|g_e\ (H_3, 7)| = 1$

$\mathcal{G}\ (H_3, x) = \sum_{i=\ g(G)}^{|V(G)|}\ g_e\ (H_3, i)\ x^i$

$\mathcal{G}\ (H_3, x) = g_e\ (H_3, 3)\ x^3\ + g_e\ (H_3, 4)\ x^4\ + g_e\ (H_3, 5)\ x^5 + g_e\ (H_3, 6)\ x^6\ + g_e\ (H_3, 7)\ x^7$

$\mathcal{G}\ (H_3, x) =\ x^4 + 4x^5 + 6x^6 + x^7$.

The geodetic polynomial of H_3 is $\mathcal{G}\ (H_3, x) = x^4 + 4x^5 + 6x^6 + x^7$.

IV. Detour geodetic polynomial of Helm Graphs

In this section we find detour geodetic polynomial of helm graphs

Defintion 4.1:

Let D $\mathcal{G}$ (G,i) be the family of detour Geodetic sets of the graph G with cardinality i and let $Dg_e\ (G, i) = |D\mathcal{G}\ (G, i)|$. Then the Detour geodetic polynomial $D\mathcal{G}\ (G, x)$ of G is defined as $D\mathcal{G}\ (G, x) = \sum_{i=\ \mathrm{d}g(G)}^{\mathrm{dg+(G)}}\ Dg_e\ (G, i)\ x^i$ Where $d_g(G)$ is the Detour number of G.

Theorem 4.2

The Detour geodetic polynomial of the helm graph H_n is $nx^2 + nx^3$

i.e D $\mathcal{G}$ (H_n ,x) $= nx^2 + nx^3$, $n \geq 4$

Proof:

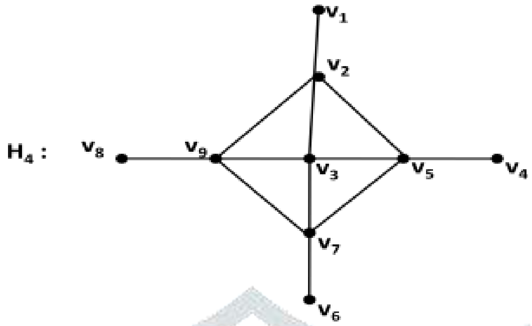
The Helm graph H_n has 2n+1 vertices, in which there is n pendant vertices and one centre vertex. There is n detour set with cardinality 2, and n detour set with cardinality 3. $d_g(G) = 2$, $dg^+(G) = 3$.

Hence the detour geodetic polynomial of the helm graph is $nx^2 + nx^3$,n ≥ 4.

Hence the proof.

Example 4.3

Let H_4 is the Helm graph with 9 vertices.



$DS_1 = \{ v_1\ v_4\}$ $DS_2 = \{ v_4\ v_6\}$ $DS_3 = \{ v_6\ v_8\}$, $DS_4 = \{ v_8\ v_1\}$,

$DS_5 = \{ v_1\ v_9\ v_4\}$, $DS_6 = \{ v_4\ v_2\ v_6\}$, $DS_7 = \{ v_6\ v_5\ v_8\}$, $DS_8 = \{ v_1\ v_7\ v_8\}$,

$d_g(G) = 2$, $dg^+(G) = 3$

$Dg_e\ (H_4, 2) = [\ \{ v_1\ v_4\},\{ v_4\ v_6\}\ , \{ v_6\ v_8\},\{ v_8\ v_1\}\ ]$

$|Dg_e\ (H_4, 2)| = 4$

$Dg_e\ (H_4, 3) = [\{ v_1\ v_6\ v_5\},\ \{ v_5\ v_6\ v_9\},\{ v_9\ v_2\ v_7\},\ \{ v_7\ v_4\ v_1\}\ ]$

$|Dg_e\ (H_4, 3)| = 4$

Detour geodetic polynomial of Helm Graph H_4 is

$D\,\mathcal{G}\ (H_4, x) = \sum_{i=2}^{3} Dg_e\ (H_4, i)\ x^i$

$D\,\mathcal{G}\ (H_4, x) = 4x^2 + 4x^3$.

The detour geodetic polynomial of ladder graph H_4 is

$D\,\mathcal{G}\ (H_4, x) = 4x^2 + 4x^3$.

Conclusion

Here Radial graph of Helm graph and geodetic polynomial, detour geodetic polynomial of helm graph have been studied. Further we can find the detour geodetic polynomial of other graphs.

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